Dissipative Landau-Zener-Stückelberg-Majorana gates

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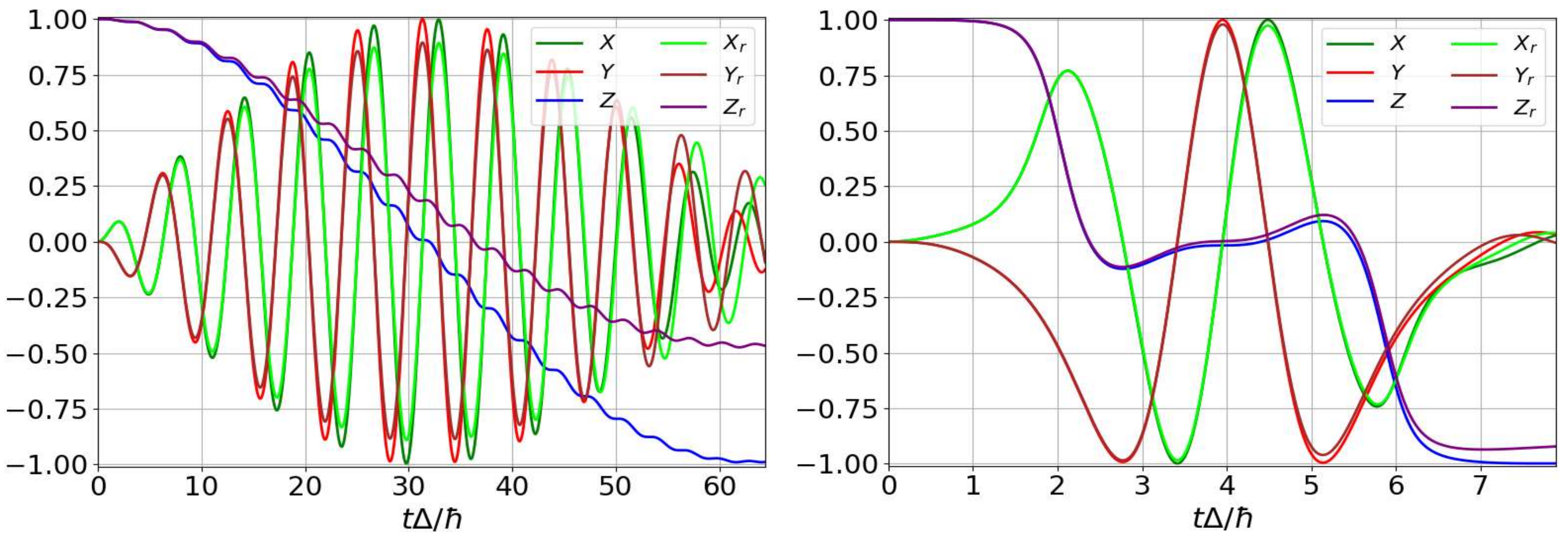
Introduction

Achieving high-precision quantum computing requires a variety of approaches, ranging from selecting the optimal physical form for the qubit to developing advanced error-correction methods. We are exploring a method for controlling the qubit state using non-adiabatic transitions, known as the **Landau-Zener-Stückelberg-Majorana (LZSM) transitions**. Experimental studies show that such gates allow implementing **extremely fast operations** [1] and also demonstrate **sufficient robustness to low-frequency noise** [2]. This method overcomes most of the drawbacks, such as the strict **driving frequency limitations** and the strong **correlation between accuracy and speed**, associated with commonly used **Rabi oscillations-based gates**. Numerical simulation results, shown in Figures on the right, also confirm the shorter duration and lower error rates of LZSM-based gates compared to Rabi oscillations-based gates (*cf.* difference between the Bloch vector components Z and Z_r). A method for finding the control parameters of a system for implementing a desired quantum gate is proposed in Ref. [3]. However, only ideal (isolated) qubits were considered in that study. We consider the **LZSM transitions of real systems taking decoherence into account**, analyze the resulting distribution of errors and look for ways to minimize them.

