

# Structure and properties of the boundary between graphene-like and Lieb lattices

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The boundary between two topological conductors can be characterized by a number of properties that arise due to the irremovable difference in the topological characteristics of the two media, and are therefore resistant to external influences. The boundary between graphene-like and Lieb lattices is a system of this kind. Both lattices are characterized by a linear electron energy spectrum, but differ in the Berry phase, which is absent in the Lieb lattice. As was shown previously for a simple model [1], specific local edge states arise in such a system, and the transmission coefficient is independent of the magnitude of the wave vector  $|\mathbf{k}|$  or the energy  $E$ , retaining only the angular dependence. In this way, the effective barrier cannot be characterized by any dimensional parameters such as width or potential energy, which emphasizes its topological nature. Naturally, a number of properties characteristic of topological materials are sensitive to the structure of the boundary between topological conductors. For example, supertunneling is not realized within the model [1], which is a consequence of the limitations of the model used. However, in another related system with a changing Berry phase [2], supertunneling occurs even with a relatively simple structure of the boundary. Thus, it is important to distinguish between constraints imposed by the model used and unavoidable general constraints, such as the "no-go" theorem [3].

In the present research, the influence of the boundary structure between graphene-like and Lieb lattices on tunneling and effective boundary conditions is investigated. Different boundary configurations between two lattices are considered under the tight-binding approximation and under the condition of maximum similarity of the conical energy spectrum of charge carriers in both conductors in the choice of model parameters. The results for the model with a two-parameter description of the boundary in accordance with Fig.1 are shown below.

