Magneto-controlled static and dynamic properties of ferro-nematic liquid crystals

$\frac{d}{dt}\theta_0(t)=\alpha_1\theta_0(t)+\alpha_3\theta_0^3(t)$

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1. Features of magnetic field-controlled molecular orientation and optical properties

Magneto-nematic liquid crystals represent a class of composite soft materials formed by dispersing magnetic nanoparticles into nematic liquid crystal matrices. Such systems combine the long-range orientational order typical for nematic phases with magnetic sensitivity introduced by the embedded magnetic particles. As a result, magneto-nematic systems exhibit enhanced responses to external magnetic fields compared with conventional nematic liquid crystals. This property makes them promising materials for magnetically controlled photonic and optoelectronic devices. The optical properties of nematic liquid crystals are determined by the orientation of the director, which defines the anisotropic dielectric tensor of the medium. In magneto-nematic systems ,the magnetic nanoparticles interact with the nematic director through surface anchoring and elastic distortions of the liquid crystal matrix. When an external magnetic field is applied, the magnetic moments of the nanoparticles tend to align with the field. Due to the coupling between the nanoparticles and the director field, this alignment induces a reorientation of the nematic director, which leads to changes in the optical anisotropy of the medium.In the static regime, the reorientation of the director modifies the birefringence of the magneto-nematic layer and therefore changes the intensity of transmitted light in a typical polarizer–liquid crystal–analyzer configuration. The threshold conditions for magnetically induced reorientation depend on several parameters, including the elastic constants of the nematic phase, the magnetic susceptibility of the nanoparticles, the particle concentration, and the strength of the anchoring between nanoparticles and the liquid crystal molecules. In the dynamic regime, the director response to time-dependent magnetic fields becomes important. The relaxation and oscillatory dynamics of the director are governed by the balance between magnetic torques, elastic restoring forces, and viscous damping in the liquid crystal. Magnetic nanoparticles can significantly modify the characteristic response times due to additional magnetic torques and particle–director coupling mechanisms. As a consequence, magneto-nematic systems may exhibit enhanced sensitivity and faster response to magnetic stimuli compared with pure nematic liquid crystals..

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2. Model Hamiltonian and free energy density

