Combined effect of Fermi liquid and spin-orbit interactions on electron transport

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Abstract

We present the gradient-invariant kinetic equations for a system of correlated electrons with spin-orbit interaction which generalize the Landau - Silin equations. In the Landau – Silin theory, in the absence of an external magnetic field, the parameters of the Fermi-liquid interaction enter to the kinetic equations only in gradient terms, which disappear in the case when high-frequency electromagnetic field is uniform. We show that when the spin-orbit interaction is taken into account, the kinetic coefficients explicitly depend on the constants of the Fermi-liquid interaction even in a uniform alternating electromagnetic field.

Introduction

The spin-orbit interaction (SOI) forms a connection between orbital and spin dynamics of conduction electrons and spin direction becomes dependent on electron momentum. This leads to phenomena which are absent in system where SOI can be neglected. At present, a significant number of works devoted to electron and spin transport in SOI systems are based on the Boltzmann kinetic equation for the diagonal in spin polarization s components of the density matrix. However, if the SOI energy is comparable of the Fermi energy, then the diagonal and off-diagonal in s components can give the same order of magnitude contribution to the kinetic coefficients. Taking into account the Fermi liquid interaction can lead to new physical effects, even in the case of weak SOI. A proven method for describing systems of interacting electrons in usual metals is the Landau-Silin Fermi liquid theory. In the Landau-Silin theory SOI is neglected and in the absence of an external magnetic field, the Landau function enters into the kinetic equations only in the gradient terms, which vanish when high-frequency electromagnetic field is uniform. In this case the kinetic coefficients do not depend explicitly on the parameters of Fermi liquid interaction. We present the gradient-invariant kinetic equations for the single-particle density matrix of a system of correlated electrons with SOI, which generalize the Landau-Silin equations, and show that when SOI is taken into account, the conductivity of 2D interacting electrons system includes constants of the Fermi liquid interaction even in a uniform alternating electromagnetic field. This is a result of the contribution to the conductivity of the density matrix components that are off-diagonal in spin polarization quantum number. Other kinetic coefficients have the same property.

Kinetic equations

Let us write the one-particle electron Hamiltonian $\hat{e}$ with spin-orbit coupling placed in a uniform static magnetic field $\mathbf{B}_0$ as a sum of spin-independent $\hat{e}^{(0)}$ and spin-dependent $\hat{e}^{(s)}$ parts.

$$\hat{e}(\hat{\mathbf{p}}) = \hat{e}^{(0)}(\hat{\mathbf{p}}) + \hat{e}^{(s)}(\hat{\mathbf{p}})$$

Here $\hat{\mathbf{p}} = -i\hbar\partial/\partial\mathbf{r} - (e/c)\mathbf{A}_0(\mathbf{r})$ is kinematic momentum operator, $\mathbf{A}_0(\mathbf{r}) = (\mathbf{B}_0 \times \mathbf{r})/2$ is vector potential of a uniform magnetic field $\mathbf{B}_0$. The operator $\hat{e}^{(s)}(\hat{\mathbf{p}})$ may include the operators of SOI energy $\hat{e}^{(so)}(\hat{\mathbf{p}})$, whose form depends on the specific crystal structure, and the Zeeman energy $\hat{e}^{(m)} = -\hat{\boldsymbol{\mu}}\mathbf{B}_0 \circ \hat{\boldsymbol{\sigma}}\mathbf{h}$, where $\hat{\boldsymbol{\mu}} = -(g m_B/2)\hat{\boldsymbol{\sigma}} \circ - m\hat{\boldsymbol{\sigma}}$ is the electron magnetic moment operator, m_B is the Bohr magneton, g is an effective g-factor, $\hat{\boldsymbol{\sigma}} = (\hat{s}_x, \hat{s}_y, \hat{s}_z)$ are the Pauli matrices, e is the electron charge, c is the light velocity.

