Field-controlled thermal spin transport in rutile-type altermagnets

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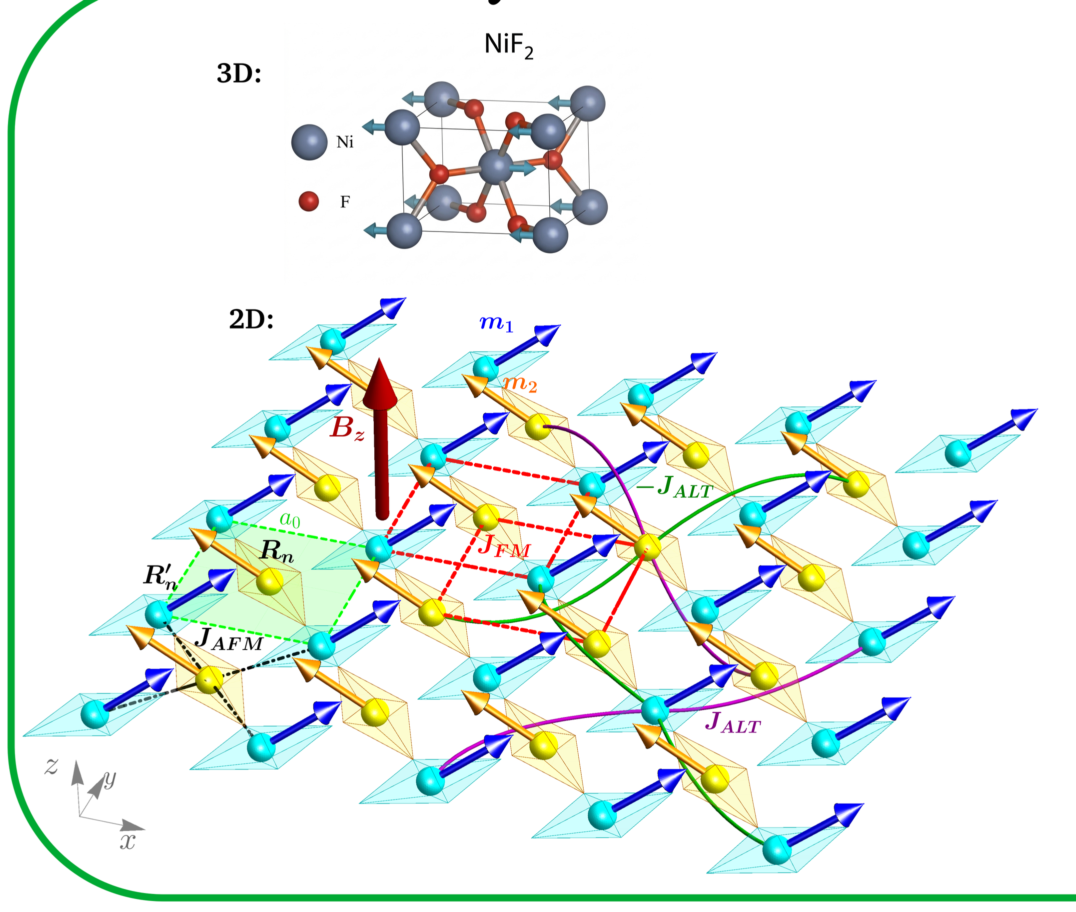
Model of Easy-Plane d-wave altermagnet



Hamiltonian of the system:

$$\mathcal{H} = \mathcal{H}_{\mathrm{AFM}} + \mathcal{H}_{\mathrm{ALT}} + \mathcal{H}_{\mathrm{FM}} + \mathcal{H}_{\mathrm{AN}} + \mathcal{H}_{\mathrm{Z}}$$

$$\mathcal{H}_{\mathrm{AN}} = K\left[\sum_{\boldsymbol{R_n}} m_{1z}^2(\boldsymbol{R_n}) + \sum_{\boldsymbol{R_n}} m_{2z}^2(\boldsymbol{R_n})\right]$$