Dynamics in ideal (no relaxation; the components of the Bloch vector X, Y, Z) and real (with relaxation; the components of the Bloch vector $X_\mathrm{r}, Y_\mathrm{r}, Z_\mathrm{r}$) systems with $T_{1,2}\Delta/\hbar = 100$ for Rabi-based (left panel) and LZSM-based (right panel) X-gate



[1] H. Zhang et al., Phys. Rev. X 11, 011010 (2021).
[2] D. L. Campbell et al., Phys. Rev. X 10, 041051 (2020).
[3] A. I. Ryzhov, O. V. Ivakhnenko, S. N. Shevchenko, M. F. Gonzalez-Zalba, Franco Nori, PRResearch, 033340 (2024).

Traditional (Rabi-based) quantum gates and their disadvantages

Consider qubits based on spin-½ particles. The system is controlled by a magnetic field $H(t) = -\frac{1}{2}(\Delta\sigma_x + \varepsilon(t)\sigma_z)$. Let the signal be harmonic with $\varepsilon(t) = A\sin\omega t$ and $\Delta E = \sqrt{\Delta^2 + \varepsilon^2}$. When ω equals to the Larmor frequency $\omega_\mathrm{q} = \Delta E/\hbar$ the occupation probability of the upper energy level oscillates with a frequency $\Omega_R = \frac{A\Delta}{2\hbar\Delta E}$. However, to achieve this effect, certain conditions must be met, which leads to the emergence of associated disadvantages.

Conditions for the realization of Rabi oscillations

Smallness of the tunneling amplitude: $A \ll \Delta E$

Zero-energy detuning: $\hbar\omega \approx \Delta E, \delta\omega = \omega - \omega_q \ll \omega$

Consequences of compliance with the conditions

Experimental complexity of setting the value of A

The greater error δA → The greater error $\delta\Omega_R^{(0)}$ → Less fidelity

Correlation between accuracy and speed

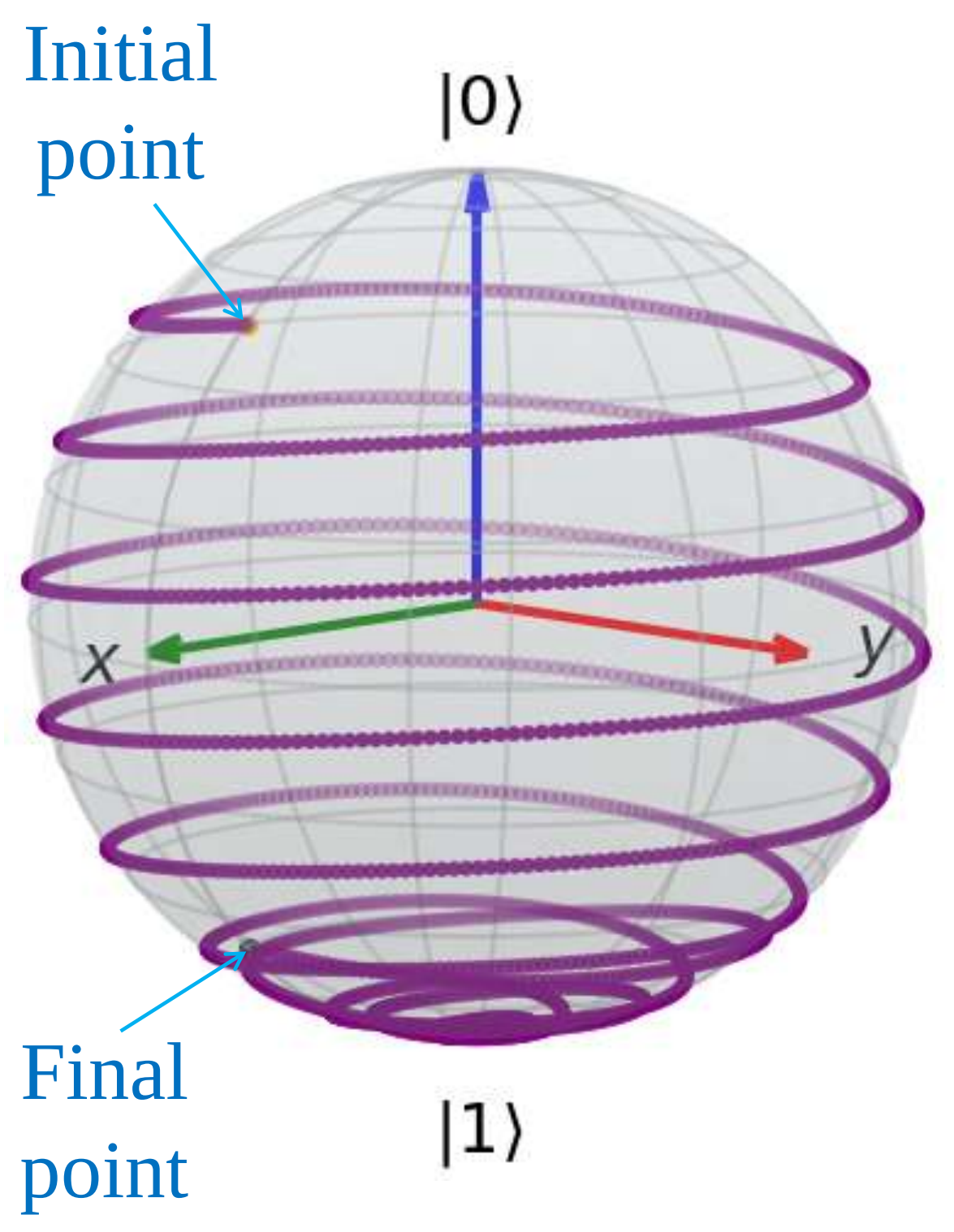
Smaller A → Smaller $\Omega_R^{(0)}$ → Less gate speed → Less fidelity