We will assume that the operators $\hat{e}$ and $\hat{e}^{(s)}$ commute, then they have a common system of eigenfunctions. The first-rank spinors $y(\mathbf{r}, s, n)$, satisfying the equation

$$\hat{e}y(\mathbf{r}, s, n) = e_s(n) y(\mathbf{r}, s, n)$$

can be chosen so that they simultaneously satisfy the equation

$$\frac{\hat{e}^{(s)} y(\mathbf{r}, s, n)}{\left| e_s^{(s)}(n) \right|} = s y(\mathbf{r}, s, n) \, .$$

Here $e_s^{(s)}(n)$ is eigenvalue of operator $\hat{e}^{(s)}$, $e_s(n)$ is the energy of a conduction electron in an individual state with a set of quantum numbers $s = \pm 1$ and n determining the spin state of the electron and its spatial motion, respectively. Since the spin projection operators do not commute with the Hamiltonian, the projection of the spin onto an arbitrary direction is not conserved and is not a quantum number describing the state of the electron. The spin polarization of an electron is described by the quantum number s. The quantity $s/2$ can be interpreted as the projection of the spin onto the direction of some effective magnetic field $\mathbf{B}^{å}$ which is the sum of the external field $\mathbf{B}_0$ and the virtual field B caused by SOI, with components depending on the occupation numbers n. For example, in a 2D electron gas based on semiconductor heterostructures with Rashba spin-orbit coupling in the absent of external magnetic field, the virtual field is $\mathsf{B} = g(\mathbf{n} ´ \, \mathbf{p})/m$, here g is the Rashba SOI constant, $\mathbf{n}$ is normal to the conduction plane, $\mathbf{p}$ is electron momentum.

Kinetic equation for the single-particle density matrix of a system of electrons in a weak alternating electromagnetic field with a vector potential $\mathbf{A}^{\sim}(\mathbf{r}, t)$ and scalar potential $j(\mathbf{r}, t)$ in the representation of occupation numbers $l = \{s, n\}$, take form, [D.I. Stepanenko, EPL, **151** 46005 (2025)]

$$\frac{\partial f_{\lambda\lambda'}}{\partial t} + \frac{i}{\hbar}\sum_{\lambda_1}\left\{ f_{\lambda\lambda_1}\left(\varepsilon^{(r)}_{\lambda_1\lambda'} + \varepsilon^{\sim}_{\lambda_1\lambda'}(\mathbf{A}',\varphi')\right) - f_{\lambda_1\lambda'}\left(\varepsilon^{(r)}_{\lambda\lambda_1} + \varepsilon^{\sim}_{\lambda\lambda_1}(\mathbf{A}',\varphi')\right)\right\} = I_{\lambda\lambda'}(\hat f) + I^{*}_{\lambda'\lambda}(\hat f). \qquad (1)$$

$$I_{\lambda\lambda'}(\hat f) = \frac{\pi}{2\hbar}\sum_{\lambda_1\lambda_2}\left\{\hat V^{(im)}_{\lambda\lambda_2}\hat V^{(im)}_{\lambda_1\lambda'} f_{\lambda_1\lambda_2}\delta_-(\varepsilon_\lambda - \varepsilon_{\lambda_2}) + \hat V^{(im)}_{\lambda'\lambda_1}\hat V^{(im)}_{\lambda_2\lambda} f_{\lambda_2\lambda_1}\delta_-(\varepsilon_{\lambda'} - \varepsilon_{\lambda_1}) - \left(\hat V^{(im)}_{\lambda_2\lambda_1}\hat V^{(im)}_{\lambda_1\lambda'} f_{\lambda_2\lambda} + \hat V^{(im)}_{\lambda\lambda_1}\hat V^{(im)}_{\lambda_1\lambda_2} f_{\lambda'\lambda_2}\right)\delta_-(\varepsilon_{\lambda_1} - \varepsilon_{\lambda_2})\right\}$$