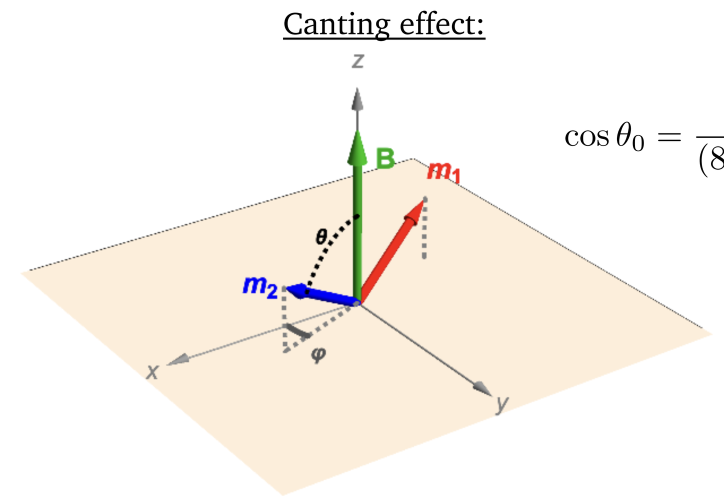
$$\mathcal{H}_{\mathrm{AFM}} = J_{\mathrm{AFM}} \sum_{\langle \boldsymbol{R_n}, \boldsymbol{R'_n}\rangle} \boldsymbol{m}_1(\boldsymbol{R_n}) \cdot \boldsymbol{m}_2(\boldsymbol{R'_n})$$

$$\mathcal{H}_{\mathrm{ALT}} = \frac{J_{\mathrm{ALT}}}{2} \sum_{\boldsymbol{R_n}} \Bigg\{ \sum_{\sigma\in\{\nwarrow,\searrow\}} \left[\boldsymbol{m}_1(\boldsymbol{R_n})\cdot\boldsymbol{m}_1(\boldsymbol{R_n}+2\delta\boldsymbol{R}_\sigma) - \boldsymbol{m}_2(\boldsymbol{R_n})\cdot\boldsymbol{m}_2(\boldsymbol{R_n}+2\delta\boldsymbol{R}_\sigma)\right] - \sum_{\sigma\in\{\nearrow,\swarrow\}} \left[\boldsymbol{m}_1(\boldsymbol{R_n})\cdot\boldsymbol{m}_1(\boldsymbol{R_n}+2\delta\boldsymbol{R}_\sigma) - \boldsymbol{m}_2(\boldsymbol{R_n})\cdot\boldsymbol{m}_2(\boldsymbol{R_n}+2\delta\boldsymbol{R}_\sigma)\right] \Bigg\}$$

$$\mathcal{H}_{\mathrm{Z}} = -B\mu_s\left[\sum_{\boldsymbol{R_n}} m_{1z}(\boldsymbol{R_n}) + \sum_{\boldsymbol{R'_n}} m_{2z}(\boldsymbol{R'_n})\right]$$

$$\mathcal{H}_{\mathrm{FM}} = -\frac{J_{\mathrm{FM}}}{2} \sum_{\boldsymbol{R_n}} \sum_{\sigma\in\{\rightarrow,\leftarrow,\uparrow,\downarrow\}} \left[\boldsymbol{m}_1(\boldsymbol{R_n})\cdot\boldsymbol{m}_1(\boldsymbol{R_n}+\delta\boldsymbol{R}_\sigma) + \boldsymbol{m}_2(\boldsymbol{R_n})\cdot\boldsymbol{m}_2(\boldsymbol{R_n}+\delta\boldsymbol{R}_\sigma)\right]$$

Canting effect:



$$\cos\theta_0 = \frac{B_z\mu_s}{(8J_{AFM} + 2K)}$$

Notations:

