Latest Advances in Theory of Logarithmic Fluids: Popycrystalline Metals and Superfluid Stars

K.G. Zloshchastiev

Institute of Systems Science, Durban University of Technology, Durban 4000, South Africa
kostiantynz@dut.ac.za

An up-to-date review of past, current and future studies of the logarithmic fluid hydrodynamic models is presented. We begin from a pedagogical introduction into a large class of condensate-like strongly-interacting materials and many-body systems, which allow description in terms of a single macroscopic function, at least effectively or in a leading-order approximation. Recently proposed statistical mechanics arguments [1] and previously known Madelung hydrodynamical presentation [2] reveal that the logarithmic nonlinearity occurs in equations describing such matter. From the viewpoint of classical fluid mechanics, the resulting equations describe in the simplest case the irrotational and isothermal flow of a two-phase barotropic compressible inviscid fluid with internal capillarity and surface tension [3]. We demonstrate the emergence of Hilbert space and spontaneous symmetry breaking in this class of fluids, which leads to a number of wave-mechanical and topological effects [4].

The applications of such fluids can be found in both classical and quantum physics. In quantum physics, such fluids can be used for describing strongly-interacting quantum fluids [5-9], including He II, a superfluid component of He-4 [8,9]. In classical realm's applications, one can show that the logarithmic fluid models can be used for describing various Korteweg-type materials, such as magmas in volcanic conduits [3,4]. Logarithmic nonlinearity can be used in hydrodynamic models of metals and alloys, which undergo liquid-solid or liquid-gas phase transitions. One of the predictions of the theory is a periodic pattern of density inhomogeneities occurring in the form of either bubbles (topological phase), or cells (non-topological phase). Such inhomogeneities are described by soliton solutions of the logarithmic wave equation, gaussons and kinks, in the vicinity of the liquid-solid phase transition. During the solidification process, these inhomogeneities become centers of nucleation. The theory thus predicts a Gaussian profile of material density inside such a cell, which should manifest in Gaussian-like profiles of microhardness inside a grain. We report experimental evidence of large-scale periodicity in the structure of grains in the ferrite in steel, copper, austenite in steel, and aluminium-magnesium alloy; and also Gaussian-like profiles of microhardness inside an averaged grain in these materials [10].

Yet another range of applications of the "logarithmic" matter can be found in relativistic astrophysics. One can demonstrate the existence of equilibria in self-gravitating logarithmic fluid, described by spherically symmetric nonsingular finite-mass asymptotically-flat solutions. Unlike other Bose liquid star models known to date, these equilibrium configurations are shown not to have scale bounds for their gravitational mass or size. Therefore, they can describe massive dense astronomical objects, such as bosonized superfluid stars or cores of neutron stars [11].

[1] K.G. Zloshchastiev, Z. Naturforsch. A **73**, 619 (2018).
[2] Y.A. Rylov, J. Math. Phys. **40**, 256 (1999).
[3] S. De Martino, M. Falanga, C. Godano and G. Lauro, Europhys. Lett. **63**, 472 (2003).
[4] K.G. Zloshchastiev, Europhys. Lett. **122**, 39001 (2018).
[5] A. Avdeenkov and K.G. Zloshchastiev, J. Phys. B: At. Mol. Opt. Phys. **44**, 195303 (2011).
[6] B. Bouharia, Mod. Phys. Lett. B **29**, 1450260 (2015).
[7] K.G. Zloshchastiev, Z. Naturforsch. A **72**, 677 (2017).
[8] K.G. Zloshchastiev, Eur. Phys. J. B **85**, 273 (2012).
[9] T.C. Scott and K.G. Zloshchastiev, Low Temp. Phys. 45, 1231 (2019).
[10] M. Kraiev, K. Domina, V. Kraieva and K. G. Zloshchastiev, JPCS **1416**, 012020 (2019).
[11] K.G. Zloshchastiev, FNT **47**, 103 (2021).