Peculiarities of phase diagram for dense superfluid neutron matter with spin-triplet anisotropic pairing in superstrong magnetic fields

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Within the framework of the generalized nonrelativistic Fermi-liquid approach, analytical expressions for the phase transition temperatures of neutron matter of supranuclear densities from normal to superfluid states with anisotropic spin-triplet *p*-wave pairing in steady and uniform extremely strong magnetic fields (from the range $10^{17}\div 2\cdot 10^{18}$ G) have been obtained. These phase transition temperatures depend nonlinearly on both the neutron matter density and the magnetic field. The expressions for them contain both terms with symmetric and linear field-dependent splitting (as it was previously derived for "moderately strong" fields $H \leq 10^{17}$ G), and new additional terms that depend nonlinearly on the superstrong magnetic field (from the interval 10^{17} G $\leq H \leq 2\cdot 10^{18}$ G), which leads to asymmetry and nonlinearity of the splitting of phase transition temperatures relative to the phase transition temperature at zero field. Phase transitions to superfluid states of this type can occur at nuclear and supranuclear densities in the liquid outer core of magnetars (strongly magnetized neutron stars).

1. Introduction

Previously, in our articles [1, 2], we studied analytically the effects of superstrong magnetic fields with $H = Z\cdot 10^{17}$ G (where $1 \leq Z$) on the magnetic properties and on the splitting of the gap in the energy spectrum of superfluid neutron matter (SNM) with spin-triplet anisotropic *p*-wave neutron pairing (similar to the spin-triplet pairing of ^{3}He atoms in anisotropic superfluid phases ^{3}He-A$_1$ and ^{3}He-A$_2$; see [3] and (1) in [4]) at supranuclear densities ($n > n_0$=0.17 fm^{-3}) in the limit of zero temperature. In [1, 2] was used the so-called generalized non-relativistic Fermi-liquid approach (see reviews [5, 6] and articles [7, 8]). Note also, that weak splitting (nonlinearly increasing under a "moderately strong" magnetic field from the range 10^{16} G $\leq H \leq 10^{17}$ G) of the energy gap in SNM was calculated numerically in [9] (for the unconventional BSk21 parametrization of the effective Skyrme forces) as nonlinear function of density n (at $n \leq n_0$) in the limiting case of zero temperature. By contrast, this report is devoted to theoretical study (analytically) the influence of a superstrong constant in time and spatially uniform magnetic field $H = Z\cdot 10^{17}$ G (with $1 \leq Z \leq 20$) on the phase transition temperatures from normal state of neutron matter (NM) to SNM at nuclear and supranuclear densities ($n \geq n_0$), at which the spin-triplet superfluidity with p - wave pairing still exists (see numerical estimate (58) in [1] and explanations there).

Let us consider here the case of temperatures close to the PT temperature T_c (n, H=0; E_c) $\equiv$

$T_{c0}(n; E_c)$ (with cutoff energy $E_c \ll \varepsilon_F(n)$, where $\varepsilon_F(n)$ is the Fermi energy of NM at $n \geq n_0$) from the NM to the SNM with spin-triplet anisotropic p - wave neutron pairing at nuclear and supranuclear densities. We have studied in [4, 10, 11] (at sub- and supranuclear densities) *linearly field dependent splitting* of PT temperatures in SNM with spin-triplet p - wave pairing of neutrons with unconventional (generalized) Skyrme forces BSk18 and BSk20 due to effect of spatially uniform "moderately strong" magnetic fields $H \equiv Z \cdot 10^{17}$ (G) with $0<Z\leq 1$. These unconventional forces are from the so - called BSk family, which contain three terms depending on the density, proposed by the Brussels - Montreal group; see, e.g., [12, 13, 14]. Note also, that previously we had already investigated [15, 16] linear splitting of PT temperatures in SNM with traditional (conventional) Skyrme forces (with only one term nonlinearly dependent on density n) in the presence of the uniform fields $H \equiv Z \cdot 10^{17}$ (G) with $0<Z\leq 1$.

As outlined in [12-14], unphysical spin (and spin-isospin) instabilities of homogeneous neutron (nuclear) matter, including the transition to a polarized state in neutron-star matter, are eliminated with the unconventional BSk forces (see also [17]). In contrast to BSk, for neutron matter with traditional Skyrme forces ferromagnetic phase transition results at nuclear and supranuclear densities for different conventional parameterizations (see, e.g., [18, 19]).

It is necessary to stress that those unconventional BSk Skyrme forces [13, 14] which lead to rather rigid equations of state (EOSs) of dense NM are consistent with the recently measured values (with sufficiently high accuracy as a result of astronomical observations; see [20-22] for details) for the masses of heavy pulsars: the pulsar PSR J1614-2230 has mass $M \approx (1.97 \pm 0.04) \cdot M_{Sun}$ [20], the mass of pulsar PSR J0348-0432 is $M \approx (2.01 \pm 0.04) \cdot M_{Sun}$ [21] and the mass of pulsar PSR J0740 + 6620 is $M \approx (2.08 \pm 0.07) \cdot M_{Sun}$ [22]. Really, let us note in this connection that maximum neutron-star mass, which is still stable against collapse to black hole, is $M_{max} \approx 2.28 \cdot M_{Sun}$ for models with BSk21 [13] and BSk24 [14] Skyrme forces (see Table I in [14]).