- $K > 0$ - ease planar case
- J_{AFM} - antiferromagnetic exchange coefficient
- J_{FM} - ferromagnetic exchange coefficient
- J_{ALT} - coefficient of the altermagnetic exchange contribution

Schrödinger-type Formalism for Spin Waves

Landau-Lifshitz equations

$$\partial_t \boldsymbol{m}_\nu(\boldsymbol{R_n}) = \frac{\gamma}{\mu_s}\left[\boldsymbol{m}_\nu(\boldsymbol{R_n}) \times \frac{\partial\mathcal{H}}{\partial \boldsymbol{m}_\nu(\boldsymbol{R_n})}\right], \qquad \nu = 1, 2$$

Holstein-Primakoff-Tyablikov representation

$$\boldsymbol{m}_\nu(\boldsymbol{R}) \to \psi_\nu(\boldsymbol{R})$$

$$\boldsymbol{m}_\nu = \boldsymbol{m}_\nu^0\left(1 - |\psi_\nu|^2\right) + \sqrt{2 - |\psi_\nu|^2}\left(\boldsymbol{T}_\nu\psi_\nu + \boldsymbol{T}_\nu^*\psi_\nu^*\right)$$

The way to obtain the dispersion relation for magnons

$$i\dot{\hat{\boldsymbol{\Psi}}}_{\boldsymbol{k}} = \hat{\eta}\hat{\mathbb{H}}_{\boldsymbol{k}}\hat{\boldsymbol{\Psi}}_{\boldsymbol{k}} \qquad \hat{\eta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\hat{\boldsymbol{\Psi}}_{\boldsymbol{k}} = [\hat{\psi}_1(\boldsymbol{k}), \hat{\psi}_2(\boldsymbol{k}), \hat{\psi}_1^*(-\boldsymbol{k}), \hat{\psi}_2^*(-\boldsymbol{k})]^{\mathrm{T}}$$

Notations:

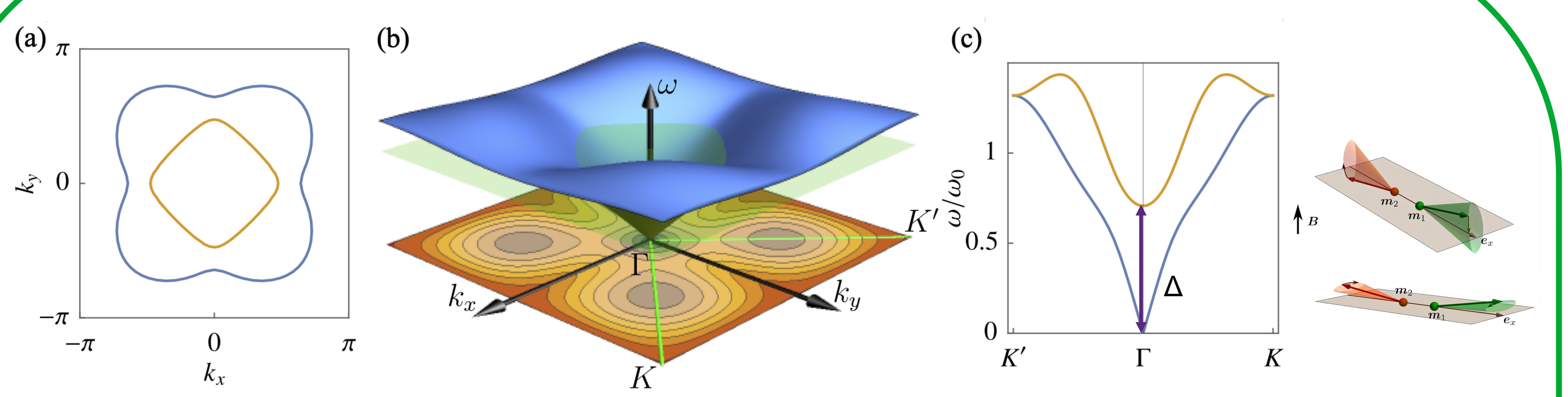
γ is the electron gyromagnetic ratio

$\boldsymbol{\Psi}_k$ is the four-component spinor

$\hat{\mathbb{H}}_k$ is some Hermitian matrix

η is the Pseudo-Euclidean metric.

