Phenomenological model of the influence of Majorana states in the vortex structure of a nonconventional superconductor film on the microwave absorption features

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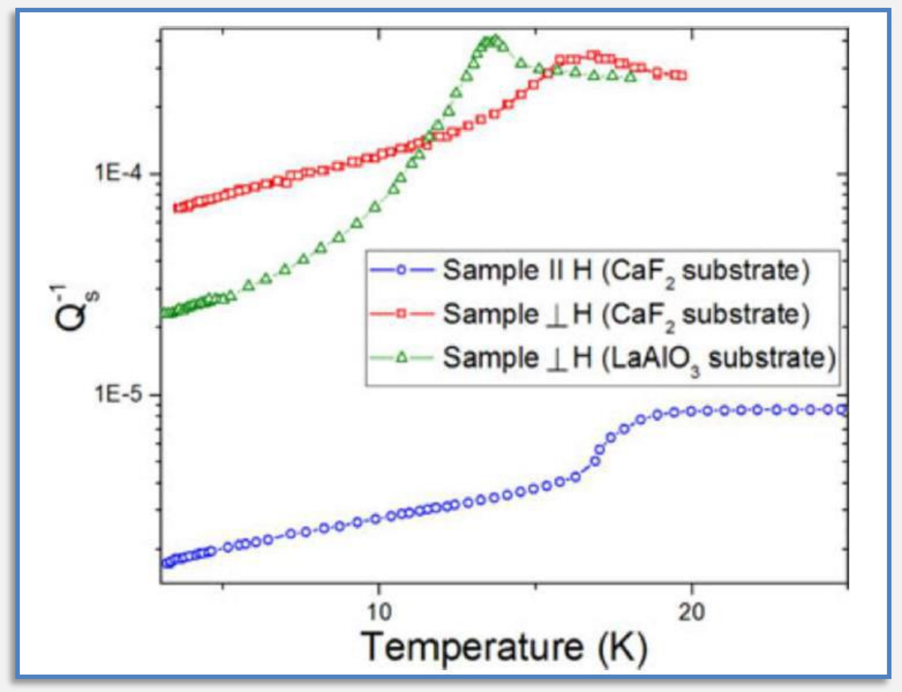
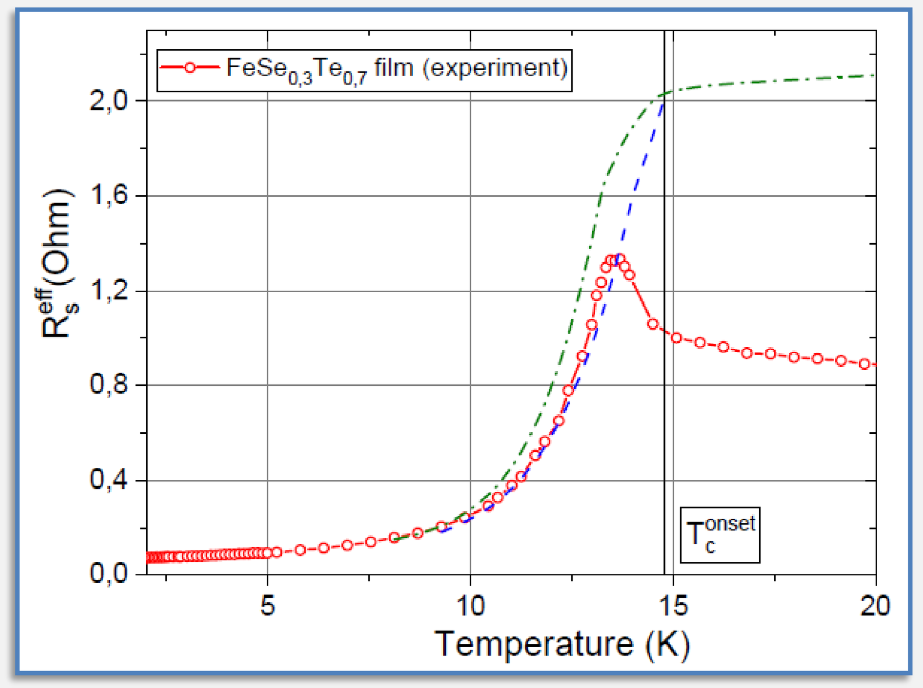
In [1], the effect of anomalous temperature dependence of losses in a thin-film HTSC disk (FeSe1-xTex) was discovered when it was placed in a perpendicular microwave field near the critical temperature. At the same time, in similar experiments with other types of HTSC or parallel orientation of the magnetic field relative to the disk, such an effect was absent. In this regard, a hypothesis was put forward that this effect is related to the emergence of 0-th Majorana modes, previously discovered by other methods in this type of HTSC [2]. The aim of this work was to develop a phenomenological model of this effect based on general quantum mechanical ideas about the properties of entangled vortices that form a Majorana pair.



