The influence of hysteresis absorption locality in a microwave nonlinear HTS transmission line on its properties

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In an experimental study of the properties of a microwave (MW) transmission line of the coplanar waveguide type based on high-temperature superconductors (HTS) at certain values of the input power P_{in} and direct current (DC) I_{dc}, the effect of strong (abrupt) changes in MW losses at a certain waveguide temperature $T < Tc$, where Tc is the critical temperature, was discovered [1].

TABLE I
CHARACTERISTICS OF YBCO EPITAXIAL FILMS ON THE SINGLE CRYSTAL MGO SUBSTRATES

Film thickness (nm)	Critical temperature (K)	Critical current density at 77K (MA/cm^2)
150	86.5	3.6
75	85.8	3.0

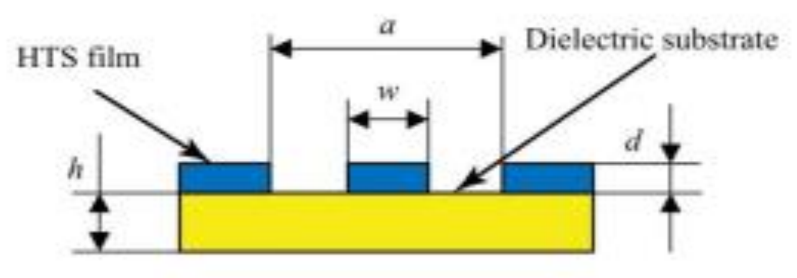


Fig. 1. Cross section of YBCO-film-based coplanar waveguide.

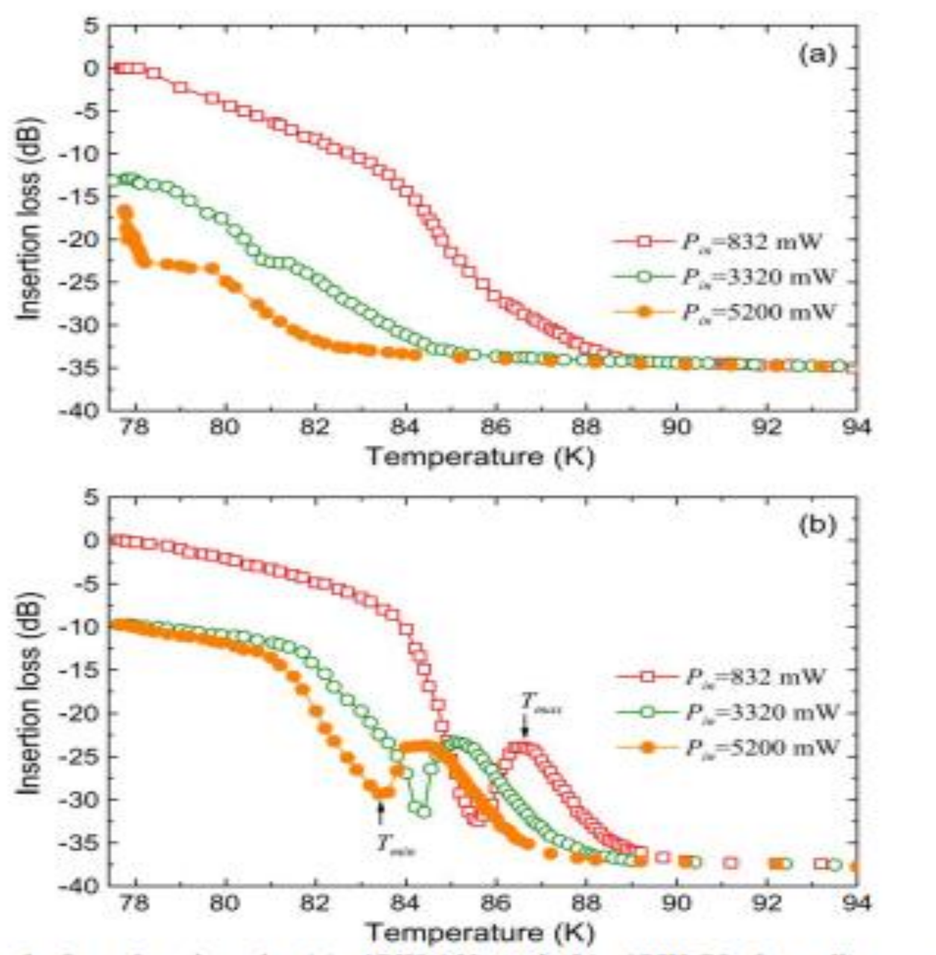


Fig. 2. Insertion loss in (a) CPW-150 and (b) CPW-75 depending on temperature at different input power of 5-μs-long MW pulses.