Greater A → 1-st condition is violated → Less fidelity

Resonance problems

Leaks to outer energy levels ← Resonant ω → Possible destruction of the qubit

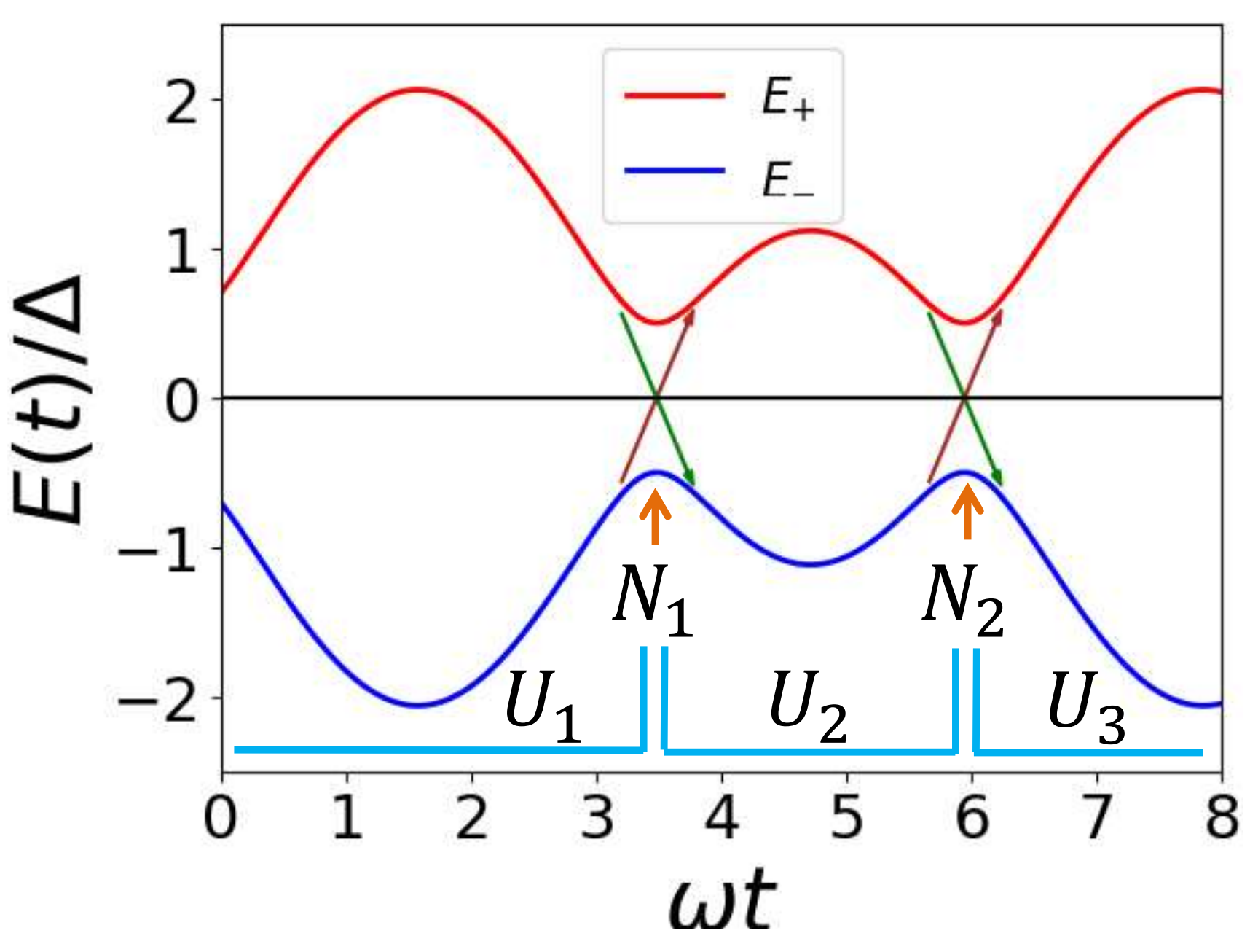
Dynamics of Rabi-based X-gate on Bloch sphere (initial state $\vec{R} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$)