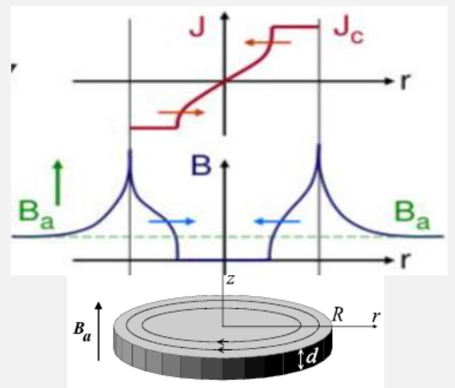
Temperature dependence of the inverse Q-factor of the resonator (decrement) with FeSe1-xTex film for the ⊥ orientation (x = 0.5 and 0.7) and for the ||- orientation (x = 0.5).

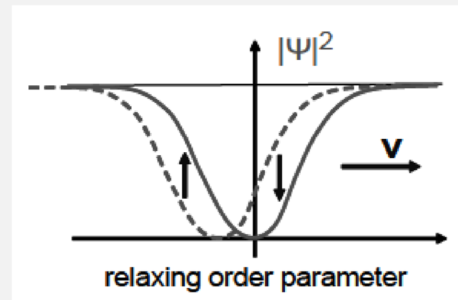


Temperature dependence of the effective surface resistance of the FeSe1-xTex film with ⊥ orientation; 0 - $R_{S\perp}^{eff}(T)$, dashed line - $R_{S\perp 0}^{eff}(T)$, dash-dotted line - $\tilde{R}_{S\perp}^{eff}(T)$

We have previously shown that the mechanism of energy dissipation in a HTSC thin disk under the action of a perpendicular MW field can be described as a consequence of the reorientation of vortices within the Bean model [3].

A moving vortex core means that at a given position the order parameter $|\psi|^2$ decreases and increases again. If we assume a delay in the recovery of $|\psi|^2$ for the relaxation time $\tau \approx \hbar \cdot \Delta \approx (300\ GHz)^{(-1)}$, we obtain additional order dissipation. At ultrahigh frequencies, the estimates give a very small vortex displacement per period. Therefore, the mechanism of establishing a new distribution of vortices with the opposite direction of the magnetic field is associated with their "reversal" (or sequential destruction and creation of new ones).





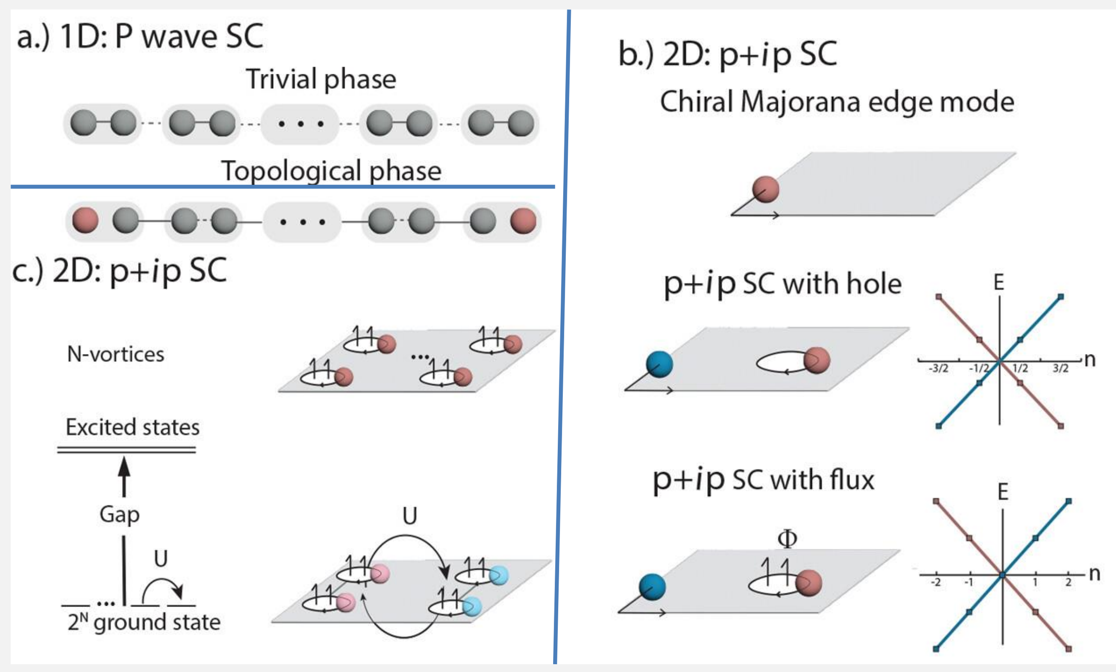
However, in the case of the emergence of 0-th Majorana modes, entangled quantum states arise, both between pairs of spatially distant vortices and between vortices and surface current states [4]. Such quantum entanglement leads to the fact that the transition to another quantum state (in particular, due to the reorientation of vortices, which leads to the dissipation of MW energy) occurs consistently for each Majorana pair of current states. As a result, the main approximation of the Bean model - the absence of a change in the vortex structure in regions of subcritical surface current density values - may be violated.