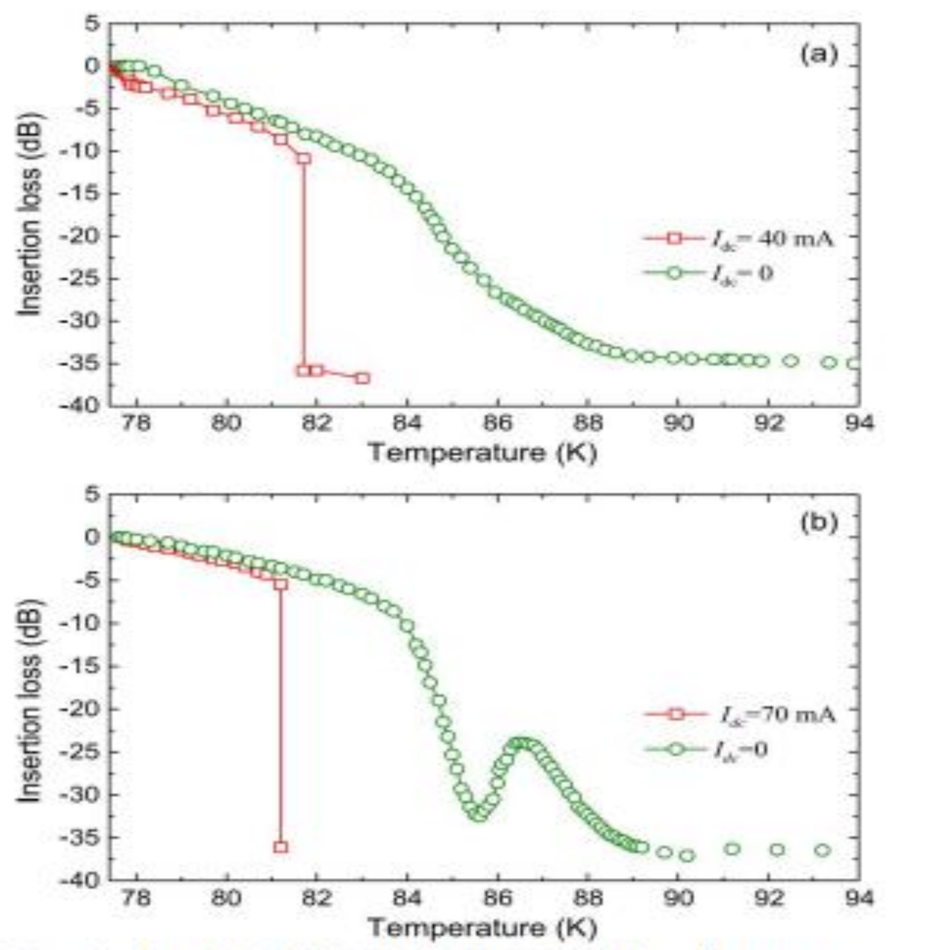
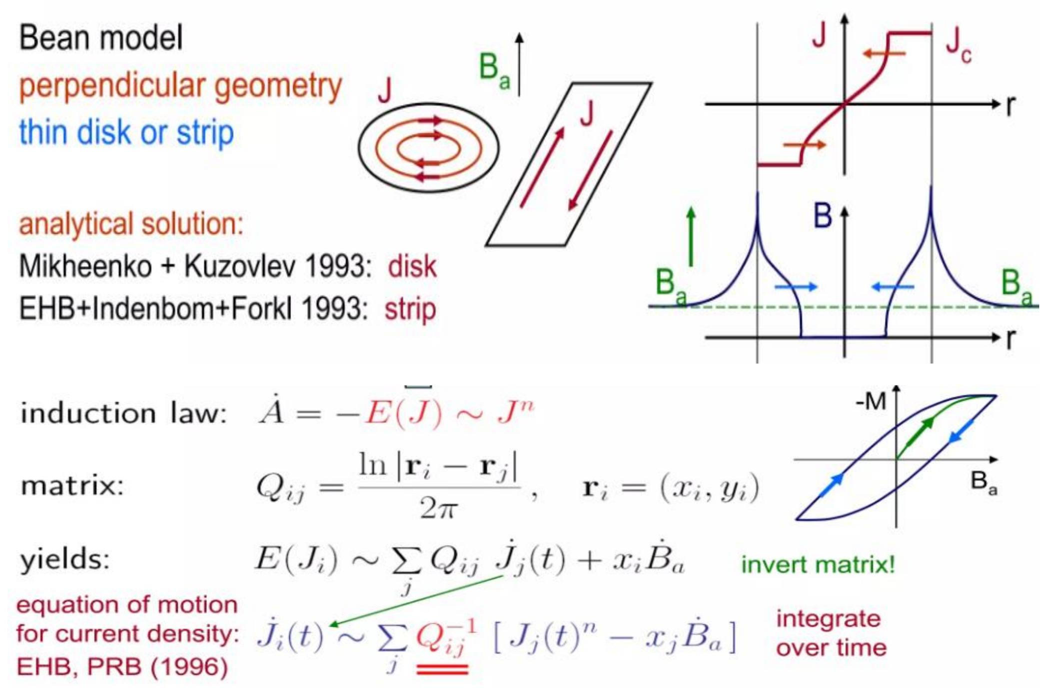