Alternative (LZSM-based) quantum gates their description using the adiabatic-impulse model

There is always a non-zero probability of a qubit spontaneously transitioning from one energy level to another. However, when $A \gg \Delta$ and $\varepsilon(t) = 0$ (so-called quasi-intersection points) this probability becomes large enough that it significantly influences the state of the system. The duration of diabatic transitions is quite short compared to the time during which qubit evolves adiabatically. Considering the overall evolution of a qubit as a sequence of these processes is called the transfer-matrix method.

Dynamics of a two-level system in the instantaneous basis



Diabatic evolution

$$N = \begin{pmatrix} Re^{i\phi_S} & T \\ -T & Re^{-i\phi_S} \end{pmatrix}, N_{inv} = N^\mathrm{T}$$

Probability for ground-state initial point:

$$P_{LZSM} = \exp(-2\pi\delta), \delta = \Delta^2/4\hbar A\omega$$

Reflection: $R = \sqrt{1 - P_{LZSM}}$, transition: $T = \sqrt{P_{LZSM}}$

Stokes Phase: $\phi_S = \frac{\pi}{4} + \delta(\ln\delta - 1) + \mathrm{Arg}[\Gamma(1 - i\delta)]$

Adiabatic evolution

$$U(t_1, t_2) = e^{-i\zeta(t_1,t_2)\sigma_z}$$

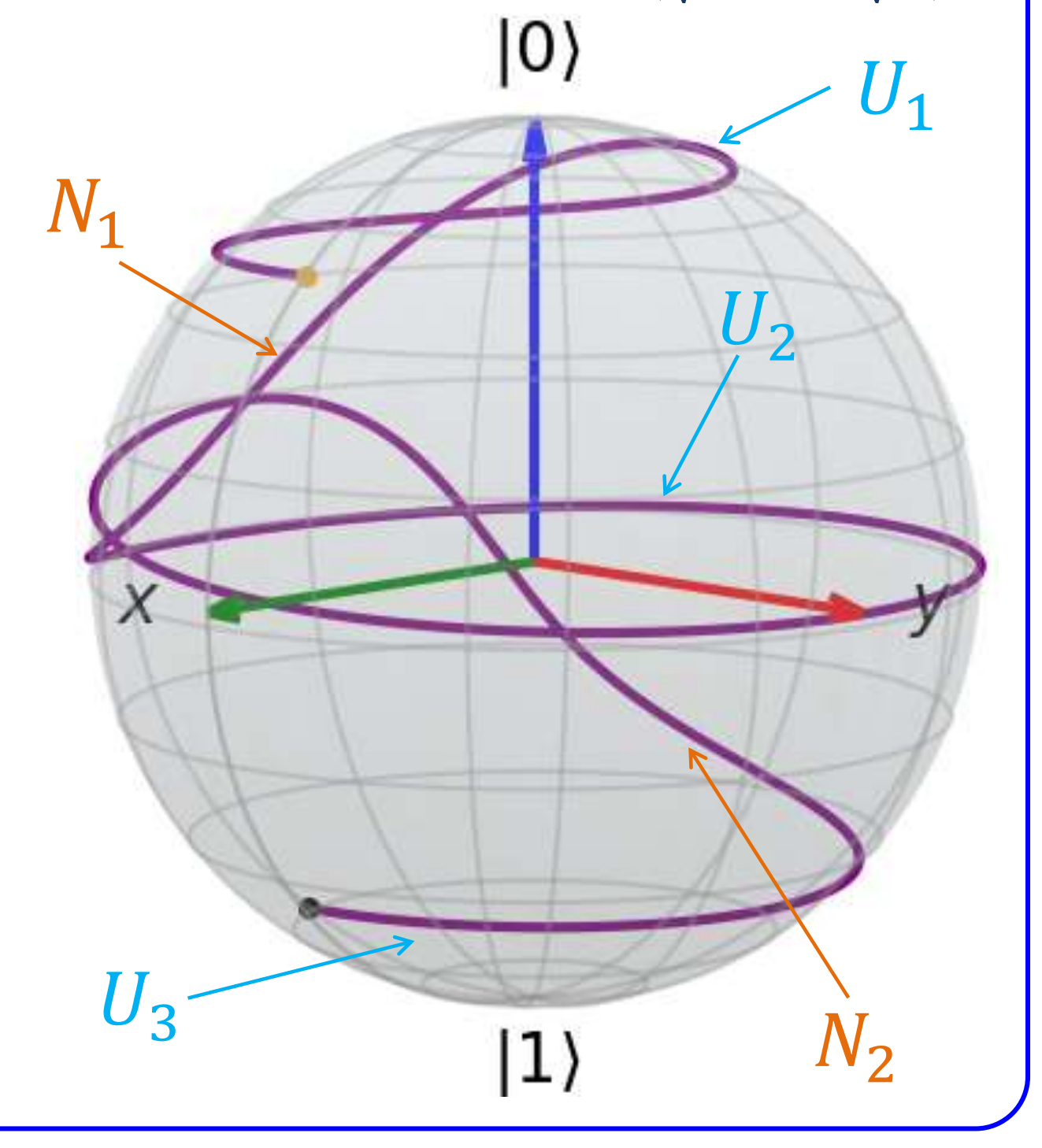
$$\zeta(t_1, t_2) = \frac{1}{2\hbar}\int_{t_1}^{t_2}\Delta E(t)dt$$

Total evolution

$$U_{gate} = U_3 N^\mathrm{T} U_2 N U_1$$

(the simplest option)

Dynamics of LZSM-based X-gate on Bloch sphere (initial state $\vec{R} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$)



Comparison of optimized Rabi-based gates and non-optimized LZSM-based with dissipation

