

# Distinguishing transport characteristics of ferromagnetic metal–magnetic quantum dot–superconductor (F-mQD-SC) nanoscale structures

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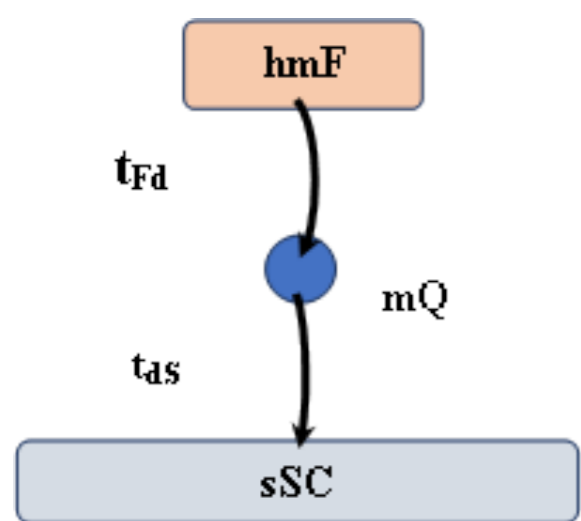
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Spin-dependent resonant tunneling through a quantum dot (QD), a small system characterized by discrete electronic states, coupled with a ferromagnetic normal metal (F) and an s-wave superconductor (S), the F-QD-S system, has been a subject of significant experimental and theoretical interest. Beyond their fundamental importance, such hybrid meso-nanostructured systems offer potential applications in future electronic devices that exploit both the charge and spin degrees of freedom of electrons. This motivated us to investigate nonequilibrium electron tunneling through a ferromagnetic normal metal–magnetic quantum dot–s-wave superconductor (F-mQD-SC) Ref. [1]. We assumed that the mQD has spin-split discrete levels (magnetic QD, mQD), with this splitting controllable by an external magnetic field. Using the Keldysh nonequilibrium Green's function method, we derived and analyzed expressions for the tunneling current,  $I$ , and the probability of Andreev reflection (AR) versus energy,  $T_A(\omega)$ , for the F-mQD-S structure. It was demonstrated that, in contrast to a system with a non-magnetic QD,  $T_A(\omega)$  exhibits a series of additional peaks caused by the splitting of the mQD levels by an effective (external and proximity-induced) magnetic field [1].

In this work, we have studied the distinguishing transport characteristics of the F-mQD-SC nanoscale structure in a non-topological state. We investigate the behavior of Andreev, quasiparticle, and total currents versus mQD gate voltage and bias voltage at different values of external magnetic field, F-lead current spin polarization, the F-lead linewidth, and the level of mQD spacing.

## Contact structure



$$H_T(\tau) = \sum_{k\sigma} \{ (t_{Fd} a_{k\sigma}^+ d_{i\sigma} + h.c.) + (t_{Sd} e^{ieV_S \tau} b_{k\sigma}^+ d_{i\sigma} + h.c.) \}$$

## Current through a quantum dot

$$I_{tot} = I^{(A)} + I_{q\uparrow} + I_{q\downarrow}$$

The tunnel current can be presented as a sum of three different contributions, where  $I^{(A)}$  arises from the AR processes and  $I_{q\uparrow}$  and  $I_{q\downarrow}$  are the quasiparticle currents with spin “up” (“down”)

$$I^{(A)} = \frac{2e}{\hbar} \int \frac{d\omega}{2\pi} \Gamma_{F\uparrow} \Gamma_{F\downarrow} \times \{ |G_{\uparrow\downarrow}^r(\omega)|^2 [n_{F\uparrow}(\omega + eV_F - eV_S) - n_{F\downarrow}(\omega - eV_F + eV_S)] + |G_{\downarrow\uparrow}^r(\omega)|^2 [n_{F\downarrow}(\omega + eV_F - eV_S) - n_{F\uparrow}(\omega - eV_F + eV_S)] \}, G_{\uparrow\downarrow}^r(\omega) = G_{\uparrow\uparrow}^r(\omega) \frac{i}{2} \Gamma_S \frac{\Delta}{\sqrt{\omega^2 - \Delta^2}} A_{\uparrow\downarrow}^r(\omega)$$

