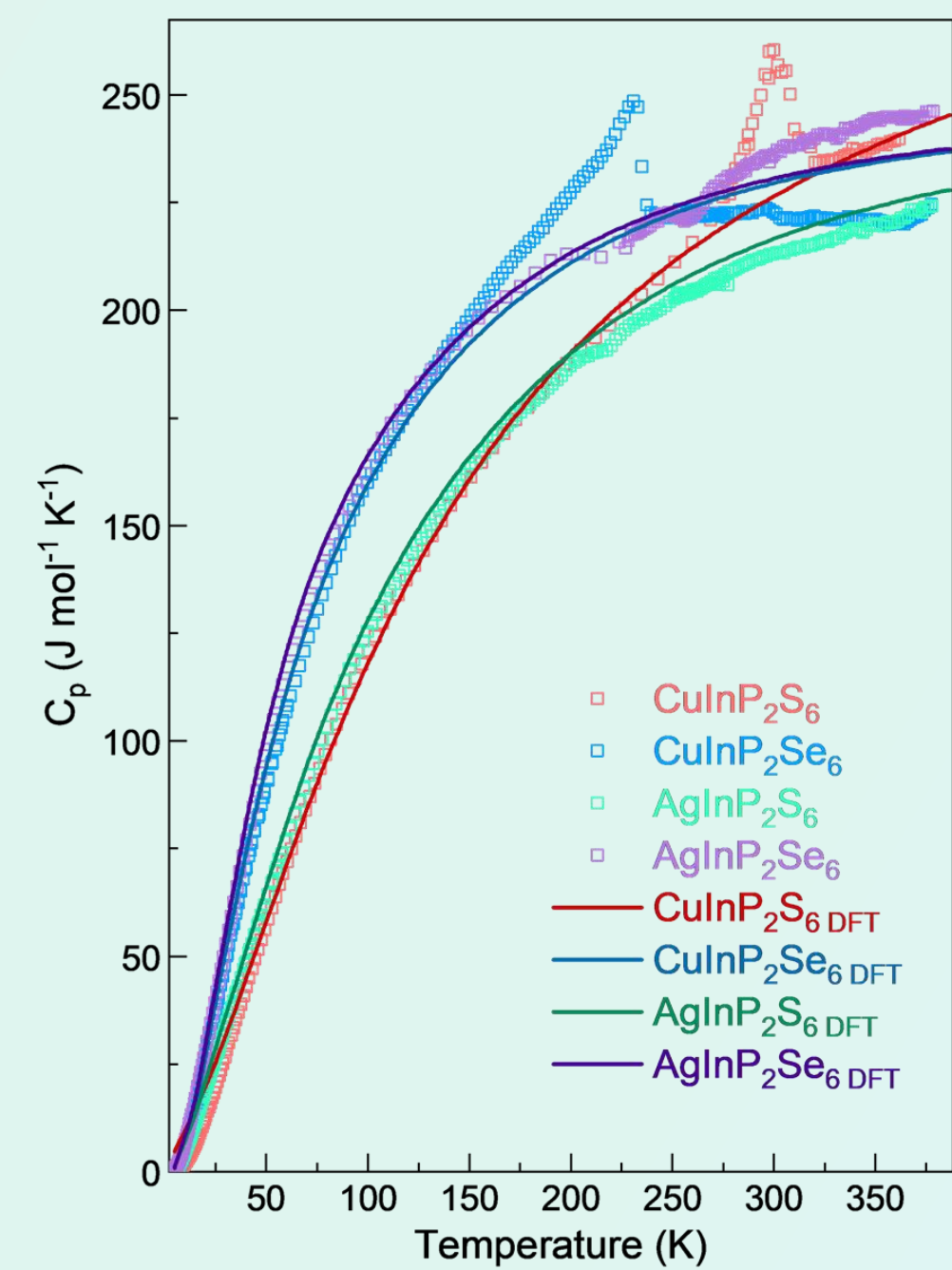


## Introduction

It has recently been established that the phenomenon of ferroelectricity in CuInP<sub>2</sub>S<sub>6</sub> can be considered within the mixed Ising model with spins  $s = 1/2$  and  $S = 1$  [1]. These spins (or rather pseudospins) are associated with the ordering dynamics of Cu<sup>+</sup> cations in the double-well local potential and with the ordering/displacive dynamics of In<sup>3+</sup> cations in the three-well local potential. Therefore, the occurrence of strong lattice anharmonicity is attributable to the presence of the indium sublattice, which manifests itself below 150 K. Conversely, at higher temperatures, the anharmonicity due to the copper sublattice activates the ionic conductivity and leads to the first order ferroelectric-paraelectric phase transition at  $T_C \approx 315$  K, as well as to the existence of fluctuated polar clusters above  $T_C$ . The present study focuses on the temperature evolution of the heat capacity of Cu(Ag)InP<sub>2</sub>S(Se)<sub>6</sub> crystals. The temperature range