Magnon Spectrum & d-wave Magnetic Moment



Notations

$\omega_0 = 4J_{\mathrm{AFM}}\gamma/\mu_s$

$b_z = B_z\mu_s/(4J_{\mathrm{AFM}})$

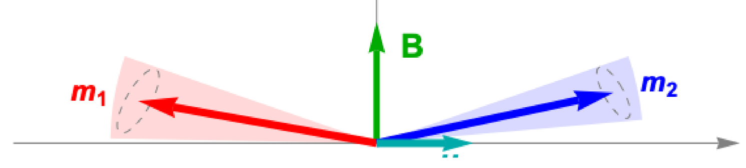
$\kappa = K/J_{\mathrm{AFM}}$

Magnon gap:

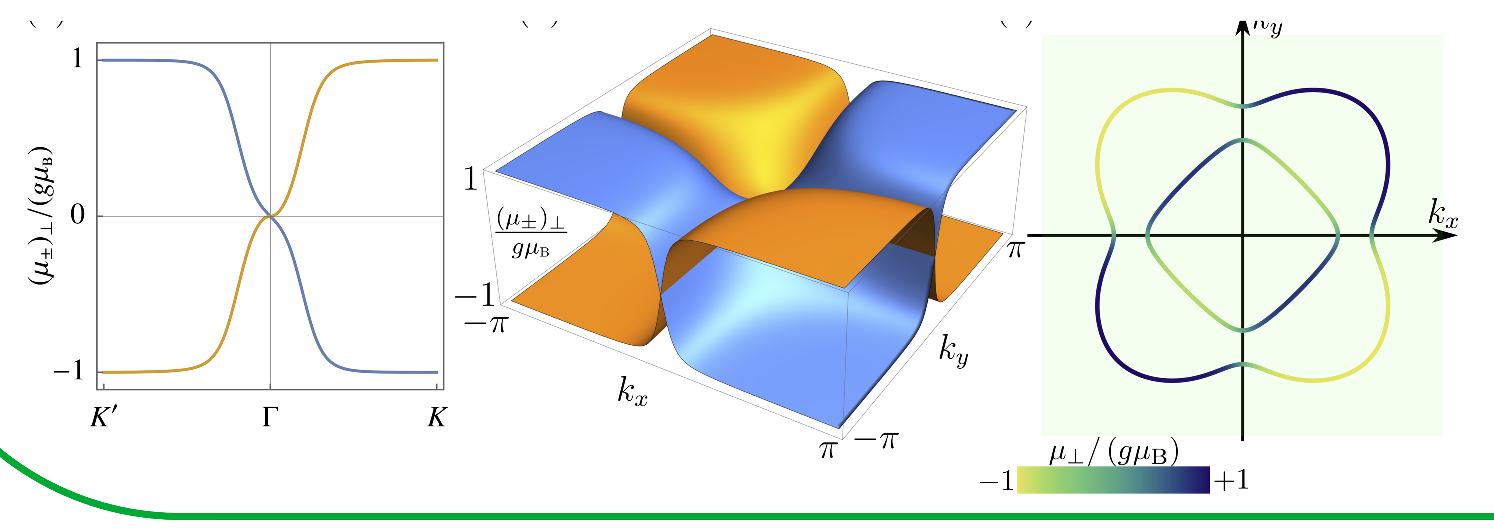
$$\Delta = \sqrt{\kappa - \frac{4b_z^2(\kappa-4)}{(\kappa+4)^2}} \approx \sqrt{\kappa - b_z^2}$$