$$I_{q\uparrow} = \frac{2e}{\hbar} \int \frac{d\omega}{2\pi} \Gamma_{F\uparrow} \Gamma_S \tilde{\rho}_S(\omega) \{ |G_{\uparrow\uparrow}^r(\omega)|^2 + |G_{\uparrow\downarrow}^r(\omega)|^2 - \frac{2\Delta}{|\omega|} \text{Re}[G_{\uparrow\uparrow}^r(\omega) G_{\uparrow\downarrow}^r(\omega)^*] \times [n_{F\uparrow}(\omega + eV_F - eV_S) - n_S(\omega)] \},$$

$$I_{q\downarrow} = \frac{2e}{\hbar} \int \frac{d\omega}{2\pi} \Gamma_{F\downarrow} \Gamma_S \tilde{\rho}_S(\omega) \{ |G_{\downarrow\downarrow}^r(\omega)|^2 + |G_{\downarrow\uparrow}^r(\omega)|^2 - \frac{2\Delta}{|\omega|} \text{Re}[G_{\downarrow\downarrow}^r(\omega) G_{\downarrow\uparrow}^r(\omega)^*] \times [n_{F\downarrow}(\omega + eV_F - eV_S) - n_S(\omega)] \}$$

$$G_{\uparrow\uparrow}^r(\omega) = \left[ \left( \sum_i \frac{1}{\omega - \epsilon_{i\downarrow}(h) + eV_F - eV_g} \right)^{-1} + \frac{i}{2} \Gamma_{F\downarrow} + \frac{i}{2} \Gamma_S \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} \right]^{-1}$$

$$G_{\uparrow\downarrow}^r(\omega) = G_{\downarrow\downarrow}^r(\omega) \frac{i}{2} \Gamma_S \frac{\Delta}{\sqrt{\omega^2 - \Delta^2}} A_{\uparrow\uparrow}^r(\omega)$$

$$G_{\downarrow\downarrow}^r(\omega) = \left[ \left( \sum_i \frac{1}{\omega - \epsilon_{i\downarrow}(h) + eV_F - eV_g} \right)^{-1} + \frac{i}{2} \Gamma_{F\downarrow} + \right.$$

$$\left. \frac{i}{2} \Gamma_S \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} + \frac{\Gamma_S^2}{4} \Gamma_S \frac{\Delta^2}{\omega^2 - \Delta^2} A_{\uparrow\uparrow}^r(\omega) \right]^{-1}$$

$$A_{\uparrow\uparrow}^r(\omega) = \left[ \left( \sum_i \frac{1}{\omega - \epsilon_{i\uparrow}(h) + eV_F - eV_g} \right)^{-1} + \frac{i}{2} \Gamma_{F\uparrow} + \frac{i}{2} \Gamma_S \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} \right]^{-1}$$

The resonant Andreev reflection probability  $T_A(\omega)$  vs the energy. The dependence on the spin-splitting of the mQD levels under the effect of the effective (proximity induced and external) Zeeman energy  $E_{Ze}$  (Fig. 1) and on the gate voltage  $V_g$  (Fig. 2).