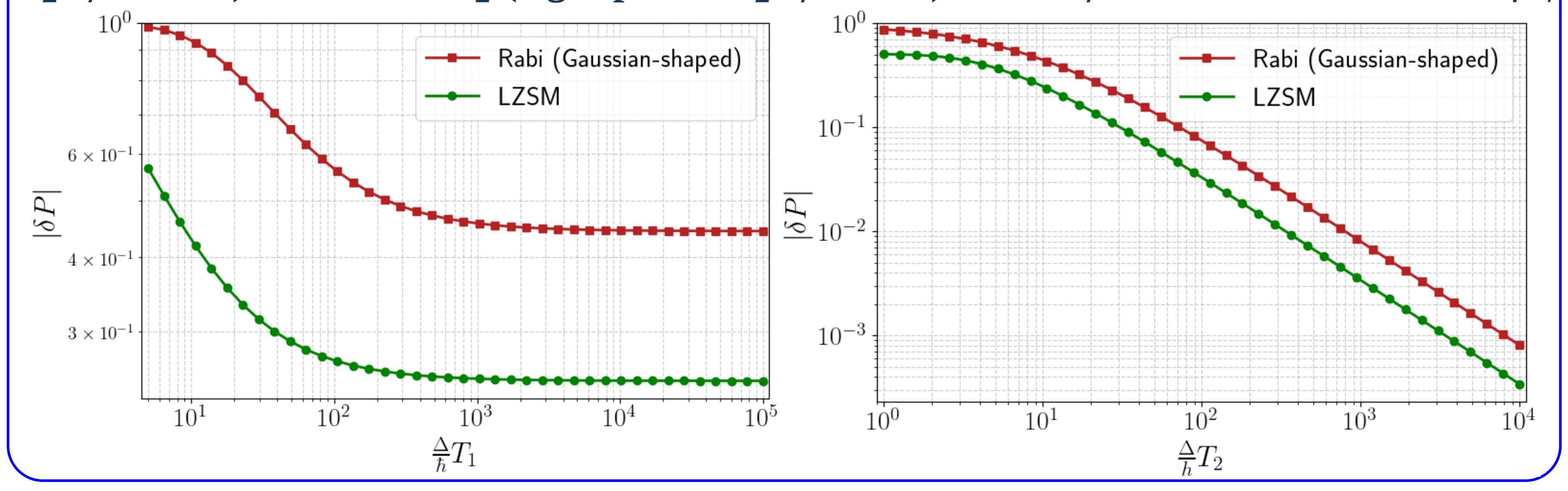
In practice, any quantum system cannot be completely isolated from its environment. Interactions between them lead to energy exchange processes (energy relaxation) and phase blurring (decoherence). These processes have corresponding characteristic times $T_{1,2}$, and from the Lindblad equation it follows that $T_2 \le 2T_1$.

About optimization of the Rabi method

Rabi-based gates require small tunneling amplitudes, so A must be small at least at the points $\omega t = \pi n$. To increase computing speed, a larger amplitudes can be used between them. The most common optimization method suggests using Gaussian-shaped envelope:

$$\varepsilon(t) = A(t)\sin\omega t, \qquad A(t) = A_0\exp\left(-\frac{(t - 0.5T)^2}{2\sigma^2}\right)$$

Dependence of the probability error $|\delta P| = |P - P_\mathrm{r}|$ of the X-gate on the T_1 (left panel, $T_2\Delta/\hbar = 10$) and on the T_2 (right panel, $T_2\Delta/\hbar = \infty$). Here $T/\sigma = 5.4$, initial state is $|0\rangle$