Magnon magnetic moment:

$$\boldsymbol{\mu}_\nu = -\hbar(\partial\omega_\nu/\partial\boldsymbol{B})$$



$$\mu_\pm = -g\mu_{\mathrm{B}}\varepsilon\cdot\Omega_{\mathrm{ALT}}(\mathbf{k})\cdot\frac{\mathcal{F}(\kappa,\eta,B_z,\theta_0) \pm \Delta\mathcal{F}(\kappa,\eta,B_z,\theta_0)}{\mathcal{G}(\kappa,B_z,\theta_0)\cdot\sqrt{\mathcal{Q}(\kappa,\eta,B_z,\varepsilon,\theta_0) \mp 8\sqrt{\mathcal{R}(\kappa,\eta,B_z,\theta_0)}}}$$



Notations

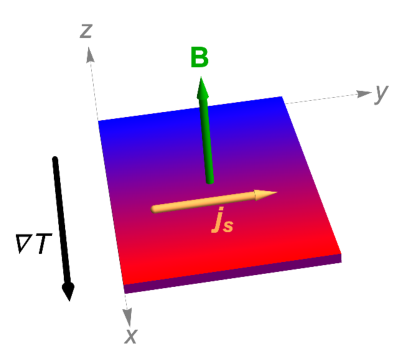
$b_z = B_z\mu_s/(4J_{\mathrm{AFM}})$

$\kappa = K/J_{\mathrm{AFM}}$

$\varepsilon = J_{\mathrm{ALT}}/J_{\mathrm{AFM}}$

$\eta = J_{\mathrm{FM}}/J_{\mathrm{ALT}}$

Spin Transport: Kinetic Approach



Boltzmann theory within the relaxation time approximation

$$j_\alpha = \frac{1}{L_xL_y}\sum_{\mathbf{k},\nu}\mu_{\mathbf{k},\nu}(v_{\mathbf{k},\nu})_\alpha\delta n_{\mathbf{k},\nu} = \sigma_{\alpha\beta}\partial_\beta T$$

The thermal spin conductivity tensor

$$\sigma_{\alpha\beta} = \frac{\tau_{\mathrm{rlx}}}{L_xL_y}\sum_{\mathbf{k}\in 1.\mathrm{BZ}}\sum_{\nu=\pm} c_{\mathbf{k},\nu}(v_{\mathbf{k},\nu})_\alpha(v_{\mathbf{k},\nu})_\beta$$

$$c_{\mathbf{k},\nu} = \mu_{\mathbf{k},\nu}\frac{\partial n^0_{\mathbf{k},\nu}}{\partial T} = \frac{\mu_{\mathbf{k},\nu}E_{\mathbf{k},\nu}}{k_BT^2}\frac{e^{\beta E_{\mathbf{k},\nu}}}{(e^{\beta E_{\mathbf{k},\nu}} - 1)^2}$$

Notations:

j_α - spin current density

$\delta n_{\mathbf{k},\nu} = -\tau_{\mathrm{rlx}}(\mathbf{v}_{\mathbf{k},\nu}\cdot\nabla T)\partial_T n^0_{\mathbf{k},\nu}$ - linear response to thermal gradient.

