V.N. Karazin Kharkiv National University
B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine

International Conference

Geometry, Differential Equations and Analysis

in memory of Aleksei Vasilyevich Pogorelov to emphasize his great contribution to Geometry, Geometric Partial Differential Equations and to celebrate his 100th birthday anniversary

June 17 - 21, 2019

Book of Abstracts

Kharkiv 2019

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Part II. Differential Equations and Analysis

Biographical sketch

A.V. Pogorelov was born on March 3, 1919, in Korocha town near Belgorod (Russia). On his farther's "farm"there was just one cow and one horse. During the collectivization they were taken from him. Once his father came to the collective-farm stable and found his horse exhausted, dying from thirst, while the stableman was drunk. Vasily Stepanovich hit the stableman, a former pauper. This incident was reported as if a kulak (rich man) has beaten a peasant, and Vasily Stepanovich was forced to escape the town, with wife Ekaterina Ivanovna, without even taking the children. A week later Ekaterina Ivanovna has secretly returned for the children. This is how A.V. Pogorelov came to Kharkov, where his father became a construction worker on the building of the tractor factory. A.V.Pogorelov told me the story of how his parents have suffered during the collectivization I have heard from him only in 2000. In my opinion, these events had a strong influence on his life and on the way of his public behavior. He was always very cautious in expressions and liked to quote his mother who kept saying: silence is gold. However, he never did the things contradicting his political views. Several times, he successfully escaped becoming a member of the Communist Party (which was almost compulsory for a person of his scale in the USSR). As far as I know, he never signed any letters of condemnation of dissidents, but, again, any letters in their support, as well. Several times he was elected to the Supreme Soviet of Ukraine (although, as he said later, against his will).

The mathematical abilities of A.V. Pogorelov became apparent already at school. His school nickname was Pascal. He became the winner of one of the first school mathematical competitions organized by the Kharkov University, and then of several All-Ukrainian Mathematical Olympiads. Another talent of A.V.Pogorelov was the painting. The parents did not know, which profession to choose for him. His mother asked the son's mathematics teacher for advice. He had a look at the paintings and said that the boy has brilliant abilities, but in the time of industrialization the painting will not give the resource for life. This advice determined their choice. In 1937, Aleksei Vasilyevich became a student of the Department of Mathematics at the Faculty of Physics and Mathematics of the Kharkov University.

His passion to mathematics immediately drew the attention of the teachers. Professor P.A. Solovjev gave him the book by T. Bonnezen and V. Fenchel "Theory of convex bodies". From that moment and for the rest of his life, geometry became the main interest of Aleksei Vasilyevich. His study was interrupted by the War. He was conscripted and sent to study at the Air Force Zhukovski Academy. But he still thinks about geometry. In August 1943, in a letter to Professor Ya.P. Blank he says: "Very much I regret, that I left in Kharkov the abstracts of Bonnezen and Fenchel on the convex bodies. There are many interesting problems in geometry "in the large"... Do you have any interesting problem of geometry "in the large"or of geometry in general in mind?"

After graduation from the Academy in 1945, A.V. Pogorelov starts his work as a designer engineer at the Central Aero-Hydrodynamic Institute. But the desire to finish his university education (he finished four out of five years) and to work in geometry brings him to Moscow University. A.V. Pogorelov asks academician I.G. Petrovsky, the head of the Department of Mechanics and Mathematics, whether he can finish his education. When Petrovski learnt that Aleksei Vasilyevich has already graduated from the Zhukovski Academy, he decided that there was no need in the formal completion of the university. When A.V. Pogorelov expressed his interest in geometry, I.G. Petrovski advised him to contact V.F. Kagan. V.F. Kagan asked, what area of geometry was Aleksei Vasilyevich interested in, and the answer was: convex geometry. Kagan said that this is not his field of expertise and suggested to contact A.D. Aleksandrov who was in Moscow at that time preparing to a mount climbing expedition at the B.N. Delone apartment (A.D. Aleksandrov was a Master of Sports on mount climbing, and B.N.Delone was the pioneer of Soviet mount climbing).

The first audition lasted for ten minutes. Sitting on a backpack, A.D. Aleksandrov asked Aleksei Vasilyevich the following question: *is it true, that on a closed convex surface of the Gauss curvature $K \leq 1$, any geodesic segment of lengths at most π is minimizing?* It took A.V. a year to answer this question (in affirmative) and to publish the result in 1946. The multidimensional generalization of his theorem is a well-known theorem of Riemannian geometry, which was proved in 1959 by W. Klingenberg: *on a complete simply connected Riemannian manifold M^2n of sectional curvature satisfying $0 < K_\sigma \leq \lambda$, a geodesic of the length $\leq \pi/\sqrt{\lambda}$ is minimizing.* In the odd-dimensional case, one needs a two-sided bound for the curvature to obtain the same result, namely $0 < \frac{1}{4}\lambda \leq K_\sigma \leq \lambda$ (and the inequality cannot be improved).