encompassed from 2 K to 370 K, thereby covering both the low-temperature region where quantum fluctuations play a pivotal role and the high-temperature region, where classical thermal fluctuations are predominant.

## Experimental Results

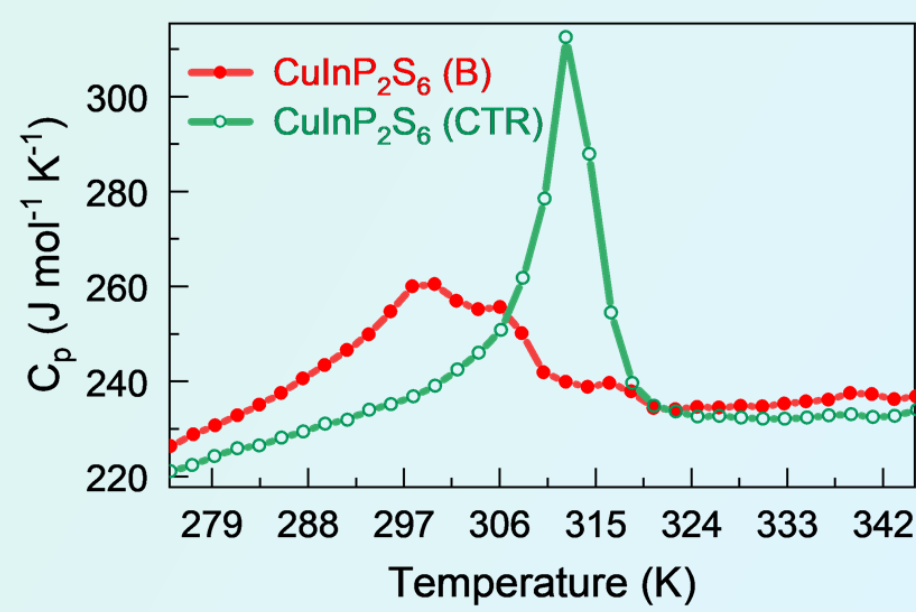
For the study, CuInP<sub>2</sub>S<sub>6</sub>, CuInP<sub>2</sub>Se<sub>6</sub>, AgInP<sub>2</sub>S<sub>6</sub> and AgInP<sub>2</sub>Se<sub>6</sub> single crystals were grown using the Bridgman method.



**Figure 1.** Experimental molar heat capacity (symbols) and DFT calculation (solid lines) of CuInP<sub>2</sub>S<sub>6</sub>, CuInP<sub>2</sub>Se<sub>6</sub>, AgInP<sub>2</sub>S<sub>6</sub> and AgInP<sub>2</sub>Se<sub>6</sub> crystals

As demonstrated in Fig. 1, the experimental measurements of the molar heat capacity of CuInP<sub>2</sub>S<sub>6</sub> and CuInP<sub>2</sub>Se<sub>6</sub> crystals confirm the presence of a phase transition at 304 K and 240 K, respectively, whereas no phase transition anomalies were observed in AgInP<sub>2</sub>S<sub>6</sub> and AgInP<sub>2</sub>Se<sub>6</sub> crystals.