Here, symbol * denotes complex conjugation,

$$\varepsilon^{(r)}_{\lambda\lambda_1} = \varepsilon_\lambda\delta_{\lambda\lambda_1} + \sum_{\lambda_2\lambda_2'}\left\langle\lambda_2,\lambda_1\middle|\hat V^{(ee)}\middle|\lambda_2',\lambda\right\rangle f^{\sim}_{\lambda_2\lambda_2'} \qquad (2)$$

is the electron energy renormalized by Fermi-liquid interaction, $\hat V^{(im)}(\mathbf{r})$ and $\hat V^{(ee)}(\mathbf{r}_1,\mathbf{r}_2)$ the interaction operators of an electron with an impurity and between electrons, $f^{\sim}_{\lambda_2\lambda_2'} = f_{\lambda_2\lambda_2'} - f_0(\varepsilon_{\lambda_2})\delta_{\lambda_2\lambda_2'}$ is the deviation of the single-particle density matrix from the equilibrium value.

$$\hat\varepsilon^{\sim}_{\lambda\lambda_1}(\mathbf{A}',\varphi') \equiv \left\langle\lambda\middle|\hat\varepsilon^{\sim}(\mathbf{A}',\varphi')\middle|\lambda_1\right\rangle = -(1/c)\int d\mathbf{r}'\mathbf{A}'(\mathbf{r}',t)\hat{\mathbf{j}}_{\lambda\lambda_1}(\mathbf{r},\mathbf{r}') + \int d\mathbf{r}'\varphi'(\mathbf{r}',t)\hat\rho^{(e)}_{\lambda\lambda_1}(\mathbf{r},\mathbf{r}')$$

is the matrix element of the operator $\hat\varepsilon^{\sim}(\mathbf{A}',\varphi')$ of interaction of an electron with an alternating electromagnetic field,

$$\hat{\mathbf{j}}(\mathbf{r},\mathbf{r}') = (e/2)\left\{\hat{\mathbf{v}}(\hat{\mathbf{p}})\delta(\mathbf{r}-\mathbf{r}') + \delta(\mathbf{r}-\mathbf{r}')\hat{\mathbf{v}}(\hat{\mathbf{p}})\right\} + c\,\mathrm{rot}\,\hat{\boldsymbol{\mu}}_0\delta(\mathbf{r}-\mathbf{r}'),\quad \rho^{(e)}(\mathbf{r},\mathbf{r}') = e\delta(\mathbf{r}-\mathbf{r}')$$

are the operators of current density and electric charge, $\hat{\mathbf{v}}(\hat{\mathbf{p}}) = \partial\hat\varepsilon(\hat{\mathbf{p}})/\partial\hat{\mathbf{p}}$ is the operator of electron velocity.

With the help of the gauge transformation in the Hilbert space:

$$\hat U = \exp\left\{\frac{ie}{c\hbar}\int d\mathbf{r}\,\Phi(\mathbf{r},t)\hat\psi^{+}(\mathbf{r},s)\hat\psi(\mathbf{r},s)\right\},\quad \Phi(\mathbf{r},t) = \int_0^1 dx\,\mathbf{r}\mathbf{A}^{\sim}(x\mathbf{r},t),$$

under which the field operators $\hat\psi^+(\mathbf{r},\sigma),\hat\psi(\mathbf{r},\sigma)$ and the Hamiltonian $\hat H(\mathbf{A}^{\sim},\varphi)$ undergo transformations

$$\hat U\hat\psi(\mathbf{r},\sigma)\hat U^+ = e^{-i\frac{e}{c\hbar}\Phi(\mathbf{r},t)}\hat\psi(\mathbf{r},\sigma),\quad \hat U\hat\psi^+(\mathbf{r},\sigma)\hat U^+ = e^{i\frac{e}{c\hbar}\Phi(\mathbf{r},t)}\hat\psi^+(\mathbf{r},\sigma),\ \hat U^+\hat H(\mathbf{A}^{\sim},\varphi)\hat U = -i\hbar\frac{\partial\hat U^+}{\partial t}\hat U + \hat H(\mathbf{A}',\varphi')$$

and kinetic equation is reduced to a gradient-invariant form, i.e. the kinetic equation for the single-particle density matrix includes only the electromagnetic field, and not the potentials $\mathbf{A}^{\sim}(\mathbf{r},t)$ and $\varphi(\mathbf{r},t)$. Here