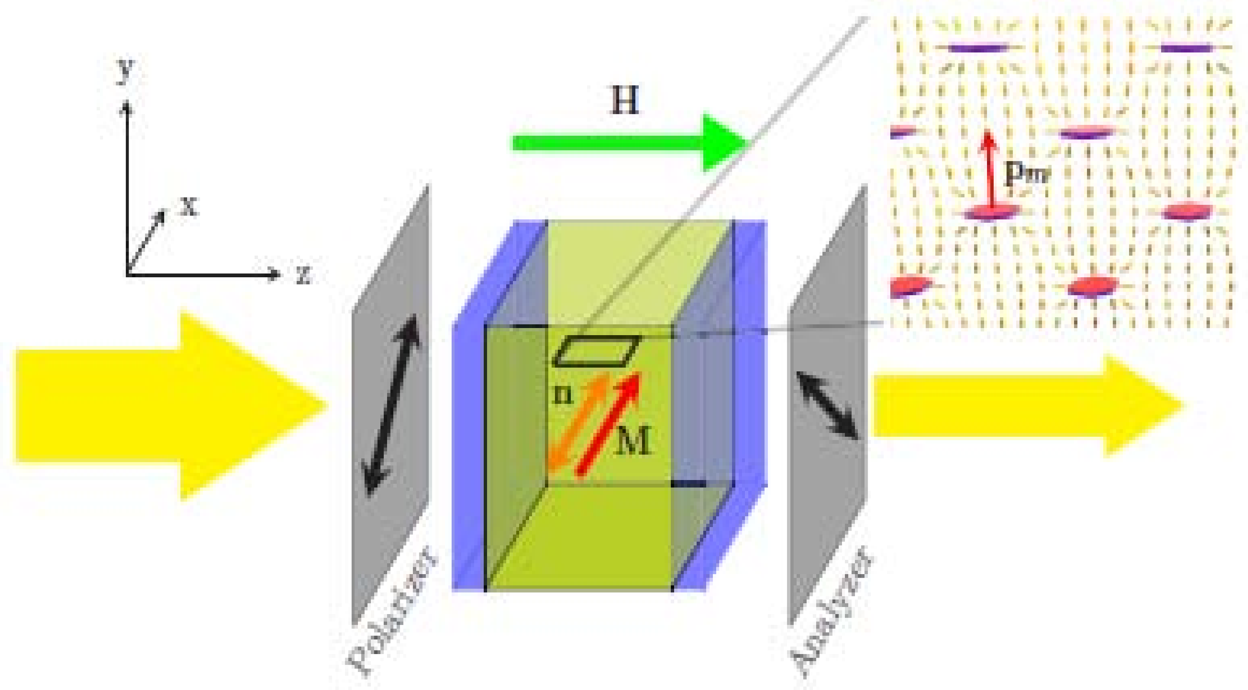

Fig. 1. Crystal optic model birefringer system between crossed polarizes. Monochromatic light is marked yellow.. $\boldsymbol{H}$ Is the magnetic field, $\boldsymbol{n}$ is the nematic director, $\boldsymbol{M}$ is the magnetization.

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The studied magneto-optical system consists of the liquid crystal between crossed polarizers (Fig. 1). The macroscopic free energy density f of the ferrzeonematic system is derived by statistical averaging of microscopic Hamiltonian $H = H_{N-N} + H_{N-m} + H_{M-B}$ within the molecular field approximation:

$$f = f_F - \frac{1}{2}\mu_0^{-1}\chi_a^{(n)}(\boldsymbol{n}\cdot\boldsymbol{B})^2 - M(\boldsymbol{m}\cdot\boldsymbol{B})$$

$$-\frac{1}{2}\mu_0^{-1}\chi_a^{(P)}(\boldsymbol{m}\cdot\boldsymbol{B})^2 + \frac{W}{2}(\boldsymbol{m}\cdot\boldsymbol{n})^2 - \frac{W_S}{2}\left(\boldsymbol{n}\cdot\boldsymbol{n}_s\right)^2, \quad (1)$$

where $f_F = \frac{1}{2}\left[K_1(\mathrm{div}\,\boldsymbol{n})^2 + K_2(\boldsymbol{n}\,\mathrm{rot}\,\boldsymbol{n})^2 + K_3(\boldsymbol{n}\times\mathrm{rot}\,\boldsymbol{n})^2\right]$ is the Frank elastic energy associated with nematic director distortions, $\boldsymbol{n}$ is the nematic director, $\boldsymbol{m}$ is the unite magnetization vector, and $\boldsymbol{B}$ is the external magnetic field. Statics behavior of the system is described by the Euler-Lagrange equation

$$\frac{\delta F}{\delta\theta} = \frac{\partial f}{\partial\theta} - \frac{\partial}{\partial z}\left(\frac{d f}{d\theta_Z}\right) = 0, \quad \frac{\delta F}{\delta\beta} = \frac{\partial f}{\partial\beta} - \frac{d}{dz}\left(\frac{\partial f}{\partial\beta_Z}\right) = 0. \qquad (2)$$

following from the condition of the minimum of the free energy *F*.

3. Static and dynamic behavior of the difference phase and output light

For the studied system, the nematic director $\boldsymbol{n}$, under uniaxial symmetry, is defined by the tilt angle θ as $\boldsymbol{n} = (\sin\theta, 0, \cos\theta)$. In accordance with equations (2), the static behavior of the system is described by the coupled system of equations:

$$\frac{\delta F}{\delta\theta} = 0 = -K_3\theta_{zz} - \frac{1}{2}\mu_0^{-1}\chi_a^{(n)}B^2\sin 2\theta + \frac{W}{2}\sin 2(\theta-\beta),$$

$$\frac{\delta F}{\delta\beta} = 0 = BM\sin\beta + \frac{1}{2}\mu_0^{-1}\chi_a^{(p)}B^2\sin 2\theta - \frac{1}{2}W\sin 2(\theta-\beta), \qquad (3)$$

for the nematic director and magnetization, and the Maxwell equation of the form

$$\nabla\times\nabla\times\boldsymbol{E}(\boldsymbol{r}) = k_0^2\boldsymbol{\varepsilon}(\boldsymbol{r})\cdot\boldsymbol{E}(\boldsymbol{r}), \ \nabla\cdot\left(\boldsymbol{\varepsilon}(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r})\right) = 0. \qquad (4)$$