The high-temperature critical  $C_p$  anomaly for a Bridgman-grown CuInP<sub>2</sub>S<sub>6</sub> crystal was compared with the one of the crystal grown by the chemical transport reactions (CTR) method. As demonstrated in Fig. 2, the  $C_p$  anomaly of Bridgman-grown CuInP<sub>2</sub>S<sub>6</sub> was more blurred in comparison to the compound obtained by the CTR method.

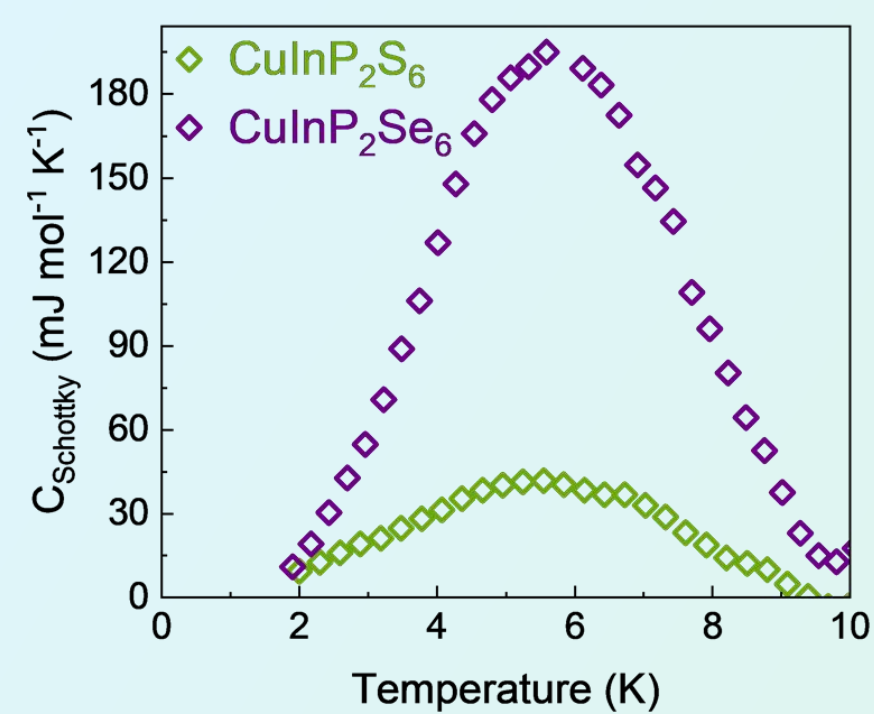


**Figure 2.** Phase transition in CuInP<sub>2</sub>S<sub>6</sub> crystals grown by Bridgman method (B) and chemical transport reactions (CTR)

## Discussion

For all CuInP<sub>2</sub>S<sub>6</sub>, CuInP<sub>2</sub>Se<sub>6</sub>, AgInP<sub>2</sub>S<sub>6</sub> and AgInP<sub>2</sub>Se<sub>6</sub> crystals, a so-called “boson peak” was observed which is attributed to the specific nature of shear modes, which underlies the low-temperature heat capacity anomaly. CuInP<sub>2</sub>S(Se)<sub>6</sub> compounds also show an anomalous increase in  $C_p/T^3$  dependence upon cooling below the “boson peak”. Such phenomenon is not observed in AgInP<sub>2</sub>S(Se)<sub>6</sub> crystals. This anomaly may be classified as the appearance of a Schottky-like peak [2]. The existence of such an anomaly in CuInP<sub>2</sub>S(Se)<sub>6</sub> is consistent with the mixed Ising model [3] and can be explained by fluctuations of  $S = 1$  pseudospins that are related to the In<sup>3+</sup> cations anharmonic dynamics in the three-well local potential. In particular, when silver replaces copper, in the centrosymmetric structure, the Ag<sup>+</sup> cations become firmly fixed at the center of the structural layer, thereby directly constraining the role of indium fluctuations at low temperatures. This assumption is corroborated by the temperature dependence of heat capacity calculated within the framework of the quantum anharmonic oscillator model [1]. The analysis indicated that the most significant increase in entropy within the ultralow-temperature region is linked to the anharmonic dynamics of indium. The model under consideration also predicts that the correlated relaxations related to Cu<sup>+</sup> cations  $s = 1/2$  pseudospins can induce the ferroelectric clusters that occur above the transition from the ferroelectric to paraelectric state.