$$\mathbf{A}'(\mathbf{r},t) = \mathbf{A}^{\sim}(\mathbf{r},t) - \frac{\partial\Phi(\mathbf{r},t)}{\partial\mathbf{r}} = \int_0^1 dx\, x\left(\mathbf{B}^{\sim}(x\mathbf{r},t)\times\mathbf{r}\right),$$

$$\varphi'(\mathbf{r},t) = \varphi(\mathbf{r},t) + \frac{1}{c}\frac{\partial\Phi(\mathbf{r},t)}{\partial t} = -\int_0^1 dx\,\mathbf{r}\mathbf{E}(x\mathbf{r},t),\ \ \mathbf{E}(\mathbf{r},t) = -\frac{\partial\varphi(\mathbf{r},t)}{\partial\mathbf{r}} - \frac{1}{c}\frac{\partial\mathbf{A}^{\sim}(\mathbf{r},t)}{\partial t}\ ,\ \ \mathbf{B}^{\sim}(\mathbf{r},t) = \mathrm{rot}\,\mathbf{A}^{\sim}(\mathbf{r},t).$$

The kernel of the integral operator in the right-hand side of Eq. (2) is the scattering amplitude in the Born approximation up to constant factor. To generalize Eqs. (1), to the case of strong interaction, the Born amplitude can be replaced by the total scattering amplitude $F(\lambda_2',\lambda_1'\,|\,\lambda_2,\lambda_1)$. Considering $|\lambda_2,\lambda_1\rangle$ and $|\lambda_2',\lambda_1'\rangle$ as initial and final states of a system of two electrons, we have $F(\lambda_2',\lambda_1'\,|\,\lambda_2,\lambda_1) = const\langle\lambda_2',\lambda_1'\,|\hat T|\,\lambda_2,\lambda_1\rangle$, where the matrix elements of the operator $\hat T$ are represented as an infinite series of perturbation theory in powers of $\hat V^{(ee)}$. The exact expression for $F(\lambda_2',\lambda_1'\,|\,\lambda_2,\lambda_1)$ is unknown. Therefore, to obtain practical results, it is necessary to use a model expression for $\hat T$ whose form depends on the specific features of electron system. It may be represented as a linear combination of the operators of the Coulomb, exchange, spin-orbit, and spin-spin interactions of electrons with coefficients that are certain functions of the coordinates. The phenomenological parameters appearing in the scattering amplitude as a result of integrating these functions should be determined from a comparison with the experimental data.

In the case where the influence of the external magnetic field $\mathbf{B}_0$ can be considered as a perturbation and the quantization of the electron energy can be ignored, the state of an electron is described by spin polarization σ and momentum $\mathbf{p}$. If in addition the characteristic dimensions of spatial inhomogeneities greatly exceed both the interaction radius and the de Broglie wavelength of an electron, and the SOI is negligibly small, then Eqs. (1), (2) are reduced to the Landau-Silin equations after the transition to the Wigner representation and expansion in gradients.

High-frequency conductivity

We will use the model expressions for the 2D electron Hamiltonian with Rashba SOI in conducting plane xy

$$\hat{\varepsilon}(\hat{\mathbf{p}}) = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \gamma(\hat{\sigma}_x \hat{p}_y - \hat{\sigma}_y \hat{p}_x)\,, \tag{3}$$

here m is an effective electron mass, γ is the Rashba SOI constant. The wave function and energy spectrum of an electron have the form

$$\psi_s(\mathbf{r},\mathbf{p}) \equiv |s,\mathbf{p}\rangle = \frac{1}{\sqrt{L_x L_y}} e^{i\frac{\mathbf{pr}}{\hbar}} \frac{1}{\sqrt{2}} \begin{pmatrix} s e^{i\theta(\mathbf{p})} \\ 1 \end{pmatrix} \equiv \frac{1}{\sqrt{L_x L_y}} e^{i\frac{\mathbf{pr}}{\hbar}}\,, \quad \varepsilon_{s\mathbf{p}} = \frac{p^2}{2m} + s\gamma p\,,$$

where $e^{i\theta(\mathbf{p})} = (p_y + ip_x)/p, p = \sqrt{p_y^2 + p_x^2}, p_x = p\sin\theta, p_y = p\cos\theta$, $s = \pm 1$ and L_x, L_y are linear dimensions of the conductor in the x,y directions.