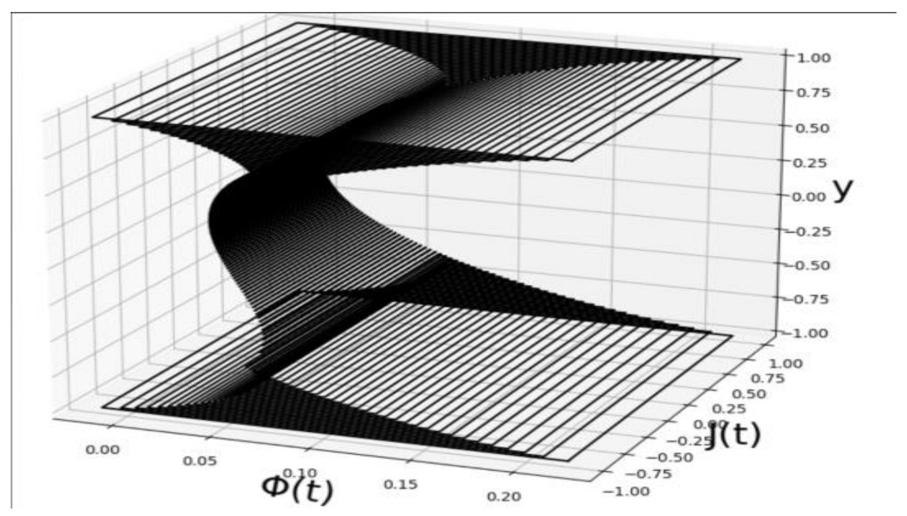