Here it should be also particularly emphasized that the spin - triplet pairing of fermions (neutrons) is not suppressed by action of strong magnetic fields in contrast to the case of the spin - singlet pairing in sufficiently strong magnetic fields (see, e.g., [23, 24]). Note, that spin - triplet pairing of neutrons in such huge fields $H = Z \cdot 10^{17}$ G (with $1 \leq Z \leq 20$) might exist inside the outer part of the liquid cores of magnetars (see, e.g., [23-32]), which are highly magnetized neutron stars.

2. Solutions of general nonlinear integral equation for the effective magnetic field in the SNM with unconventional Skyrme interaction and spin – triplet anisotropic *p* - wave pairing of neutrons in a superstrong magnetic field at temperatures in the vicinity of T_{c0}

We study here the SNM in superstrong magnetic fields $H = Z \cdot 10^{17}$ G (with $1 \leq Z \leq 20$) at temperatures close to the PT temperature $T_c(n, H=0; E_c) \equiv T_{c0}(n; E_c)$ from the NM to the SNM with spin-triplet anisotropic p-wave neutron pairing at nuclear and supranuclear densities. For so strong magnetic fields the following "strong" inequalities are valid [1]:

$$T \sim T_{c0} << \xi << \mu_0 = \varepsilon_F \, . \quad (1)$$

Here $\xi \equiv |\mu_n| \cdot H_{\text{eff}}$, $H_{\text{eff}}(n,H)$ is the effective magnetic field (which is renormalized in dense NM owing to the influence of Fermi − liquid effects; see more details in [1]) and $\mu_n \approx -1.91304 \cdot \mu_N \approx$ $\approx -0.603077 \cdot 10^{-17}$ MeV/G <0 is the neutron magnetic moment, $\mu_N = e\hbar/2m_pc$ is nuclear magneton [33]; $\mu_0(n) = \mu(n,T,H)|_{T=0,\ H=0} = \varepsilon_F(n)$ is the Fermi energy, which coincides with the chemical potential $\mu_0(n)$ of NM in the limit of zero temperature and in the absence of magnetic field. Due to inequalities (1) the dependence of μ on temperature T can be neglected and that is why we can use below the same approximate formula for the chemical potential of neutron matter in a superstrong magnetic field as in [1, 2]:

$$\mu(\xi) \approx \varepsilon_F \cdot \left[1 - \left(\frac{\xi}{2\varepsilon_F}\right)^2\right]. \tag{2}$$

Now the general nonlinear integral equation for the strong effective magnetic field in SNM with spin-triplet anisotropic p-wave pairing of neutrons can be represented as equation for the function $\xi(n,H)$ at temperatures $T \sim T_{c0}$ (see equation (2) in [4]):

$$\xi(T,p,H,n) = -\mu_n H + \left(r(n) + s(n) \cdot p^2\right) \cdot K_2\left(T,\xi\right) + s(n) \cdot K_4\left(T,\xi\right), \tag{3}$$

(compare this equation with (7) in [1], where the same SNM in strong magnetic field was considered in the limiting case of zero temperature). Here, $r(n)$ and $s(n)$ are functions depending on the neutron number density ($n \equiv y \cdot n_0$, n_0=0.17 fm^{-3}), which are expressed through the generalized Skyrme parameters as follows:

$$r(n) \equiv t_0' + \left(t_3'/6\right) \cdot n^{\alpha}, \quad s(n) \equiv \left(t_1'(n) - t_2'(n)\right) \Big/ \left(4\hbar^2\right), \tag{4}$$

where

$$t_0' = t_0 \cdot (1 - x_0), \quad t_3' = t_3 \cdot (1 - x_3), \tag{5}$$

$$t_1'(n) = t_1 \cdot (1 - x_1) + t_4 \cdot (1 - x_4) \cdot n^{\beta}, \tag{6}$$

$$t_2'(n) = t_2 \cdot (1 + x_2) + t_5 \cdot (1 + x_5) \cdot n^{\gamma}. \tag{7}$$

The dimentional Skyrme parameters t_0 , t_1 , t_2 , t_3 , t_4 , t_5 and dimensionless ones x_0 , x_1 , x_2 , x_3 , x_4, x_5 , α, β, γ take specific numerical values for each unconventional parameterization of Skyrme forces from the BSk family of parameterizations, which include, for example, BSk20, BSk21 (see in [13] and [4, 11]), BSk22, ..., BSk26 (see in [14]). In this section of the report, for the sake of greater generality, the parameterization of Skyrme forces is not specified.