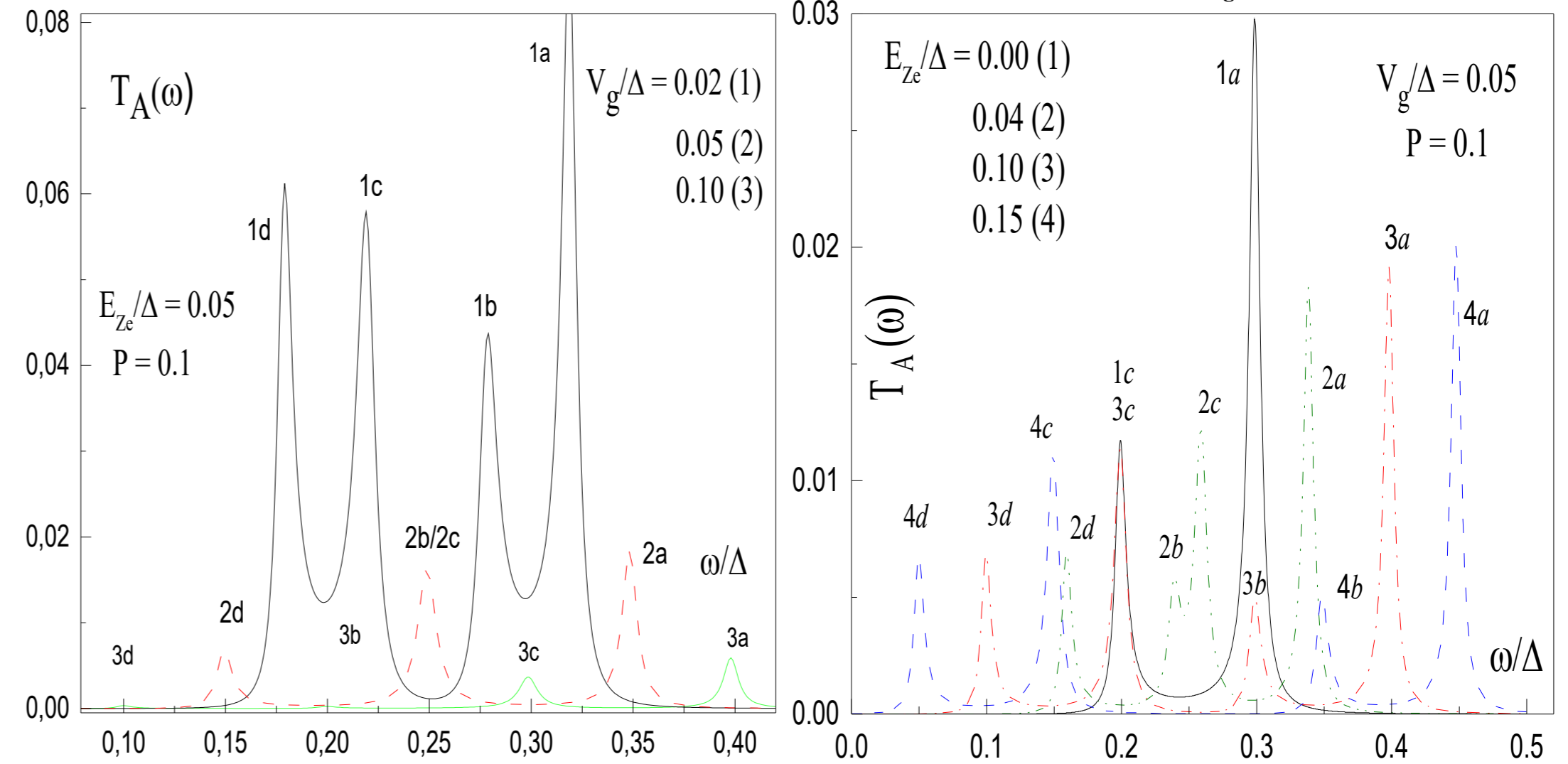


Fig.1. The mQD has two states  $\epsilon_1^0 = 0.25\Delta$ ,  $\epsilon_2^0 = -0.25\Delta$ ,  $P = 0.1$ ,  $V_g = 0.05\Delta$  and  $E_{Ze} = 0.0-0.15\Delta$ ,  $\Gamma_{F0} = \Gamma_S = 0.01$ .

Fig.2. The mQD has two states  $\epsilon_1^0 = 0.25\Delta$ ,  $\epsilon_2^0 = -0.25\Delta$ ,  $P = 0.1$ ,  $V_g = 0.02\Delta-0.1\Delta$  and  $E_{Ze} = 0.05\Delta$ ,  $\Gamma_{F0} = \Gamma_S = 0.01$ .

The tunnel current  $I$  versus the gate voltage  $V_g$  of mQD behaviour for  $V < \Delta$  (Fig. 3 and 4) and for  $V > \Delta$  (Fig. 5 and 6).