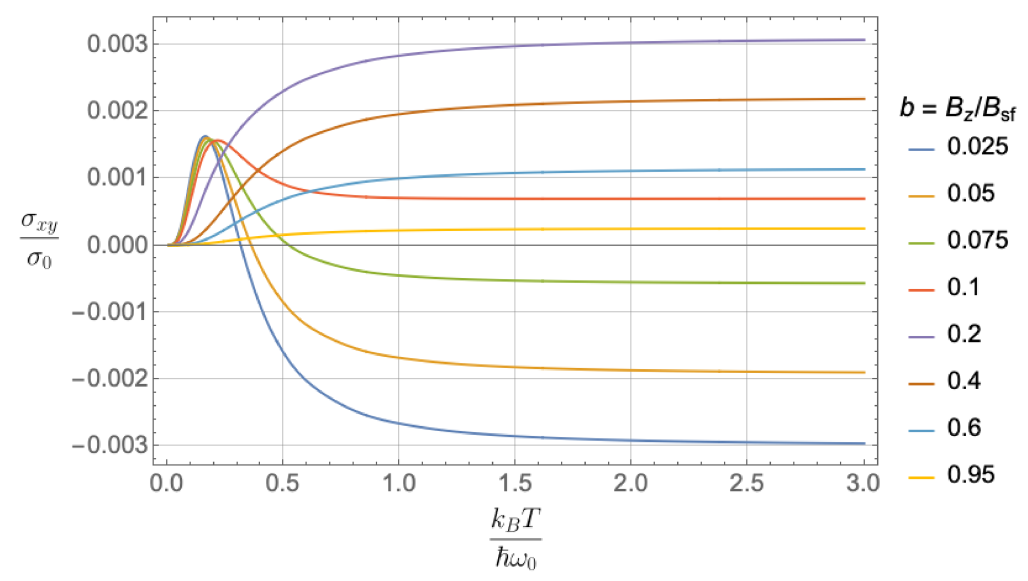
τ_{rlx} - the relaxation time

$n^0_{\mathbf{k},\nu}$ - the equilibrium Bose-Einstein distribution

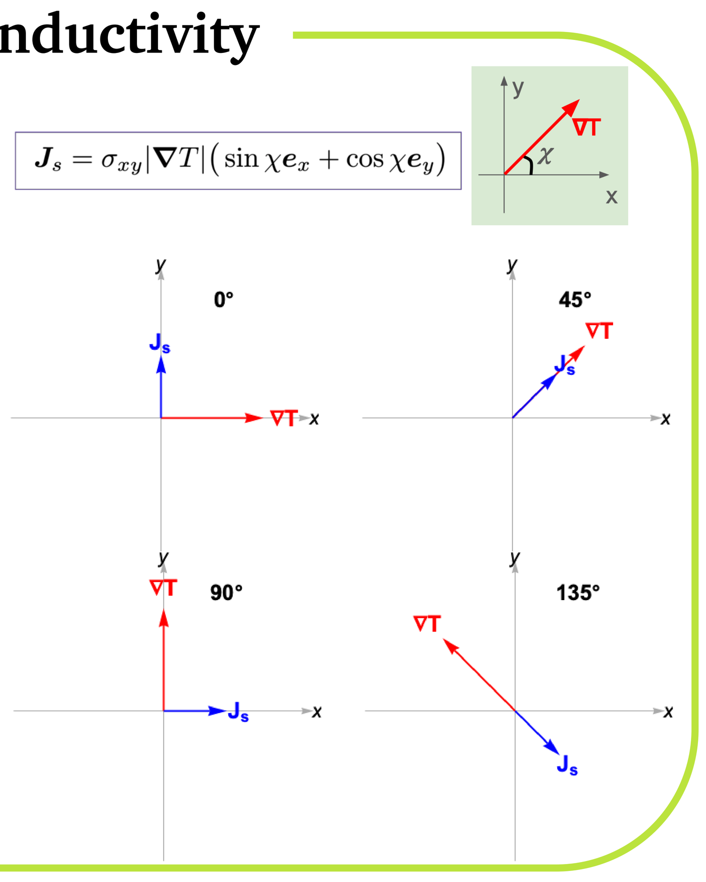
$v_\alpha = \partial\omega_\nu(\mathbf{k})/\partial k_\alpha$ - magnon group velocity

$c_{\mathbf{k},\nu}$ - spin capacity: Thermal derivative of the Bose-Einstein distribution weighted by the magnon magnetic moment.