Let us assume that the electromagnetic wave propagates along the normal to the conducting plane xy. Then the density matrix $f_{s\mathbf{p},s'\mathbf{p}'} = f_{ss'}(\mathbf{p})\delta_{\mathbf{pp}'}$ and the matrix of the electron velocity operator $\langle s\mathbf{p}|\hat{\mathbf{v}}|s'\mathbf{p}'\rangle = \langle s|\mathbf{v}(\mathbf{p})|s'\rangle\delta_{\mathbf{pp}'}$ are diagonal in momentum.

From Eq. (1) follows the kinetic equation for the linear in alternating electric field $\mathbf{E}(\mathbf{r},t) = \mathbf{E}e^{-i\omega t + ik_z z}$ correction to the single-particle density matrix $f^{\sim}_{ss'}(\mathbf{p})$

$$\frac{\partial \tilde f_{\sigma\sigma'}(\mathbf{p})}{\partial t}+\frac{i}{\hbar}(\varepsilon_{\sigma'\mathbf{p}}-\varepsilon_{\sigma\mathbf{p}})\tilde f_{\sigma\sigma'}(\mathbf{p})+\frac{i}{\hbar}\left(f_0(\varepsilon_{\sigma\mathbf{p}})-f_0(\varepsilon_{\sigma'\mathbf{p}})\right)\sum_{\sigma_1,\sigma_1',\mathbf{p}_1}\langle\sigma_1\mathbf{p}_1,\sigma\mathbf{p}|\hat V^{(ee)}|\sigma_1'\mathbf{p}_1,\sigma'\mathbf{p}\rangle \tilde f_{\sigma_1\sigma_1'}(\mathbf{p}_1)+$$

$$e\langle\sigma|\mathbf{v}(\mathbf{p})|\sigma'\rangle\mathbf{E}\frac{f_0(\varepsilon_{\sigma\mathbf{p}})-f_0(\varepsilon_{\sigma'\mathbf{p}})}{\varepsilon_{\sigma\mathbf{p}}-\varepsilon_{\sigma'\mathbf{p}}}=I(\tilde f_{\sigma\sigma'}(\mathbf{p}))\,. \tag{4}$$

We will use the relaxation time τ approximation for the collision integral $I(\hat{\tilde f}_{\sigma\sigma'}(\mathbf{p}))=-\tilde f_{\sigma\sigma'}(\mathbf{p})/\tau$. Neglecting exchange effects and relativistic corrections in the interaction operator between electrons, we set $\hat V^{(ee)}=V(|\mathbf{r}-\mathbf{r}_1|)$. The matrix element $\langle\sigma_1\mathbf{p}_1,\sigma\mathbf{p}|\hat V^{(ee)}|\sigma_1'\mathbf{p}_1,\sigma'\mathbf{p}\rangle$ is equal to

$$\frac{1}{4L_xL_y}V(|\mathbf{p}-\mathbf{p}_1|)(1+\sigma'\sigma_1'e^{i(\theta-\theta_1)})(1+\sigma\sigma_1e^{-i(\theta-\theta_1)}), \tag{5}$$

where $\theta_1=\theta(\mathbf{p}_1), V(|\mathbf{q}|)=\int d^2\mathbf{r}V(|\mathbf{r}|)e^{i\mathbf{q}\mathbf{r}/\hbar}$.