The functionals $K_2(T,\xi)$ and $K_4(T,\xi)$ in equation (3) have the following form (compare with (12) and (13) in [1], where the limiting case T=0 was studied):

$$K_{\sigma}\left(T,\xi\right) = \frac{1}{8\pi^2\hbar^3} \int_{p_{\min}}^{p_{\max}} dq q^{\sigma} \int_0^1 dx \kappa\left(q,x;T,\xi\right), \quad (\sigma = 2, 4), \tag{8}$$

where

$$\kappa\left(q,x;T,\xi\right) = \frac{z(q,\xi) + \xi(q)}{E_+(q,x^2;T,\xi)} \cdot \tanh\left(\frac{E_+(q,x^2;T,\xi)}{2T}\right) - \frac{z(q,\xi) - \xi(q)}{E_-(q,x^2;T,\xi)} \cdot \tanh\left(\frac{E_-(q,x^2;T,\xi)}{2T}\right). \tag{9}$$

Here $p_{\max} = p_F \cdot \sqrt{1+a}$, $p_{\min} = p_F \cdot \sqrt{1-a}$, where $p_F(n)$ is the Fermi momentum, $a(n) = E_c/\varepsilon_F(n) \ll 1$

(at $n \geq n_0$) is the cutoff parameter; E_c is the constant cutoff energy, $\varepsilon_F(n) = p_F^2/2m^*(n)$, $m^*(n)$ is the effective neutron mass inside neutron matter, which depends on the neutron number density n in the SNM and on the unconventional Skyrme parameters $t_1'(n)$ and $t_2'(n)$ (see (6), (7)) according to the general formula (see, e.g., (10) in [4] and (3) in [2]):

$$\frac{m}{m^*(n)} = 1 + \frac{m\,n}{4\hbar^2}\left[t_1'(n) + 3t_2'(n)\right], \tag{10}$$

where $m=(m_p+m_n)/2 \approx 938.919$ MeV $/c^2$ is the average value of a free nucleon mass [33]. Formula (9) includes the energies $E_\pm(q,x^2;T,\xi)$ of quasiparticles (neutrons) in SNM with spin − triplet anisotropic p-wave pairing (of the ^{3}He−A type) in a magnetic field with two opposite values ±1 of the spin projection of a Cooper pair of neutrons along and against the magnetic field (similar to ^{3}He−A in a magnetic field, where for the total spin S=1 of the nuclei of ^{3}He atoms forming Cooper pairs, the projections are equal ±1, in contrast to the quasi-isotropic superfluid phase ^{3}He−B, where, in addition to ±1, there is also a projection of the spin of the pair, which is equal to zero):

$$E_\pm(q,x^2;T,\xi) = \sqrt{q^2\Delta^2_{\uparrow(\downarrow)}(T,\xi)\cdot(1-x^2) + (z(q;T,\xi) \pm \xi(q,H))^2}\,. \tag{11}$$

Here, taking into account (2) in the case of an extremely strong magnetic field, the function $z(q;T,\xi)$ has the following approximate form:

$$z(q;T,\xi) \approx z(q;T=0,\xi) = \frac{q^2}{2m^*} - \mu(\xi) \approx \varepsilon(q) - \varepsilon_F\cdot\left[1-\left(\frac{\xi}{2\varepsilon_F}\right)^2\right] \tag{12}$$

(note, that in (8) - (11) and below the dependence of functions on the density n is implied, but it is omitted in their arguments to shorten the notation of formulas). At temperatures close to the PT temperature $T_{c0}(n;\, E_c)$ the effect of functions $\Delta_{\uparrow\downarrow}(T,\xi)$ in (11) is negligible and that is why the following formulas for the energies $E_\pm(q,x^2;\, T,\xi)$ of neutrons are valid in the leading approximation with respect to this small parameter $\Delta_{\uparrow\downarrow}$:

$$E_\pm^{(0)}(q;\xi) \approx \left|z(q;\xi) \pm \xi(q,H)\right|, \tag{13}$$

that is, functions $E_\pm^{(0)}(q;\xi)$ don't depend on temperature and on the angular variable. As a consequence of this, the function $\kappa^{(0)}(q;T,\xi)$ from (9) in the leading approximation is reduced to the following form:

$$\kappa^{(0)}(q;T,\xi) = \tanh\left(\frac{z(q,\xi)+\xi(q)}{2T}\right) - \tanh\left(\frac{z(q,\xi)-\xi(q)}{2T}\right). \tag{14}$$