Schematic illustration of TSCs and Majorana-based topological quantum computing [4].

(a) 1D topological superconductor (Kitaev chain), where each conventional Fermion is a combination of two Majorana Fermions. When "intra-site" pairing between the two Majorana Fermions is stronger than the "inter-site" pairing (upper), a topologically trivial SC is obtained. When intersite interaction is stronger (lower), a 1D TSC is obtained, with two unpaired Majorana Fermions (red spheres) with zero energy at two ends.

(b) 2D $p + ip$ superconductor. (top) Just like the 1D TSC can have 0D boundary modes at two ends, the 2D TSC has 1D chiral Majorana edge modes. (middle) If we pierce one hole to create a region without superconductivity, half-integer excitation spectra are created. (bottom) If we add one magnetic flux quantum Φ to the hole to create a superconducting vortex, the energy spectra become integers and a Majorana zero mode is generated.

(c) Scheme for topological quantum computation. With 2N superconducting vortices, the ground states will have a 2^N degeneracy. The unitary transform of U, which can be used as a quantum gate, can be realized by exchanging different pairs of Majorana zero modes within the ground states.



Scheme of mathematical modeling of microwave field energy dissipation by Majorana pairs

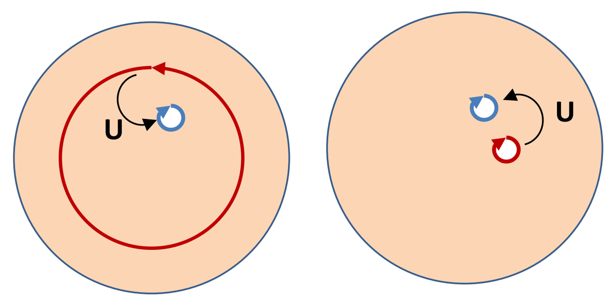
Accurate modeling of the dissipation effect in the presence of both a conventional vortex distribution and the contribution of dissipation by Majorana pairs requires the assignment of values for a large number of unknown parameters. Therefore, in this work we have limited ourselves to the analysis of those corrections that the presence of Majorana vortices gives to the general structure of the vortex distribution, calculated according to the generally accepted Bin model.

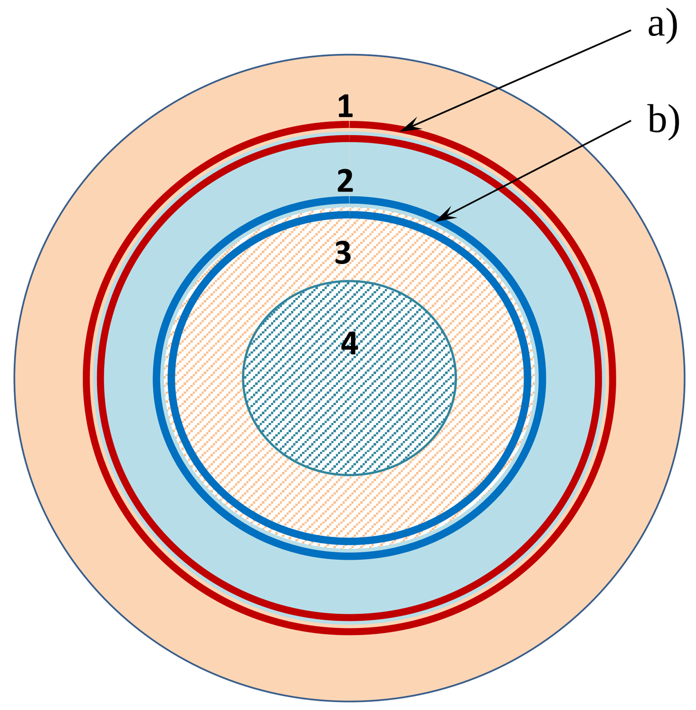
1. In the assumption of this model, a regular vortex transforms into a different energy state (disappears or flips) only when the magnetic field of the entire set of surface currents of the disk is unable to compensate for the change in the external magnetic field at the vortex location. In this case, the only limitation on the current distribution is the maximum critical value of the surface current density at any point in the disk.
2. As a result of the influence of the microwave field on the disk, a periodic structure of annular regions arises, in which:
   - only the distribution of vortices changes, and the current density is equal to the critical one;
   - only the surface current density changes, and the vortex structure is "frozen".

