Long-time asymptotics for the integrable nonlocal nonlinear Schrödinger equation

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We study the initial value problem for the integrable nonlocal nonlinear Schrödinger (NNLS) equation

$$iq_t(x,t) + q_{xx}(x,t) + 2\sigma q^2(x,t)\bar{q}(-x,t) = 0$$

with decaying (as $x \to \pm\infty$) boundary conditions. Our aim is to describe the long-time behavior of the solution of this problem. In order to do this, we develop, first, the formalism of the Inverse Scattering Transform method in the form of a multiplicative Riemann–Hilbert factorization problem for solving the Cauchy problems for the NNLS equation. Then, using this formalism, we appropriately modify the nonlinear steepest-decent method for studying the long-time solutions of the related Riemann-Hilbert problem, which allows us to obtain explicit formulas for the main term of the asymptotics for the original (Cauchy) problem. Our results show that, in contrast to the case of the conventional (local) nonlinear Schrödinger equation, where the main asymptotic term (in the solitonless case) decays to 0 as $O(t^{-1/2})$ along any ray $x/t = const$, the power decay rate in the case of the NNLS depends, in general, on x/t, and can be expressed in terms of the spectral functions associated with the initial data.