In order to extract the Schottky contribution to the  $C_p(T)$  in CuInP<sub>2</sub>S(Se)<sub>6</sub>, a method similar to the approach employed in the study of magnetic compounds was used. In that method, a nonmagnetic the lattice  $C_p$  is taken as a reference and then subtracted from the curve of the magnetic sample [4]. As the reference  $C_p(T)$  of AgInP<sub>2</sub>S(Se)<sub>6</sub> were used. In this study, the curves were normalized according to the following parameters: the temperature of the boson peak was set to 1, and the value of  $C_p/T^3(T)$  at the boson temperature was set to 1. The results are shown in Fig. 3.



**Figure 3.** Extracted Schottky contribution to  $C_p(T)$  in CuInP<sub>2</sub>S<sub>6</sub> and CuInP<sub>2</sub>Se<sub>6</sub> crystals

## Conclusions

A comprehensive study of the heat capacity of Cu(Ag)InP<sub>2</sub>S(Se)<sub>6</sub> crystals was conducted, enabling the estimation of the individual contributions to the heat capacity in these van der Waals materials.

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## Debye-Einstein Model

In order to describe the temperature dependence of the heat capacity, a combined Debye-Einstein model was used, which allows for the contribution of different types of crystal lattice vibrations. This approach is particularly relevant for complex solids, in which low-frequency acoustic phonons and high-frequency optical modes contribute differently to the total heat capacity. The molar heat capacity at constant volume was expressed as the sum of the Debye and Einstein contributions:

$$C_V(T) = n_D C_D(T) + \sum_{i=1}^m n_{E,i} C_{E,i}(T), \quad (1)$$

where  $n_D$  and  $n_{E,i}$  is a number of vibrational modes, which satisfy the normalization condition  $n_D + \sum_{i=1}^m n_{E,i} = 3N$ , where  $N$  is a number of atoms in a formula unit. The Debye contribution is

$$C_D(T) = 9n_D R \left(\frac{T}{\Theta_D}\right)^3 \int_0^{\Theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2} dx, \quad (2)$$