To solve the nonlinear integral equation (3), we can use perturbation theory in powers of a small parameter $h \equiv \frac{|\mu_n|\cdot H}{\varepsilon_F} << 1$. This parameter is small even for the case of superstrong magnetic fields H=Z·10^{17} G (with 1≤Z≤20) at nuclear and supranuclear densities n in SNM with unconventional BSk Skyrme forces [12, 13, 14]. **As a result,** we can transform $(\xi/2\varepsilon_F)^2$ (in (2) and (12)) to another form using (45) and (47) from [1], namely:

$$\left(\frac{\xi}{2\varepsilon_F}\right)^2 = \left(\frac{|\mu_n|\cdot H_{eff}(p_F,H)}{2\varepsilon_F}\right)^2 \approx \frac{h^2}{4\cdot[1-R(n)-2S(n)]^2} \ll 1\,, \tag{15}$$

where

$$R(n) \equiv \frac{\nu_F(n)}{2} \cdot r(n), \qquad S(n) \equiv \frac{\nu_F(n)}{2} \cdot s(n) \cdot p_F^2(n) \tag{16}$$

(see (4)-(7)), which exactly coincide with the definitions of these functions (13), (14) in [9]) and

$$\nu_F(n) = \frac{3n}{2\varepsilon_F(n)} = \frac{m^* \cdot p_F}{\pi^2 \hbar^3} \tag{17}$$

is the density of states on the Fermi surface in neutron matter with neutron number density n; m^* (n) is the effective mass of a neutron inside the SNM (see (10) here above).

Note that at temperatures T close to the PT temperature $T_{c0}(n;\, E_c)$ from the NM to the SNM and at densities $n \geq n_0$ the following inequalities are valid:

$$\frac{a}{2t} \equiv \frac{E_c}{2T} > \frac{E_c}{2T_{c0}} \gg 1. \tag{18}$$

Let us also note here, looking ahead, that if we choose the BSk21 [13] or BSk24 [14] as a non-traditional parametrization of Skyrme forces in NM and set the cutoff energy equal, e.g., to E_c= 10 MeV, then according to [34] (see Figs. 1-3 there) we have at densities $n \geq n_0$ that

$$t \equiv \frac{T}{\varepsilon_F} < t_{c0}(n;a) \equiv \frac{T_{c0}(n;a)}{\varepsilon_F(n)} < max\left(\frac{T_{c0}(n;a)}{\varepsilon_F(n)}\right) < 10^{-3} \tag{19}$$

(these inequalities are valid for both BSk21 and BSk24). In this case the temperature corrections (dimensionless) make a negligible contribution, which is $<10^{-6}$, and it is an excess of the permissible calculation accuracy here, since the initial parameters of the non-traditional Skyrme forces (see (5) – (7) above) are given in [12 – 14] with an accuracy of up to 6 significant digits.

3. Solutions of general integral equations for the phase transition temperatures in the SNM with unconventional Skyrme interaction and spin – triplet anisotropic *p* - wave pairing of neutrons in a superstrong magnetic field

We will not write out here in explicit form the initial integral equations (valid at arbitrary temperatures from the range of existence of spin-triplet superfluidity) for the components of the order parameter $\Delta_{\uparrow\downarrow}(T,\xi)$ in SNM with spin-triplet anisotropic p - wave pairing of neutrons, which we have already given many times before (see, e.g., equations (9) in [4], see also [10,11,17]). Note that, taking into account the approximate formula (13) for the functions $E_\pm^{(0)}(q;\xi)$, these equations are reduced to the following two equations for determining the sought temperatures of phase transitions in SNM in a superstrong magnetic field $H=Z\cdot 10^{17}$ G (with $1 \leq Z \leq 20$):

$$1 + \frac{c_3(n)}{12\pi^2\hbar^3} \int_{p_{\min}}^{p_{\max}} dq q^4 \frac{\tanh\left(\dfrac{\dfrac{q^2}{2m^*} - \mu(\xi) \pm \xi(q,H)}{2T_{c(\pm)}}\right)}{\dfrac{q^2}{2m^*} - \mu(\xi) \pm \xi(q,H)} = 0\,. \tag{20}$$

Here $c_3(n) \equiv t_2^{'}(n)/\hbar^2 < 0$ is the "interaction constant", which depends on the density n according to formula (7) (in the case of using unconventional BSk parametrizations of Skyrme forces) and leads to spin-triplet p - wave pairing of neutrons in the SNM. The upper and lower limits of integration in (20) have the same form $p_{\max} = p_F \cdot \sqrt{1+a}$ and $p_{\min} = p_F \cdot \sqrt{1-a}$, as in formula (8).

For the chemical potential $\mu(\xi)$ of neutron matter, we will use in (20) the approximate formula (2) with the correction quadratic on a strong magnetic field to the Fermi energy.

In the absence of a magnetic field, that is, in the limit at $h \to 0$, it is obvious that two equations (20) are reduced to a single equation, the solution of which is the reduced temperature $t_{c0}(n;a)$ (see (19)) of the phase transition from NM to SNM with spin-triplet anisotropic p-wave pairing of neutrons at H=0:

$$t_{c0}(n;a) \equiv \frac{T_{c0}(n;a)}{\varepsilon_F(n)} \simeq \frac{2\gamma}{\pi} \cdot a \cdot \exp\left(\frac{2}{c_3 \cdot m^* \cdot n} + \frac{3a^2}{16} + \frac{3a^4}{512} \right), \tag{21}$$

which exactly agree with the corresponding formula (22) in [34]. Here $\gamma = e^C \approx 1.781072418$ and C = 0.5772156649… is the Euler's constant.