Temperature Dependencies of spin conductivity



$$\boldsymbol{J}_s = \sigma_{xy}|\boldsymbol{\nabla}T|\left(\sin\chi\boldsymbol{e}_x + \cos\chi\boldsymbol{e}_y\right)$$



Notations

B_{sf} is the critical spin-flip field

$\sigma_0 = \tau_{\mathrm{rlx}}k_B\omega_0 g\mu_B/\hbar$

Temperature Regimes

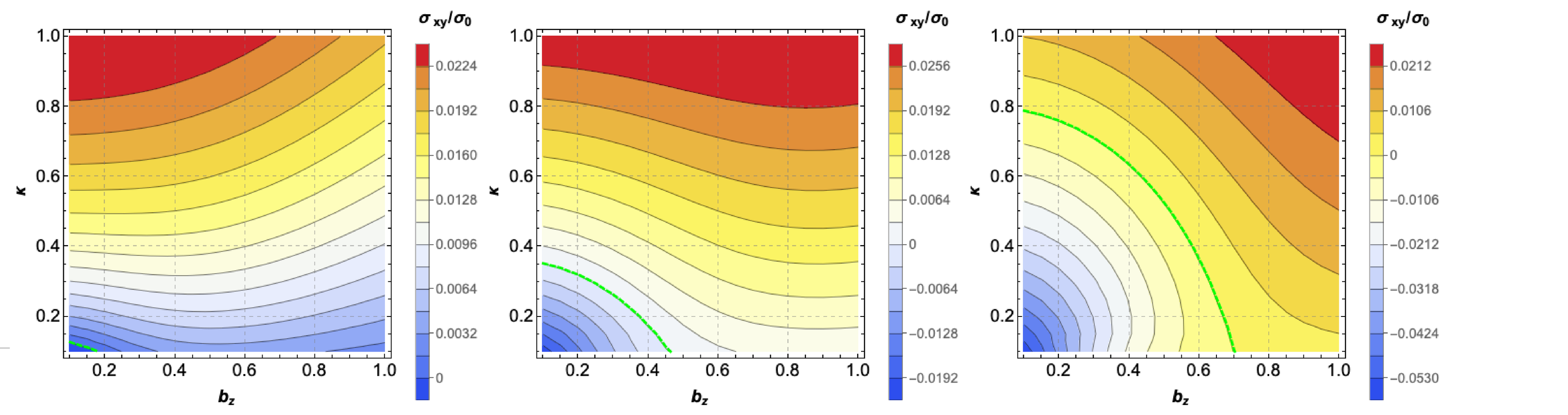
Low temperatures $k_BT \ll \hbar\omega_0$: $\sigma_{xy} \approx \sigma_0\frac{3\zeta(3)}{2\pi}\frac{\varepsilon\sqrt{1 - \frac{4b_z^2}{(4+\kappa)^2}}}{1+2\eta}\left(\frac{k_BT}{\hbar\omega_0}\right)^2$

High temperatures $k_BT \gg \hbar\omega_0$: $\sigma_{xy} \approx \frac{\sigma_0}{(2\pi)^2}\int_{BZ}\sum_{\nu=\pm}\mu_\nu(k)v_{\nu,x}v_{\nu,y}\,d^2k$

Notations

$\varepsilon = J_{\mathrm{ALT}}/J_{\mathrm{AFM}}$ $\quad b_z = B_z\mu_s/(4J_{\mathrm{AFM}})$

$\eta = J_{\mathrm{FM}}/J_{\mathrm{ALT}}$ $\quad \kappa = K/J_{\mathrm{AFM}}$



Map of spin thermal conductivity at **high temperatures** as a function of the magnetic field and the anisotropy coefficient

Conclusions

- Easy planar d wave altermagnets possess momentum-dependent magnon moment μ(k).
- Thermal gradient ∇T induces spin splitter effect of magnons.
- External magnetic field B_z enables efficient control of the spin conductivity.
- **Temperature Regimes:**
  - Low-T (T → 0): No sign reversal; conductivity scales as ~ T².
  - High-T: Temperature independent of spin conductivity; effective control of the thermal spin conductivity by magnetic field.

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