Fig. 3. Insertion loss in (a) CPW-150 and (b) CPW-75 depending on temperature at 5 μs-long pulse of P_{in} = 832 mW at dc (I_{dc} = 40 mA for CPW-150 and 70 mA for CPW-75) and without it (I_{dc} = 0).

In this paper, we analyze the specifics of modeling and calculating losses in such a line. When calculating power losses for a nonlinear waveguide line based on high-temperature superconductors in a microwave field, a standard electrical engineering approach is typically used: determining the area of the hysteresis loop arising in the waveguide cross-section when calculating the magnetization curve within the Bean model [2]. It turns out that taking into account the dependence of the critical current density on the magnetic field strength leads to significant distortions in the shape of the magnetization curve [3].



Furthermore, neglecting the fact that both the hysteresis loop shape and its initial phase are local—depending on the distance to the strip center—can lead to significant errors in describing the response dynamics and calculating losses. Therefore, when calculating the quality factor of such a line, we previously proposed first using a local calculation of the magnetization curve and then integrating the resulting power loss density values over the width of the waveguide line [4].



Three-dimensional image of the set of hysteresis loops depending on the distance to the center of the film

The main differences between the model and the classical approach to thermal analysis are:

1. ***At a phonon velocity of approximately 4000 m/s***, heat propagates over a distance of approximately 400 nm during one dissipation cycle (at a frequency of 10 GHz).

   This is 4 times the film thickness (100 nm) and 250 times smaller than its width (100 μm).

   Therefore, the classical heat conduction equation, which is based on the instantaneous and localized heat transfer approximations, is inapplicable for calculating a stable and periodically changing temperature field in a film, both across its width and thickness.

2. ***As heat propagates across the film thickness*** and is removed into the substrate, the localized heat transfer approximation is violated.

   In particular, phonon reflection from the interface with the substrate is observed, due to their different velocities on either side of the interface. This leads to the appearance of additional Kapitsa thermal resistance at the interface. This, in turn, determines the Kapitsa relaxation time, which for a 100 nm thick film at nitrogen temperatures ranges from 1 to 10 ns, significantly exceeding the microwave excitation period. This means that the substrate does not have time to respond to periodic changes in the film temperature.

   Furthermore, the film's thermal response time, $\tau_{th} \sim d/s$, is approximately $0.25 \cdot 10^{-10}$ $[s]$ – (a quarter of the microwave excitation period). This means that the delay of temperature fluctuations relative to fluctuations in the external magnetic field cannot be neglected (a phase lag is observed).

   In general, to calculate heat transfer into the substrate, it is necessary to solve a one-dimensional thermal conductivity equation with spatial nonlocality.

3. ***When heat propagates across the film width***, two factors must be considered:

   - ***Temporal nonlocality***, which is due to the fact that the rate of heat exchange between different regions is limited by the velocity of phonons.

     Generally, a model taking into account the relaxation time (e.g., the Cattaneo model) is used to calculate the heat transfer process in this case.

   - ***The motion of the heat source*** is associated with the fact that the dissipation zone, according to Bean's model, moves from the edges to the center of the strip at a velocity of about $v = a \cdot \nu \sim 10^6$ $[m/s]$.

   To account for this effect, we switch to a reference frame that is commoving with the motion of the heat source.

   Taking these factors into account reveals that the temperature distribution across the film width cannot be considered uniform. Overall, we obtain a picture of a "travelling heat wave" against a background of average temperature.

4. ***As a result***, it turns out that the thermal field in the film can be described by a complex wave-like distribution (with different values of the amplitude and phase of oscillations at different points).

   This, in turn, necessitates taking into account the local amplitude and phase of the film's temperature oscillations when calculating the field and current density distributions within the Bean model.

5. ***The critical current density***, in turn, strongly depends on the temperature near the critical point. Therefore, the amplitude and phase distribution of temperature oscillations in the film can have a significant impact on the overall distribution of currents and fields.

Therefore, the overall mutual influence of electrodynamic and thermal (phonon) processes necessitates solving a self-consistent problem. To solve this problem, we plan to construct a numerical model within the COMSOL package.

Spatial non-locality of heat transfer

Instead of classical proportionality, the heat flux density becomes the result of integral interaction:

$$q(x) = -\int k(|x - x'|)\nabla T(x')dx'$$

$k(|x - x'|)$ is a kernel function that specifies the degree of influence of temperature gradients at remote points

On the scale of superconducting nanofilms

heat transfer is not described by the diffusion equation, but by the hyperbolic equation of heat conduction (known as the Cattaneo - Vernotte model)

$$\tau\frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \chi\nabla^2 T$$

where τ is the relaxation time of the heat flow (related to the speed of phonons). The presence of τ means that with a sharp temperature gradient, heat spreads in the form of temperature waves with the speed of phonons, and does not dissipate instantly.

The equation of heat conduction with a moving source

describes the temperature distribution of the movement of heat sources with a given speed relative to the body. The general form in partial derivatives:

$$\frac{\partial T}{\partial t} = a\nabla^2 T + \frac{Q}{\rho c}\delta(x - vt)$$

At the same time, in the reference system associated with the source itself, the problem is often considered stationary

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