Neglecting the small contribution of the terms $O(a^6)$ and $O(t_{c0}^2)$ and also taking into account (15), **we obtain as a result** (see more details in [35]) the **following approximate expressions for the** sought **temperatures** $T_{c(\pm)}(n,h;a)$ **of phase transitions in SNM with spin-triplet anisotropic** ***p*** **- wave pairing of neutrons in superstrong magnetic fields** ***H=Z*·10**^{17} **G (with 1≤Z≤20) at nuclear and supranuclear densities** $n \geq n_0$**:**

$$T_{c(\pm)}(n,h;a) \simeq T_{c0}(n;a) \cdot \left\{ 1 \pm \frac{h}{(1-R-2S) \cdot I_0} \cdot \left[\left[(I_0 - 2) \cdot \left(\frac{3}{2} + S \right) - \frac{5a^2}{8} \cdot \left(1 + \frac{3a^2}{80} \right) \right] + \right. \right.$$

$$\left. \left. + \frac{h^2}{(1-R-2S)^2 \cdot I_0} \cdot \left[(I_0 - 3) \cdot S \cdot (3+S) - \frac{1}{a^2} - \frac{3}{8} - \frac{5a^2}{4} \cdot \left(S - \frac{1}{32} \right) - \frac{a^4}{64} \cdot \left(3S - \frac{5}{16} \right) \right] \right] \right\}, \tag{22}$$

where

$$I_0(n;a) \simeq -\frac{4}{c_3 \cdot m^* \cdot n} + O(a^6) > 0, \tag{23}$$

and $c_3(n) \equiv t_2'(n)/\hbar^2 < 0$ (see after (20)) and for $m^*(n)$ see (10).

It follows from formula (22) that the terms linear in the magnetic field lead to splitting of PT temperatures $T_{c(\pm)}(n,h;a)$, which is symmetrical with respect to the temperature $T_{c0}(n; E_c)$ (see (21), where the cutoff parameter $a(n)=E_c/\varepsilon_F(n) \ll 1$ at $n \geq n_0$), and the quadratic correction in (22) with respect to the small parameter h (see after (14)) leads to an asymmetry and nonlinearity in the splitting of the PT temperatures in superstrong magnetic fields, in which the role of this nonlinear correction increases. **The formula for the asymmetry** ***U*(*n*,*h*;*a*) of the splitting of the PT temperatures** $T_{c(\pm)}(n,h;a)$ **has the following form:**

$$U(n,h;a) \equiv T_{c(+)}(n,h;a) + T_{c(-)}(n,h;a) - 2T_{c0}(n;a) \simeq$$

$$\simeq \frac{2h^2 \cdot T_{c0}(n;a)}{(1-R-2S)^2 \cdot I_0} \cdot \left[(I_0 - 3) \cdot S \cdot (3+S) - \frac{1}{a^2} - \frac{3}{8} - \frac{5a^2}{4} \cdot \left(S - \frac{1}{32} \right) - \frac{a^4}{64} \cdot \left(3S - \frac{5}{16} \right) \right]. \tag{24}$$

In order to more clearly investigate the nature of the dependence of functions $T_{c(\pm)}(n,h;E_c)$ and the asymmetry of their splitting $U(n,h;a)$ on density (at $n \geq n_0$), it is necessary to specify the non-traditional BSk parameterization of the Skyrme forces in superdense neutron matter. The next section of this report is devoted to these issues.

4. Splitting of the phase transition temperatures in SNM with unconventional parameterization BSk21 of Skyrme forces at nuclear and supranuclear densities and with spin-triplet anisotropic *p* - wave pairing of neutrons in superstrong magnetic fields

To find an explicit concrete dependence of functions $T_{c(\pm)}(n,h;E_c)$ of the form (22) on the density (at $n \geq n_0$), we choose the unconventional BSk21 parameterization [13] of Skyrme forces in superdense neutron matter, as in our previous papers [1, 2]. For the case of BSk21 parameterization, we will use the following explicit expression for the Fermi energy $\varepsilon_{F,\,BSk21}(y)$ of neutron matter (see (41) from [2]):

$$\varepsilon_{F,BSk21}(y) \approx y^{2/3} \cdot \left[1 + y \cdot \left(3.97930 \cdot y^{1/12} + 0.0422618 \cdot \sqrt{y} - 3.89571\right)\right] \cdot 60.90152\ (MeV). \quad (25)$$

Here $y \equiv n/n_0$ is the reduced density of neutron matter (n_0=0.17 fm^{-3}). See Fig. 1, where the graph of dependence $\varepsilon_{F,\,BSk21}(y)$ is plotted on the interval of change of the reduced density $1.0 \leq y \leq 2.0$.