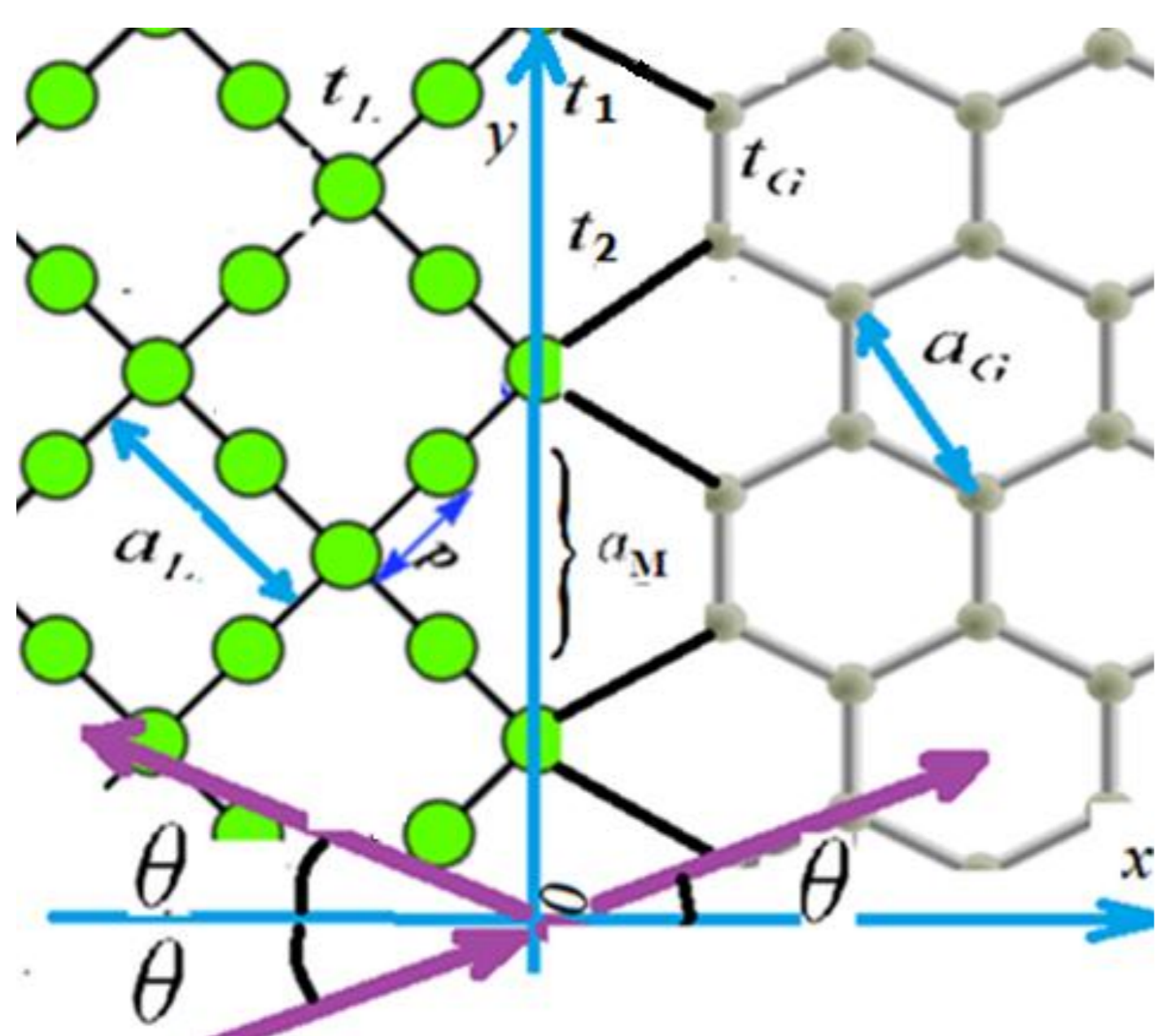


Fig.1 The boundary of two lattices and some of their parameters

**Problem formulation.** Let us consider the contact between the Lieb lattice and a graphene-like lattice, as shown in Fig. 1. within the standard nearest neighbor approximation for the electronic energy spectrum

$$E = \sum_{i,j} t_{i,j} \bar{\phi}_i \phi_j, \quad (1)$$

where  $t_{i,j} = t_L$  for Lieb lattice,  $t_{i,j} = t_G$  for the graphene-like lattice, and  $t_{i,j} = t_1$  or  $t_2$  between atoms of two types (Fig.1), the summation is carried out over all pairs of neighboring atoms,  $\phi_i$  -- amplitude of the probability of finding an electron near a site  $i$ .

The transmission coefficient in the absence of an external magnetic field is calculated:

$$T = \frac{4 \operatorname{Re} R}{|1+R|^2}, \quad R = \frac{(t_1^2 + t_2^2)\sqrt{3} - 2it_1t_2 \cos\theta}{2\sqrt{2}t_G t_L \cos^2\theta}, \quad (2)$$

where  $\theta$  is the angle of incidence of the wave on the boundary,  $t_{1,2,L,G}$  are the values of the overlap integral in accordance with Fig.1.