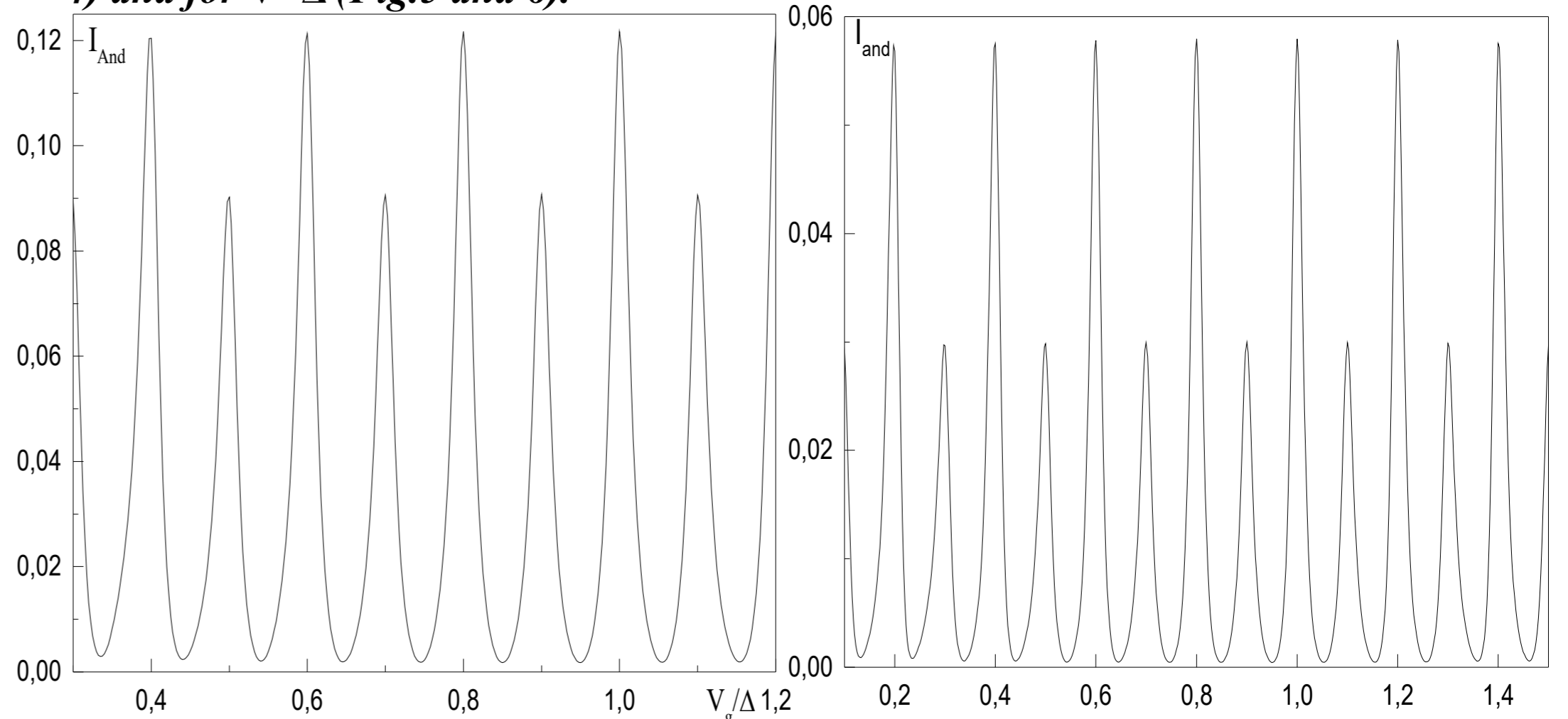


Fig.3. The current  $I$  vs the gate voltage  $V_g$  for  $V < \Delta$ . The mQD has ten states  $\epsilon_1^0 = 0$ ,  $\epsilon_i = \epsilon_1^0 + (i-1)\Delta\epsilon$ ,  $\Delta\epsilon = 0.2\Delta$ ,  $P = 0.1$ ,  $V = 0.02\Delta$  and  $E_{Ze} = 0.1\Delta$ ,  $\Gamma_{F0} = \Gamma_S = 0.01$ .

Fig.4. The current  $I$  vs the gate voltage  $V_g$  for  $V < \Delta$ . The mQD has ten states  $\epsilon_1^0 = 0$ ,  $\epsilon_i = \epsilon_1^0 + (i-1)\Delta\epsilon$ ,  $\Delta\epsilon = 0.2\Delta$ ,  $P = 0.1$ ,  $V = 0.25\Delta$  and  $E_{Ze} = 0.1\Delta$ ,  $\Gamma_{F0} = \Gamma_S = 0.01$ .