Few years ago, I asked Aleksei Vasilyevich, why the Soviet mathematicians at that time showed not much interest to the global Riemannian geometry. He answered: "We had enough interesting problems to think about". However, as V.A. Toponogov told me later, the first person who appreciated his comparison theorem for triangles in a Riemannian space was A.V. Pogorelov (in my opinion, it would be more correct to call this theorem the Aleksandrov-Toponogov theorem, since A.D. Aleksandrov discovered and proved it for general convex surfaces in the three-dimensional Euclidean space).

Aleksei Vasilyevich became a postgraduate-in-correspondence at Moscow State University under the supervision of professor N.V. Efimov. Having read the manuscript of the A.D. Aleksandrov's book "Intrinsic geometry of convex surfaces he starts his work in the geometry of general convex surfaces.

One of the main roles of a supervisor, in the opinion of N.V. Efimov, was to inspire a post-graduate student to solving difficult and challenging problems. I gave numerous talks both at the N.V. Efimov's and the A.V.Pogorelov's seminars. They were very different by style. The N.V.Efimov's seminar was long gathered, then the talk lasted for two hours or more, and the talk was always praised very warmly, so it was almost impossible to understand the real value of the result. A.V. always started on time, very punctually. The report lasted for at most an hour. A.V. did not like to go through the details of the proof(probably because in many cases, after the theorem was stated, he could prove it immediately).

In the estimation of the results he was strict and even severe. For example, in 1968, three applicants for the Doctor degree presented their theses at the Pogorelov's seminar in Kharkov. He supported only one of them, V.A. Toponogov, and rejected the other two, who went to Novosibirsk to A.D. Aleksandrov. All three theses were later successfully defended.

A.V. praised rarely, but when he did – that meant that the result was really good. He had a very fast thinking, an enormous geometric intuition, and grasped the essence of the result very fast. Many seminar participants were afraid to ask questions not to look foolish.

In 1947, A.V. Pogorelov defended his Candidate thesis. The main result of his thesis was the following theorem: every general closed convex surface possesses three closed quasi-geodesics. This theorem generalizes the Lusternik - Shnirelman theorem on the existence of three closed geodesic on a closed regular convex surface (a quasi-geodesic is a generalization of a geodesics; both the left and the right "turns"of a quasi-geodesic are nonnegative; for instance, the union of two generatrices of a round cone dividing the cone angle in two halves is a quasi-geodesic).

After defending his Candidate thesis, A.V. discharges from the military service and moves to Kharkov (probably, this was not an easy thing to do at that time: he was discharges by the same Order of the Defence Minister, as the son of M.M. Litvinov, the former Soviet Minister for Foreign Affairs). In one year, he defends his Doctor Thesis on the unique determination of a convex surface of bounded relative curvature. Soon after that, he proves the theorem on the unique determination in the most general settings.

Until 1970, A.V. Pogorelov lectured at Kharkov University. Based on this lecture notes, he published a series of brilliant textbooks on analytic and differential geometry and the foundation of geometry. Sometimes, during routine lectures, he was thinking about his research. Anecdote says that on one of such lectures reflecting on something completely different he started improvising and became lost. Then he opened the textbook with the words: "What does the author say on the topic? Oh, yes, it is obvious . . . ". In contrast, when lecturing on a topic interesting to him, A.V. Pogorelov was very enthusiastic and inspired (I remember one of his topology courses for the 4th year students). But perhaps the best of his lecturing brilliance was seen when he was presenting his own results. His talks were real fine art performances. In his opinion, one of the most valuable qualities of a mathematical result is its beauty and naturalness. That is why he usually omitted technicalities, and for the sake of simplicity and beauty was ready to sacrifice the generality.

A.V. Pogorelov was the author of one of the most popular school textbooks in geometry. This began as follows. He was a member of the commission on the school education whose head was A.N. Kolmogorov. A.V. disagreed with the textbook written by A.N.Kolmogorov and his coauthors and wrote his own manual for teachers on elementary geometry, in which he built the whole school geometry course starting with a set of natural and intuitive axioms. The manual was published in 1969 and formed a basis for his school textbook. A.V. used to say: "My textbook is the Kiselyov's improved textbook"("Elementary geometry"by A.P. Kiselyov is probably the most well-known Russian-language school geometry textbook; it was first published in 1892, with the last edition in 2002; many generations of students studied the Kiselyov's "Geometry"). The first version of the A.V. Pogorelov's textbook sparked sharp criticism from A.D. Aleksandrov whom Pogorelov deeply respected. This criticism was based on implementing the axiomatic approach as early as in year six at school: "What is the point to prove 'obvious' statements (from the student's point of view)?". After reworking of the textbook, these disagreements were resolved, and they remained in strong friendship till the last days of A.D. Aleksandrov.

Aleksei Vasilyevich was a person of the highest decency. When a five year contract with the "Prosvescheniye"Publisher was coming to an end, another publisher offered a very tempting contract to him. He refused on the unique ground that it will be unfair to the editor of the textbook. It should be noted that the money for the school textbook republishing were the main source of his living in the middle of the 90th.