   These regions periodically appear at the boundary of the disk and move towards its center with the frequency of the field. A central Meissner region may also form, in which vortices are absent.
4. For "Majorana pairs" due to the entanglement of their quantum states, an additional condition arises. It is that the direction of the magnetic flux quantum through the vortex core and the direction of the surface current entangled with it (or the direction of the flux quantum in another vortex) can change (transition to a more energetically favorable state) only simultaneously.
5. Such a possibility of simultaneous ("intertwined") change of direction of magnetic flux: vortex and surface circular current, or a pair of two differently directed vortices, is possessed only by annular regions near the boundary of two corresponding zones of the Bean model (see figure). The width of such regions is determined by the characteristic distance between paired Majorana fermions.
6. Entanglement of quantum states of unidirectional vortices (within one region of the Bean model) is also possible. However, the dissipation mechanism due to the disappearance or reversal of ordinary vortices in these regions is dominant. Therefore, the presence of Majorana pairs in these regions will have a much smaller effect on the total dissipation.
7. The emergence of 2 new annular zones will also affect the structure of vortices in other annular regions. Therefore, to calculate the corrections in the total dissipation, it is not enough to take into account the dissipation in these zones, but it is necessary to recalculate the dynamics of the change in the field and current distribution in the disk. In this case, it is necessary to take into account the presence of both ordinary and paired vortices, which generally requires numerical modeling.

Phenomenological description of the effect of microwave radiation dissipation by Majorana pairs in a thin-film superconducting disk

- The scheme above illustrates the formation of Majorana pairs between superconducting currents at the boundary of an extended superconductor and around the vortex core.
- However, in thin-film superconductors, the main part of the surface current flows not along the edge of the disk, but over the entire area of the film. This means that a Majorana pair is formed between the local currents of one of the vortices and part of the distribution of circular superconducting currents (for clarity, in the figure both distributions – around the vortex core and on the disk surface, are depicted by single circular currents).
- It is also possible a formation of a Majorana pair by the currents of two vortices (with both identical and oppositely directed magnetic fluxes).
- The quantum entanglement of two Majorana fermions means that the quantity being measured is only the total magnetic flux created by these currents.
- In the absence of an external magnetic field, the two orthogonal states of such a Majorana pair are energetically degenerate. Therefore, the transition between them does not require energy losses and the Majorana pair is in a superposition of these states.
- But as the external magnetic field increases, an energy gap appears between these two states and grows. It becomes "profitable" for the Majorana pair of currents to move to a lower energy level, releasing thermal energy equal to the size of the gap.
- In the second half-period of microwave oscillations of the external magnetic field, this state becomes energetically disadvantageous and the Majorana pair of currents transitions to the orthogonal state, again releasing thermal energy.

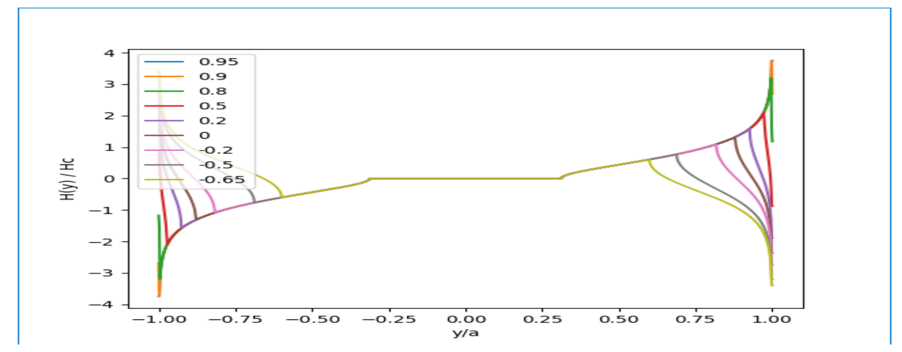




a) Region with the possibility of simultaneous ("intertwined") change in the direction of the magnetic flux of eddy and surface currents

b) Region with the possibility of simultaneous ("intertwined") change in the direction of the magnetic flux of two differently directed vortices

1. $J = J_{crit};\ B > 0$
2. $J = J_{crit};\ B < 0$
3. $J < J_{crit};\ B(t) = const$
4. $J < J_{crit};\ B(t) = 0$



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