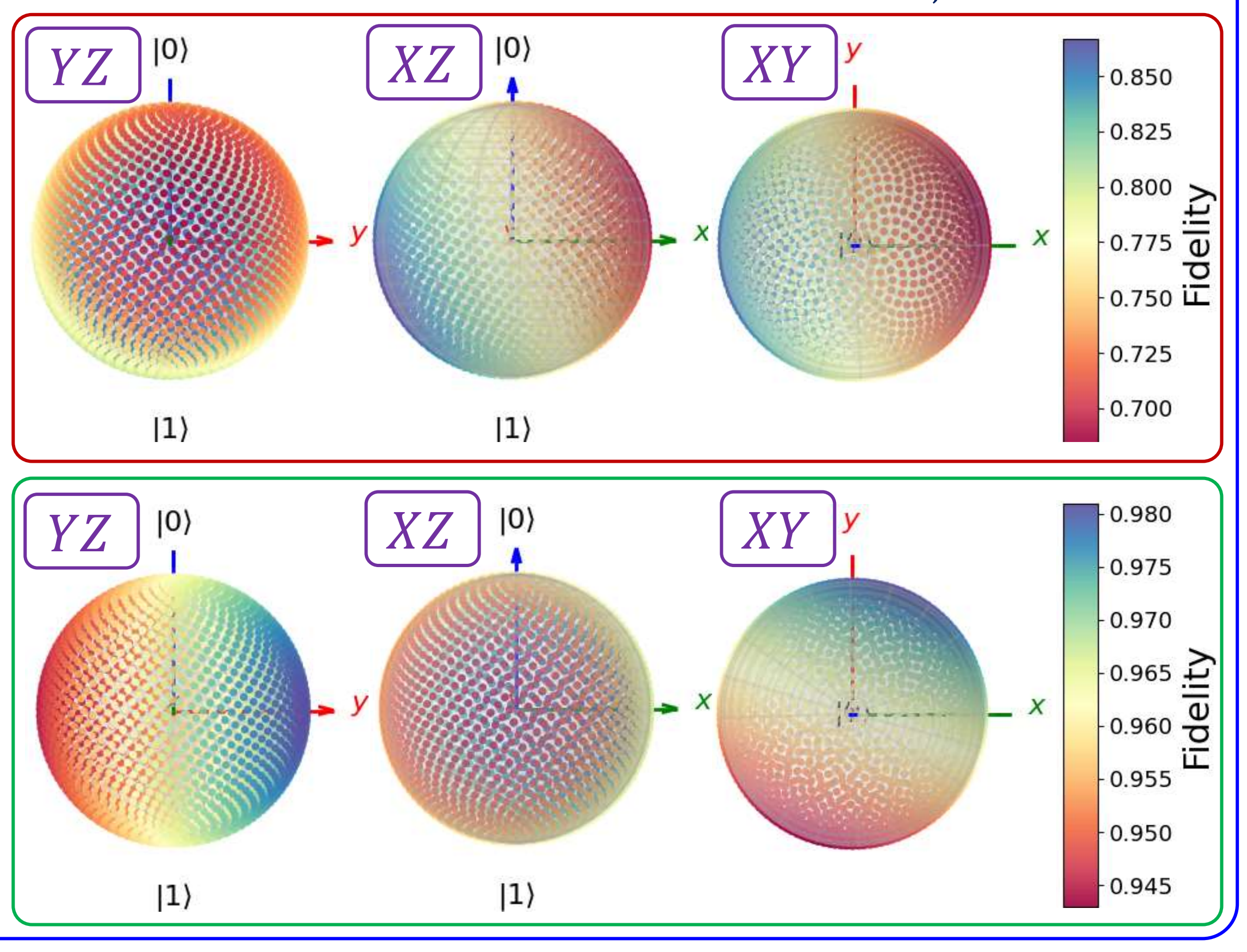
Fidelity distribution

There's a more universal way to evaluate the accuracy (fidelity) of realization of a quantum gate. In addition to the control parameters (Δ, A, ω, etc.) and $T_{1,2}$, it also depends on the initial state ρ_in of the qubit:

$$F(\rho, \rho_{in}) = \left(\mathrm{tr}\sqrt{\sqrt{\rho}U\rho_\mathrm{in}U^\dagger\sqrt{\rho}}\right)^2$$

The figures on the right show a view of the Bloch sphere from three different sides

Fidelity distribution over $N = 2000$ initial states for the optimized (at $T/\sigma = 5.4$) Rabi-based (top panel) and LZSM-based (bottom panel) X-gate with $T_{1,2}\Delta/\hbar = 100$



Conclusions

- The dependence of the probability of spontaneous (LZSM) transitions between energy levels on magnetic field parameters allows them to be used as an alternative method for controlling the states of qubits;
- Such gates are less constrained in the choice of control parameters and therefore lack the drawbacks of the traditional (Rabi-based) approach;
- Even non-optimized LZSM-based gates have significantly higher implementation speeds and higher computational fidelities compared to Gaussian-shaped optimized Rabi-based gates;
- The relaxation leads to asymmetrical fidelity distribution in the xy-planes for LZSM gates due to the peculiarities of their paths of evolution.