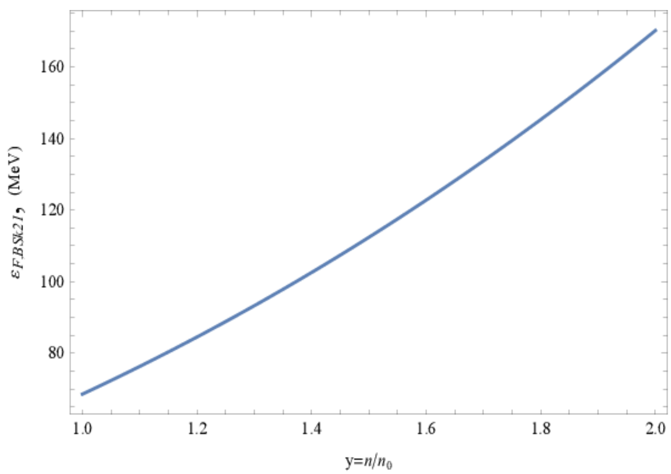

Fig. 1. Fermi energy $\varepsilon_{F,\,BSk21}(y)$ [see (25)] in NM with BSk21 Skyrme at supranuclear densities, $n>n_0$.

From formulas (16), (17) taking into account (4)-(7), the form of the dimensionless functions $R_{BSk21}(y)$ and $S_{BSk21}(y)$ follows, which are included in the general formulae (22) and (24):

$$R_{BSk21}(y) \approx -\frac{2.093544 \cdot y^{1/3} \cdot \left(0.454645 + 0.127577 \cdot y^{1/12}\right)}{1 + y \cdot \left(3.97930 \cdot y^{1/12} + 0.0422618 \cdot \sqrt{y} - 3.89571\right)} < 0, \quad (26)$$

$$S_{BSk21}(y) \approx \frac{1.537217 \cdot y \cdot \left(1.76082 + 0.0412311 \cdot \sqrt{y} - 1.29408 \cdot y^{1/12}\right)}{1 + y \cdot \left(3.97930 \cdot y^{1/12} + 0.0422618 \cdot \sqrt{y} - 3.89571\right)} > 0. \quad (27)$$

See the graphs (Fig. 2(a) and Fig. 2(b) respectively) of these functions for supranuclear NM densities in the reduced density range $1.0 \leq y \leq 2.0$.

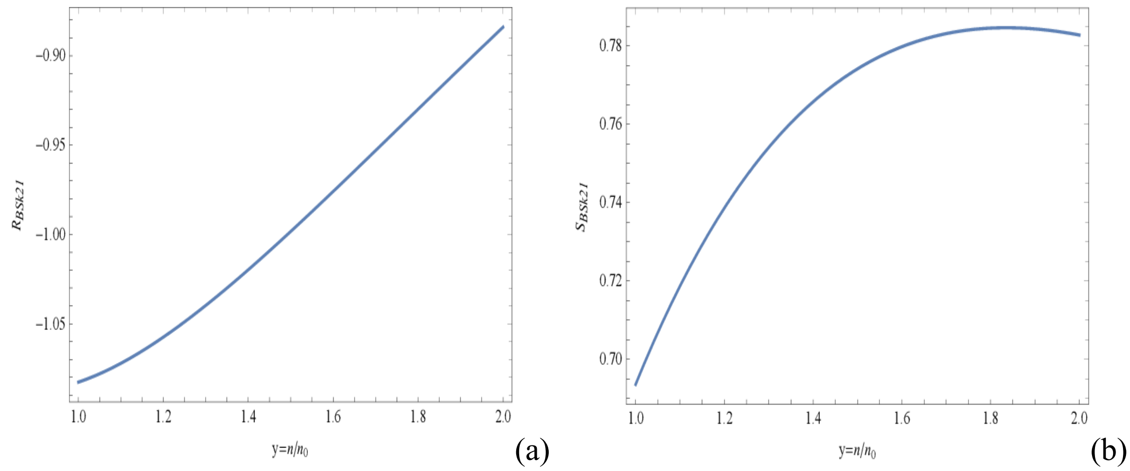

Fig. 2: (a) $R_{BSk21}(y)$ [see (26)] for NM with BSk21 Skyrme at supranuclear densities ($n>n_0$);
(b) $S_{BSk21}(y)$ [see (27)] for NM with BSk21 Skyrme at supranuclear densities.