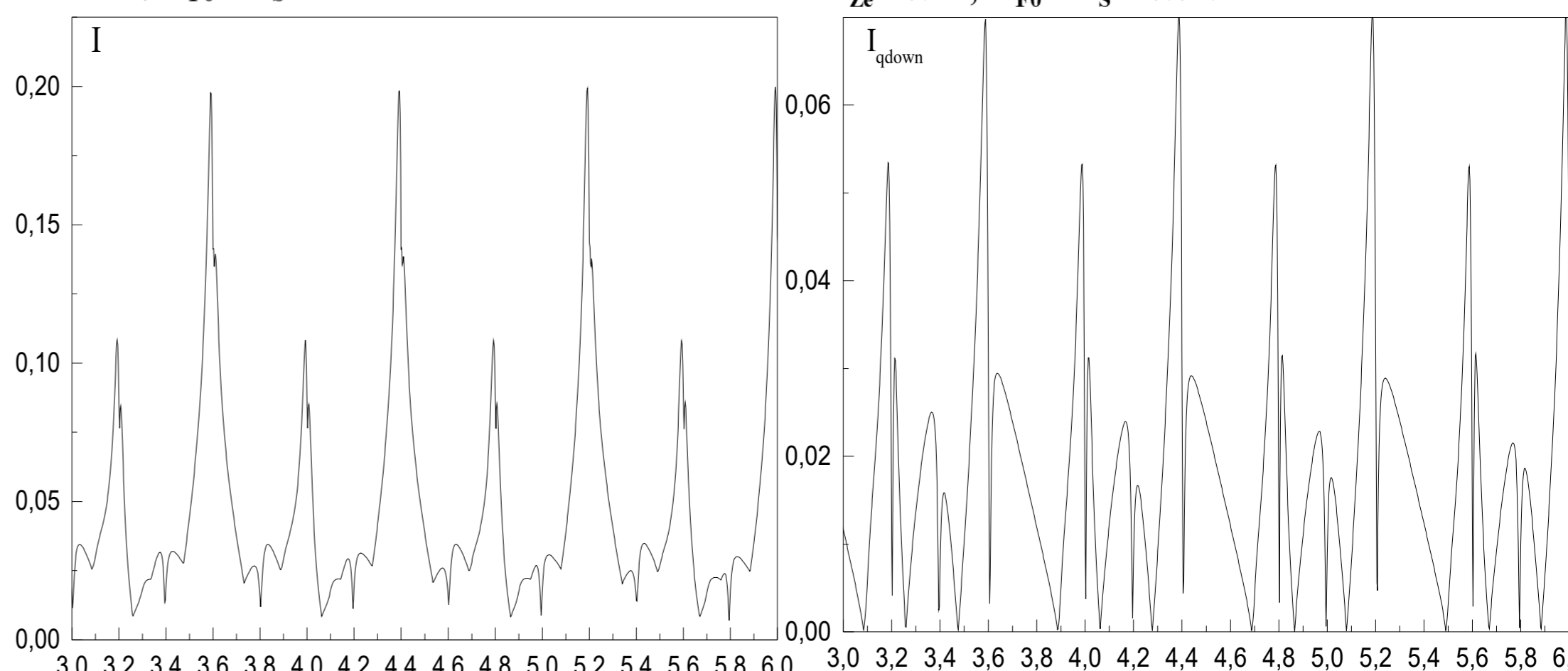


Fig.5. The total current  $I$  vs the gate voltage  $V_g$  for  $V > \Delta$ . The mQD has ten states  $\epsilon_1^0 = 0$ ,  $\epsilon_i = \epsilon_1^0 + (i-1)\Delta\epsilon$ ,  $\Delta\epsilon = 0.8\Delta$ ,  $P = 0.1$ ,  $V = 1.05\Delta$  and  $E_{Ze} = 0.1\Delta$ ,  $\Gamma_{F0} = \Gamma_S = 0.01$ .

Fig.6. The quasiparticle current  $I_{qdown}$  for one spin zone vs the gate voltage  $V_g$  for  $V > \Delta$ . The mQD has ten states  $\epsilon_1^0 = 0$ ,  $\epsilon_i = \epsilon_1^0 + (i-1)\Delta\epsilon$ ,  $\Delta\epsilon = 0.8\Delta$ ,  $P = 0.1$ ,  $V = 1.05\Delta$  and  $E_{Ze} = 0.1\Delta$ ,  $\Gamma_{F0} = \Gamma_S = 0.01$ .

## Conclusions

In this report, we discuss the electron transport through the F-mQD-SC nanoscale structure in a non-topological state. We investigate the behavior of Andreev, quasiparticle, and total currents. We found that resonant curves for tunnel current versus gate voltage of mQD have different kinds of peaks depending on (i) the value of external magnetic field and spin-splitting under the effect of the Zeeman energy, (ii) bias voltage, (iii) F-lead current spin polarization, (iv) the F-lead linewidth, and (v) the level of mQD spacing. Also, the current  $I$  vs bias voltage  $V$  behavior at different values of external magnetic field, gate voltage, F-lead current spin polarization, the F-lead linewidth, and the level of mQD spacing was studied. The results open a way for exploring such systems in tunable spintronic we considered the tunneling transport peculiarities of a ferromagnetic metal-magnetic quantum dot-superconductor hybrid structure.