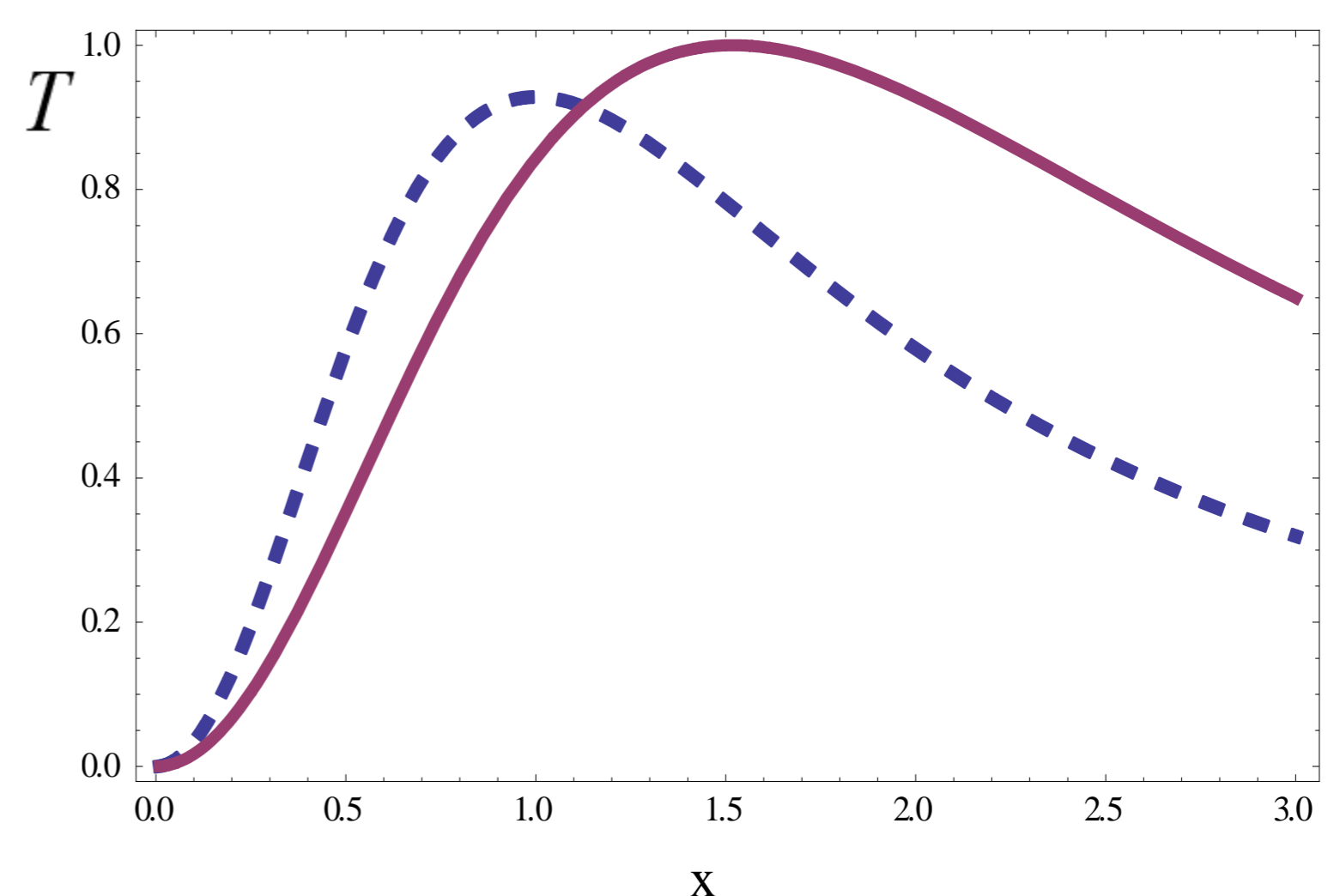


Fig.2 Transmission coefficient  $T$  for  $\theta=0$  as the function  $t_1=t_2=x$  (dashed line) and for  $t_1=0, t_2=x$  (solid line).

When one of the two overlap integrals that describe the boundary vanishes, the possibility of supertunneling arises. Thus, for a normally incident wave, supertunneling occurs for  $t_1=0, t_2=2t_L/\sqrt{3}$ .

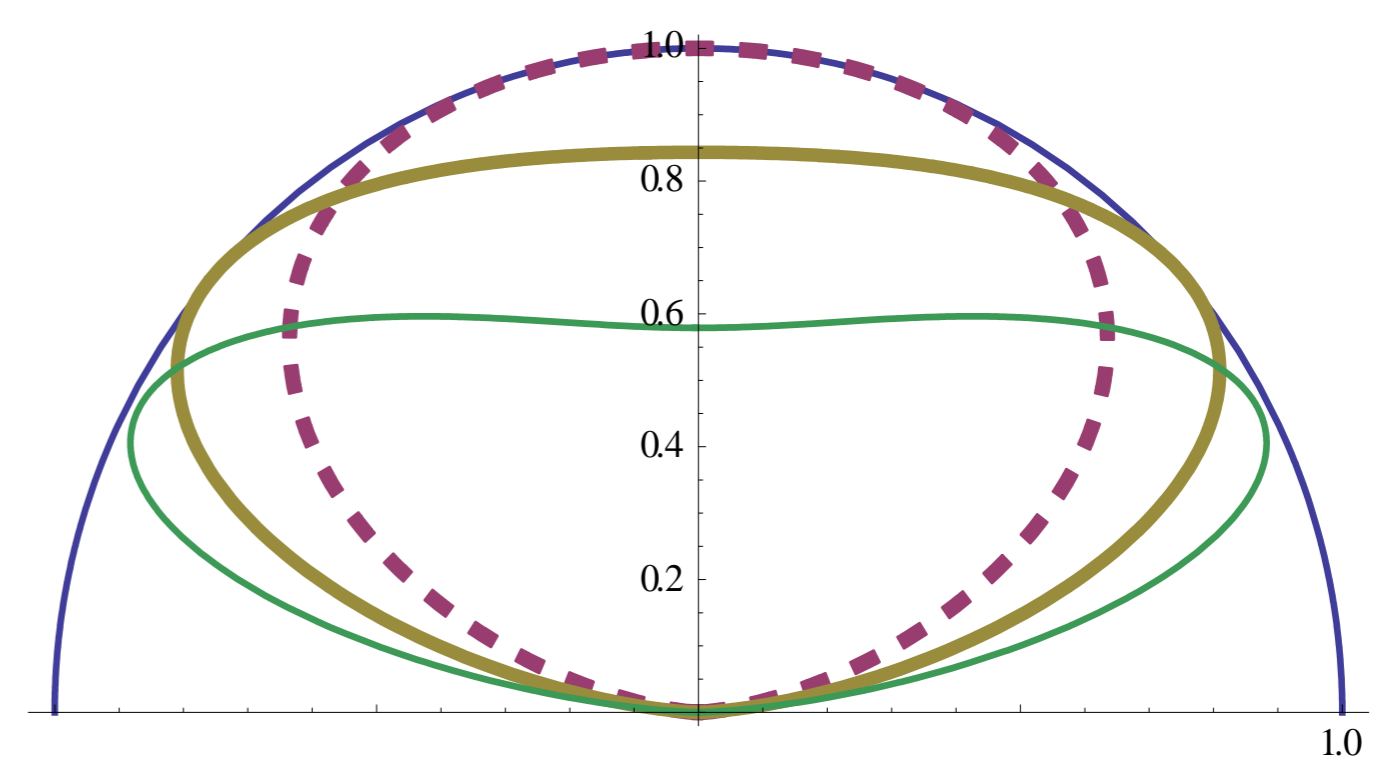


Fig.2. Angular dependence of the transmission coefficient  $T(\theta)$  for  $(t_1, t_2) = (t_L, t_L)$  (solid line),  $(2t_L/\sqrt{3}, 0)$  (dashed), and  $(t_L/2, t_L/2)$  (thin).

The research was carried out within the fundamental scientific program No. 0122U001501, NAS Ukraine.

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