Assume that the energy of the SOI $\varepsilon^{(so)}=\gamma p$ is small compared to the Fermi energy ε_F and the relation

$$f_0(\varepsilon_{\sigma'\mathbf{p}})-f_0(\varepsilon_{\sigma\mathbf{p}})=(\sigma'-\sigma)\gamma p\frac{\partial f_0(\varepsilon_{\mathbf{p}})}{\partial\varepsilon_{\mathbf{p}}}$$

is valid, here $\varepsilon_{\mathbf{p}}=p^2/(2m)$. The density matrix $\tilde f_{\sigma\sigma'}(\mathbf{p},t)=\tilde f_{\sigma\sigma'}(\mathbf{p},\omega)e^{-i\omega t}$ can be represented as the sum of the diagonal and off-diagonal parts in the quantum number σ

$$\tilde f_{\sigma\sigma'}(\mathbf{p},\omega)=-\frac{\partial f_0(\varepsilon_{\mathbf{p}})}{\partial\varepsilon_{\mathbf{p}}}\left(\varphi(\sigma)\delta_{\sigma\sigma'}+\phi\,\delta_{\sigma,1}\delta_{\sigma',-1}+\phi'\delta_{\sigma,-1}\delta_{\sigma',1}\right)\,, \tag{6}$$

the functions $\varphi(\sigma)$, f, f' depend on $\mathbf{p},\omega$, and $f'(\mathbf{p},\omega)=f^{*}(\mathbf{p},-\omega)$. At sufficiently low temperatures we may set $\partial f_0(\varepsilon_\mathbf{p})/\partial\varepsilon_\mathbf{p}=-\delta(\varepsilon_\mathbf{p}-\varepsilon_F)$. For the energy spectrum (3), the conductivity tensor is diagonal $\sigma_{xy}=0$ and $\sigma_{xx}=\sigma_{yy}=\sigma$. The complex combination $j_+=j_y+ij_x$ of the components of the electric current density is

$$j_+=e\,\mathrm{Tr}\left\{(\hat{v}_y+i\hat{v}_x)\hat{f}^{\sim}\right\}=\sigma(E_y+iE_x)\equiv\sigma E_+. \tag{7}$$

Substituting expression (6) into Eq. (7), we find

$$j_+=\frac{em}{2\pi\hbar^2}\left\{\sum_{\sigma}\left(\frac{p_F}{m}+\sigma\gamma\right)\varphi_{-1}(\sigma)+\gamma\left(f_{-1}-f'_{-1}\right)\right\}, \tag{8}$$

where $p_F=\sqrt{2m\varepsilon_F}$ and $\varphi_{-1}, f_{-1}, f'_{-1}$ are the Fourier coefficients of the functions φ, f, f',

$$f_{-1}=\frac{1}{2\pi}\int_0^{2\pi}d\theta\,e^{i\theta}f.$$

Let us expand the matrix element of the electron-electron interaction energy in a Fourier series

$$V(|\mathbf{p}-\mathbf{p}_1|)=\sum_{n=-\infty}^{\infty}V_n e^{in(\theta-\theta_1)},\quad V_n=V_{-n}. \tag{9}$$

Formula (9) is an analogue of the expansion of the Landau function in a series of Legendre polynomials in metals with a spherical Fermi surface. Substituting formulas (5), (6), (9) in Eq. (4) and moving from summation over $\mathbf{p}_1$ to integration, we find after simple calculations the system of algebraic equations for the coefficients $\varphi_{-1}(\sigma)$, f_{-1}, f'_{-1}

$$\tilde{\omega}\varphi_{-1}(\sigma) = i\frac{e}{2}\left(\frac{p_F}{m} + \sigma\gamma\right)E_+,$$

$$\left(\tilde{\omega} - \omega_\gamma\right)f_{-1} - \frac{\omega_\gamma}{4}\left\{U_{11}f_{-1} + U_{12}f'_{-1}\right\} = i\frac{e\gamma}{2}\left(1 - \frac{\omega_\gamma U_{22}}{2\tilde{\omega}}\right)E_+,$$

$$\left(\tilde{\omega} + \omega_\gamma\right)f'_{-1} + \frac{\omega_\gamma}{4}\left\{U_{11}f'_{-1} + U_{12}f_{-1}\right\} = -i\frac{e\gamma}{2}\left(1 + \frac{\omega_\gamma U_{22}}{2\tilde{\omega}}\right)E_+. \qquad (10)$$