Using (25)-(27), $I_0(n;\,a)$ from (23) with taking into account that (see (7) here and (57) from [1])

$$t^{'}_{2,\,BSk21}(y) \approx \left(-1390.38 + 1294.08 \cdot y^{1/12}\right)(MeV \cdot fm^5), \qquad (28)$$

and also the PT temperature $T_{c0}(n;\,E_c)$ (see (21), with the cutoff parameter $a(n)=E_c/\varepsilon_F(n)$), we plotted, based on formulas (22) and (24), the following graphs (**see Fig. 3(a) and Fig. 3(b)**) for $T_{c0}(n;\,E_c)$, for the sought functions $T_{c(\pm)}(n,h;E_c)$ and for the asymmetry $U_{BSk21}(n,h;E_c)$ of their splitting in SNM with BSk21 Skyrme forces at supranuclear densities in the range $1.0 \le y \equiv n/n_0 \le 2.0$ (with cutoff energy E_c=16 (MeV) < $\varepsilon_{F,\,BSk21}$(y)) in superstrong magnetic field $H = 2\cdot 10^{18}$ G.

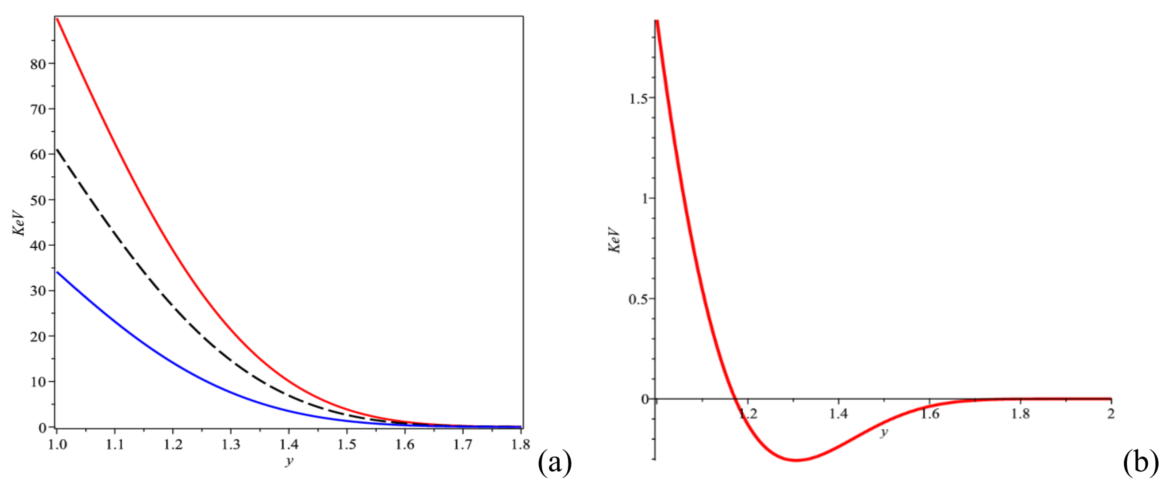

Fig. 3: (a) PT temperatures in SNM with BSk21 Skyrme (with cutoff energy E_c=16 MeV) at superstrong magnetic field $H=2\cdot 10^{18}$ G and at supranuclear densities ($y \equiv n/n_0>1$): $T_{c(+)}(n,H;E_c)$ – upper solid line and $T_{c(-)}(n,H;E_c)$ – bottom solid line [see (22)], $T_{c0}(n;E_c)$ – middle dashed line [(21)]**;**
(b) Nonmonotonic asymmetry (positive and negative) of splitting $U(n,H;E_c)$ [see (24)] of PT temperatures $T_{c(+)}(n,H;E_c)$ and $T_{c(-)}(n,H;E_c)$ in SNM with BSk21 Skyrme (with cutoff energy E_c=16 MeV) at superstrong magnetic field $H=2\cdot 10^{18}$ G and at $n>n_0$.

See also **Fig. 4(a)** for PT temperatures $T_{c0}(n;E_c)$ and $T_{c(\pm)}(n,H;E_c)$ and **Fig. 4(b)** for the asymmetry $U_{BSk21}(n,H;E_c)$ of splitting in SNM with BSk21 Skyrme forces at supranuclear densities (in the same range $1.0 \le y \equiv n/n_0 \le 2.0$ as in **Fig. 3(b)** but with another cutoff energy E_c=10 (MeV)) in strong magnetic field $H = 10^{18}$ G (at Z=10).