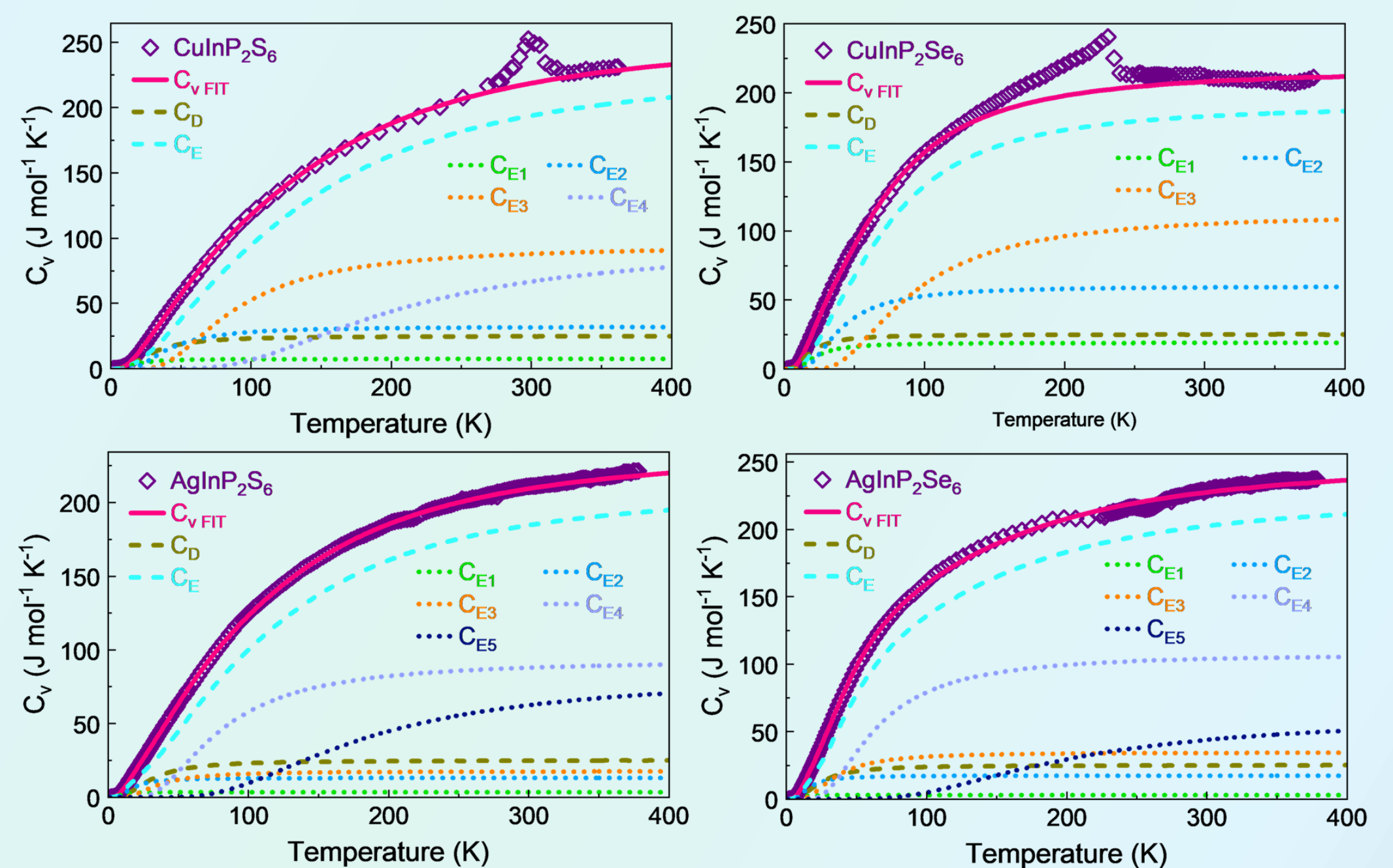
where  $R$  is gas constant,  $\Theta_D$  is the Debye temperature. The Einstein contribution is

$$C_{E,i}(T) = 3n_{E,i} R \frac{(\Theta_{E,i}/T)^2 e^{\Theta_{E,i}/T}}{(e^{\Theta_{E,i}/T} - 1)^2}, \quad (3)$$