Here $\tilde{\omega} = \omega + i/\tau$, $\omega_\gamma = 2\varepsilon_F^{(so)}/\hbar$, $\varepsilon_F^{(so)} = \gamma p_F$ is the SOI energy of an electron on the Fermi contour, $U_{11} = 2\nu_1 + \nu_2 + \nu_0$, $U_{12} = 2\nu_1 - \nu_2 - \nu_0$, $U_{22} = \nu_2 - \nu_0$, $\nu_n = mV_n/(2\pi\hbar^2), n = 0,1,2$. From Eqs. (10) and (8) we obtain the conductivity of 2D correlated electron system with SOI in a uniform alternating electromagnetic field

$$\sigma = \sigma_D + \sigma_N = i\frac{e^2 n_0}{m\tilde{\omega}}(1+\eta) + i\eta\frac{e^2 n_0}{m\tilde{\omega}}\frac{\tilde{\omega}^2 + \omega_\gamma^2\left(1+\nu_1\right)\left(\nu_0 - \nu_2\right)/2}{\tilde{\omega}^2 - \omega_\gamma^2\left(1+\nu_1\right)\left(1+(\nu_0+\nu_2)/2\right)}. \qquad (11)$$

Here $\eta = m\gamma^2/(2\varepsilon_F) \ll 1$, $n_0 = m\varepsilon_F/(\pi\hbar^2)$ is the electron density; at $\omega = 0$, $\gamma = 0$, we have $\sigma = \sigma_0 \equiv e^2 n_0 \tau/m$. The first term σ_D in formula (11), independent on parameters of the electron-electron interaction, is due to the components of density matrix that are diagonal in spin polarization quantum number. The second term σ_N caused by the off-diagonal in σ components, is a result of combined effect of spin-orbit and Fermi-liquid interactions. It includes the constants of Fermi liquid interaction and vanishes when the SOI is neglected. Other kinetic coefficients have analogous dependence on Fermi-liquid parameters. In the case when the energy of SOI is comparable to the Fermi energy, the SOI constant γ is of the order of Fermi velocity, and then diagonal and off-diagonal in σ components of the density matrix make the same order of magnitude contribution to the conductivity.

Let us consider the collisionless limit $w, w_g \ ? \ t^{-1}$. Neglecting the Fermi-liquid interaction ($u_n = 0$), the SOI correction to the conductivity s_N as a function of frequency, has a sharp maximum $s_N(w_g) \ ; \ hs_0$ and increases in wt times at the resonance frequency $w_g = (e_{1\mathbf{p}} - e_{-1\mathbf{p}})/\mathbf{h}$, which corresponds to transitions between electron energy levels with different spin polarization $s = \pm 1$ on the Fermi contour. If the parameter $x = (1+u_1)(1+(u_0+u_2)/2)$ is positive, s_N takes a resonance value

$$s_N(w_r) = hs_0(1+u_0)(1+u_1)(2x)^{-1}$$

at $w_r = w_g\sqrt{x}$. For the typical values of the SOI energy of an electron 0.1 meV, the resonance frequency $w_r \ ; \ 10^{11}$ Hz. The interaction between electrons leads to a renormalization of the resonance frequency w_r. In the high-frequency limit $w \ ? \ t^{-1}, w_g$, the dependence of the conductivity on the parameters of the Fermi liquid manifests itself in terms of high order in $(w_g/w)^2$. Under condition $e_F \ ? \ e^{(so)} \ ? \ \mathbf{h}t^{-1}$, frequency-dependent measuring the conductivity can provide information about the parameters of the Fermi-liquid interaction if the SOI constant is known from other experiments.

Conclusion

The kinetic equation (1) is applicable to study of combined effects of spin-orbit and electron-electron interactions in a wide class of electron systems including novel topological materials with nontrivial band structure, such as topological insulators, Dirac and Weyl semimetals, iridates and related materials containing 4d and 5d transition metal elements, heterojunctions, in which strong spin-orbit interaction leads to the appearance of non-trivial topology of electronic band structure and surface states. In contrast to usual metals, the single-particle Hamiltonian of conduction electron in these systems represents a matrix operator and, at calculating kinetic coefficients it is necessary to take into account the off-diagonal elements of the density matrix with respect to spin polarization or pseudospin quantum numbers.