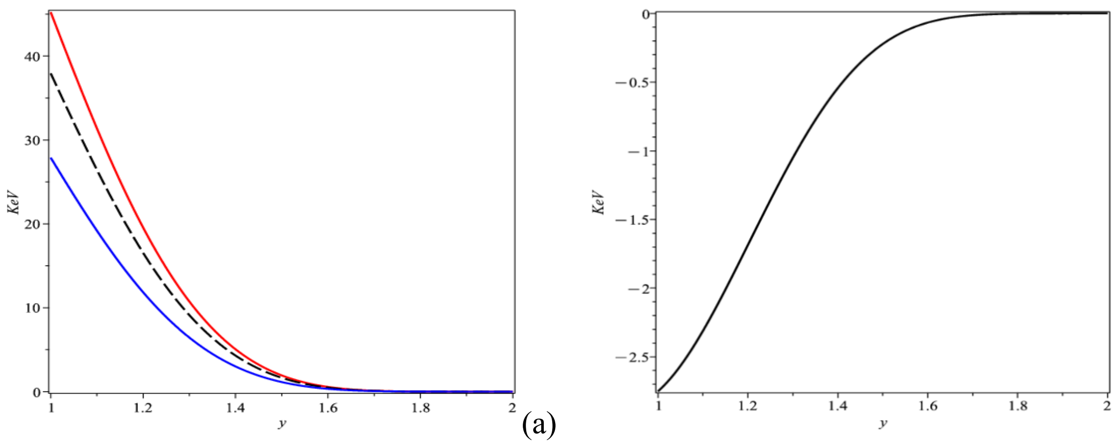

Figure 4: (a) PT temperatures in SNM with BSk21 Skyrme (with cutoff energy E_c=10 MeV) at strong magnetic field $H = 10^{18}$ G and at supranuclear densities ($y \equiv n/n_0 > 1$): $T_{c(+)}(n,H;E_c)$ – upper solid line, $T_{c(-)}(n,H;E_c)$ – bottom solid line [see (22)], $T_{c0}(n;E_c)$ – middle dashed line (21).
(b) Negative monotonic asymmetry of splitting $U_{BSk21}(n,H;E_c)$ [see (24)] of PT temperatures $T_{c(+)}(n,H;E_c)$ and $T_{c(-)}(n,H;E_c)$ in SNM with BSk21 Skyrme (with cutoff energy E_c=10 MeV) at strong magnetic field $H = 10^{18}$ G and at supranuclear densities.

5. Conclusions

Thus, as a result of calculations carried out here within the framework of the generalized Fermi-liquid approach (see [5-8]), it is shown that at temperatures close to the PT temperature $T_c(n,H{=}0;E_c) \equiv T_{c0}(n;E_c)$ from the NM to the SNM with spin-triplet anisotropic *p*-wave neutron pairing at nuclear and supranuclear densities ($n \ge n_0 = 0.17$ fm^{-3}) in ultra-strong magnetic fields $H = Z \cdot 10^{17}$ G (with $1 \le Z \le 20$), the temperatures of phase transitions $T_{c(\pm)}(n,H;E_c)$ in the SNM (see **(22)** and **Figures 3(a)** and **4(a)**) undergo splitting relative to the temperature $T_{c0}(n;E_c)$. This splitting of the PT temperatures (which is caused by the action of the magnetic field H on the SNM) depends quadratically on H in superstrong fields and asymmetrically (see (24) and **Figures 3(b)** and **4(b)**) with respect to $T_{c0}(n;E_c)$, and also depends nonlinearly on the density n of the SNM (when using non-traditional (generalized) parametrizations of the Skyrme forces from the BSk family [13, 14]). It should also be noted that the dependence of the asymmetry $U_{BSk21}(n,H;E_c)$ (see **(24)** and **Figures 3(b), 4(b)**) on the density is changed quantitatively and qualitatively (**nonmonotonic dependence on *n* at E_c=16 MeV** in **Fig. 3(b)** and **monotonic behavior at E_c=10 MeV** in **Fig. 4(b**) depending on the value of the cutoff energy used E_c.<< $\varepsilon_{F,BSk21}(y)$ (see **Fig. 1**). **General approximate analytical formulas (22) and (24)**, which are valid for an arbitrary BSk parameterization, **as well as Figures 3(a,b) and 4(a,b)** constructed on the basis of these formulas for a specific BSk21 parameterization of Skyrme forces in neutron matter, **are the main results of this report.** They generalize the analytical results obtained earlier in [4, 10] (on the linear and symmetric splitting of the PT temperatures in the case of SNM with BSk Skyrme forces [12, 13] in “moderately strong” magnetic fields $H=Z\cdot 10^{17}$ G with $0<Z\leq 1$). See also [9], where the results of numerical calculations were obtained on the weak manifestation of nonlinearity and asymmetry of the splitting of phase transition temperatures $T_{c(\pm)}(n,H;E_c)$ in SNM in fields of 10^{16} G $\leq H\leq 10^{17}$ G (i.e. at $0.1\leq Z\leq 1$) at nuclear and subnuclear density values ($n\leq n_0$). It should also be noted here that splitting and asymmetry with a nonlinear dependence on superstrong magnetic fields $H=Z\cdot 10^{17}$ G (at $1\leq Z\leq 20$) and on the density $n>n_0$ take place not only for PT temperatures, but also for the energy gap in superfluid neutron matter with anisotropic spin-triplet *p*-wave pairing in the limit of zero temperature [2]. And finally, **the obtained results of this report can be applied to the description of spin-triplet superfluidity in superdense neutron matter in the outer layer of the cores of highly magnetized neutron stars, called “magnetars”.**

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