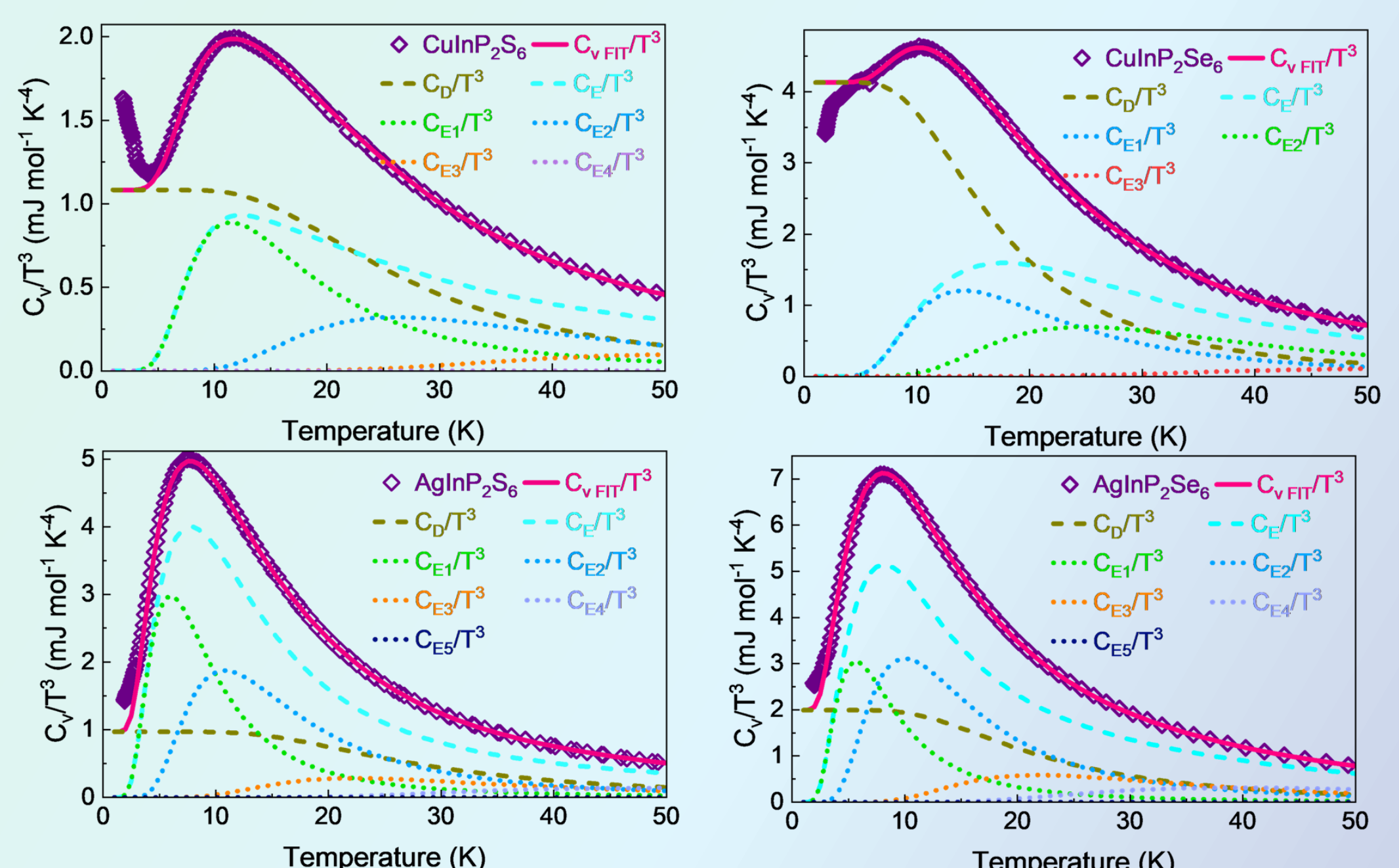
where  $\Theta_{E,i}$  is the characteristic Einstein temperature of the correspondence optical mode.

Accordingly, the model enables the concurrent evaluation of a continuous spectrum of low-energy phonon excitations (the Debye model) and discrete local atomic vibrations (the Einstein model). The number of Einstein terms,  $m$ , was selected based on the complexity of the structure and the quality of the fit to the experimental data.

The parameters  $\Theta_D$ ,  $\Theta_{E,i}$  as well as  $n_{E,i}$  were determined by means of the least-squares method, with the aim of minimizing the deviation function:  $\chi^2 = \sum_j [C_{exp}(T_j) - C_{calc}(T_j)]^2$ , where  $C_{exp}(T_j)$  are the experimental values of  $C_V$  at  $T_j$ , and  $C_{calc}(T_j)$  are values, obtained from the model. To ensure an exact fit, the experimental  $C_p$  was converted to the  $C_V$  as  $C_V = C_p - TV \frac{\alpha^2}{\beta}$ , where  $V$  is the molar volume,  $\alpha$  is the volume thermal expansion coefficient,  $\beta$  is the isothermal compressibility of the material.



**Figure 4.** Results of fitting experimental  $C_V$  of CuInP<sub>2</sub>S<sub>6</sub>, CuInP<sub>2</sub>Se<sub>6</sub>, AgInP<sub>2</sub>S<sub>6</sub> and AgInP<sub>2</sub>Se<sub>6</sub> crystals. The Debye and Einstein contributions are shown



**Figure 5.** Results of fitting experimental  $C_V$  of CuInP<sub>2</sub>S<sub>6</sub>, CuInP<sub>2</sub>Se<sub>6</sub>, AgInP<sub>2</sub>S<sub>6</sub> and AgInP<sub>2</sub>Se<sub>6</sub> crystals in low temperature region in  $C_V/T^3(T)$  scale. The Debye and Einstein contributions are shown