for the amplitude of the monochromatic light wave $\boldsymbol{E}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}(\boldsymbol{r})\exp(i\boldsymbol{k}\cdot\boldsymbol{r}), \ \boldsymbol{k} = k_0\tilde{\boldsymbol{n}}$ propagating through the system. Coupling of (3) and (4) follows from the definition of the anisotropy dielectric tensor as, $\boldsymbol{\varepsilon} = \varepsilon_\perp + \varepsilon_a(\boldsymbol{n}\otimes\boldsymbol{n}), \ \varepsilon_a = \left(\varepsilon_\parallel - \varepsilon_\perp\right)$..The two solutions, of this system represent the binary splitting of the monochromatic wave with ordinary $\tilde{n}_o$ and extraordinary $n_{ex}(\theta(z))$refractive indices. .Accordingly, the phase difference of the output monochromatic wave$\Phi = k_0\int_0^d dz\left[\tilde{n}_{ex}(z) - \tilde{n}_o\right]$ as the corresponding light intensity $I = I_0\sin^2 2\alpha\cos^2\Phi/2$, are the functions of the external magnetic field $\boldsymbol{B}$ (see Fig.2).
The description of the dynamic is based on the principle of least action in which the Rayliegh dissipative force supplements the variation of the system Lagrangian.form
In the considered model system of the FNLC cell with the Rayleigh dissipation function is quadratic in the nematic director velocity, and this variational principle

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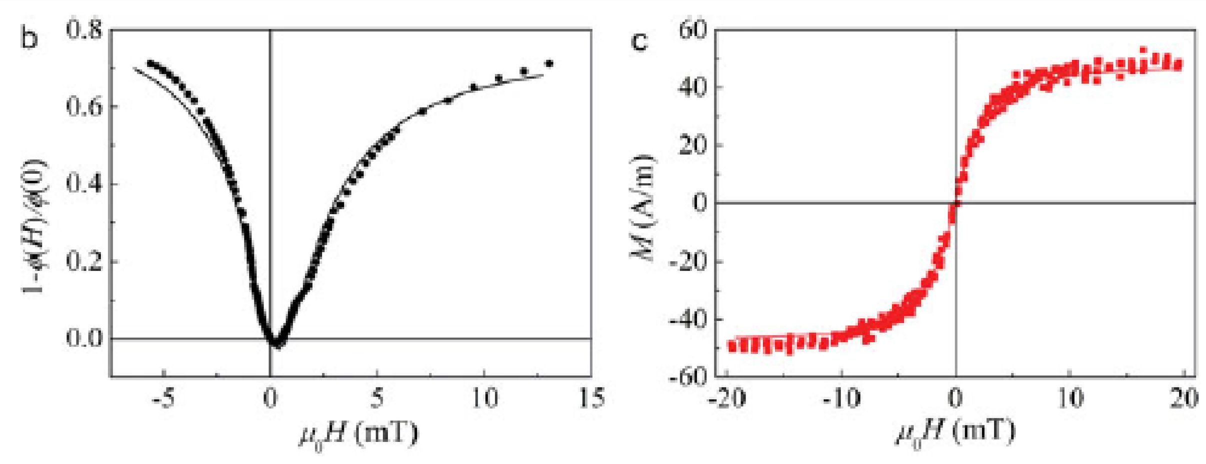

Fig. 2. (b) Normalized phase difference vs. external magnetic field *H*, *(c)* Magnetization curve.

yields the dynamic equation describing the dynamics including field-on and field-off transients.

$$\gamma_\theta\frac{\partial\theta(z,t)}{\partial t} = -\frac{\delta F}{\delta\theta(z,t)}, \quad \gamma_\beta\frac{\partial\beta(z,t)}{\partial t} = -\frac{\delta F}{\delta\beta(z,t)}, \qquad (5)$$

where γ_θ and γ_β are the relaxation coefficients associated with the nematic and magnetic subsystems, respectively. It is proposed that magnetic subsystem much faster than nematic subsystem. Then (5) is reduced to the cubic nonlinearity equation

$$\frac{d}{dt}\theta_0(t) = \alpha_1\theta_0(t) + \alpha_3\theta_0^3(t), \quad (6)$$

This equation describes the respond on magnetic field has the form of exponential growth of the tilt angle amplitude transition to a saturation state governed by cubic nonlinearity.