QUANTUM ANALOGUE OF ENERGY EQUIPARTITION THEOREM

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One of the fundamental law of classical statistical physics is the equipartition energy theorem which states that for each degree of freedom the average kinetic energy is equal to $E = kT/2$, where k is the Boltzmann constant and T is temperature of the system. For quantum systems it does not hold. We prove that for a free quantum Brownian particle coupled to thermostat consisting of harmonic oscillators of temperature T, the mean kinetic energy of the Brownian particle is equal to mean kinetic energy per one degree of freedom of the thermostat free oscillators. The averaging is twofold: (i) over the Gibbs canonical state for free oscillators of the thermostat and (ii) over frequencies of those thermostat oscillators which contribute to the Brownian particle energy according to the probability distribution, which is a cosine Fourier transform of the response function which, in turn, solves the quantum generalized Langevin equation for the Brownian particle. We show that this relation can be obtained from the fluctuation-dissipation theorem derived within the linear response theory and is universal in the sense that it holds true for any linear and non-linear systems in contact with bosonic thermostat.

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