A.V. Pogorelov told me that I.G. Petrovsky invited him to the Moscow University, I.M. Vinogradov invited to Moscow Mathematical Institute, A.D. Aleksandrov invited to Leningrad several times. He even spent one year (1955-1956) in Leningrad, but then returned to Kharkov. He preferred to stay in Kharkov, far from the fuss and noise of the capitals. In Kharkov he proved his theorems, and to Moscow and Leningrad he went to shine.

Aleksei Vasilyevich Pogorelov was a person blessed by an incredible natural talent combined with a constant tireless labor.

Alexander Borisenko

Part I. GEOMETRY

Action of the Monge-Ampere operator on polynomials

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We consider the Monge-Ampere equation of the simplest kind

$$\nabla_{22} z = z_{xx} z_{yy} - z_{xy}^2 = f(x, y)$$

with given polynomial $f(x, y)$ in right hand part.It is natural question to construct solution $z(x, y)$ also in the form of some polynomial. The difficulties at the solution of this problem begin for polynomial f of the 4-th degree.

The space of all uniform polynomials of the 4-th degree can be considered as Euclidean Space E^5. Under action of the operator ∇_{22} it passages into itself.

We obtain necessary and at some cases sufficient conditions on the uniform polynomials f of the 4-th degree for existence of the solution $z(x, y)$ at the form of uniform polynomial of the 4-th degree. We find invariant hypersurfaces in the space E^5 for the operator ∇_{22}in form of generalized cone and quadric, constructed closed chains of polynomials $\nabla_{22} z_i = z_{i+1}$ and proved, that such chain lies on some invariant hypersurface.

The polynomial z is a stable point of ∇_{22} if $\nabla_{22} z = z$. We proved that the polynomial degree of a stable point can be only 4. The space of general polynomials (not only uniform) of the degree 4 can be considered as Euclidean space E^{15}. We find the set of all polynomial stable points of ∇_{22} and proved that this set is some 4-dimensional submanifold in E^{15}.

The work is a development of the articles [1, 2]

1. Yu. Aminov, *Polynomial solutions of the Monge-Ampere equation.* Sbornik: Mathematics, **205**:11 (2014), 1529-1563.

2. Yu. Aminov, K. Arslan, B. Bayram, B. Bulca, C. Murathan, G. Öztürk *On the solution of the Monge-Ampere equation $Z_{xx} Z_{yy} - z_{xy}^2 = f(x, y)$ with quadratic right side*, Zh. Mat. Fiz. Anal. Geom. **7**:3 (2011), 203-211.

On PL-embeddings of S^2 into E^4

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The following question was posed by Yu.B. Zelinsky [1]:

Question: *Does there exist a 2-convex embedding of the two-dimensional sphere S^2 into the four-dimensional Euclidean space E^4?*

Recall that the embedding $i : C \to E^n$ is called m - convex if through each point $x \in E^n \setminus i(C)$ passes the m -dimensional plane π such that $i(C) \cap \pi = \emptyset$. Note that the ordinary notion of convexity corresponds to the case of $m = n - 1$.

Previously, we proved that there is no 2-convex C^2 -smooth embedding of S^2 in E^4 [2]. We partially generalize this result to the class of piecewise linear (PL) embeddings, i.e. embeddings whose image is a polyhedron in E^4 that homeomorphic to the two-dimensional sphere.

We proved the following theorem:

Theorem. *There is no 2-convex PL - embedding of S^2 into E^4 such that each vertex is incident with no more than 5 edges.*

1. Yu.B. Zelinskii *Convexity. Selected topics.* Kiev: Institute of mathematics NASU, 2012. (in Russian)
2. D.V. Bolotov *On embeddings S^2 into E^4*, Reports of NASU, **11** (2013), 19–22. (in Russian)

Bounded harmonic functions on negatively curved manifolds

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The existence of non-constant bounded harmonic functions on simply connected manifolds with negative curvature bounded away from zero is still an unsolved problem. In this talk we review the known result and methods. We will focus on a construction of a manifold where the Dirichlet problem is not solvable but still the manifold supports a lot of bounded non-trivial harmonic functions. It will be shown that this is not a unique phenomena, but a large class of manifolds of this type supports non-constant harmonic functions.

On cylindricity of submanifolds of nonnegative Ricci curvature in a Minkowski space

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By the Splitting Theorem of Cheeger-Gromoll, a complete Riemannian manifold M^n of non-negative Ricci curvature which contains a *line* (a complete geodesic every arc of which minimises the distance between its endpoints) is the Riemannian product $N^{n-1} \times \mathbb{R}$ (under the assumption of non-negativity of the sectional curvature, the same result was proved by Toponogov). The counterpart of this result in submanifold geometry is as follows: a complete Riemannian submanifold $M^n \subset \mathbb{R}^{n+p}$ of non-negative Ricci curvature which contains a line (of $\mathbb{R}^{n+p}$) is the cylinder $N^{n-1} \times \mathbb{R} \subset \mathbb{R}^{n+p-1} \times \mathbb{R}$. Similar results, under certain assumptions on the index of relative nullity and the type (these are two integer affine invariants of a point of the submanifold) have been established by Borisenko.

A direct translation of these results to the Finsler settings by replacing the Euclidean ambient space $\mathbb{R}^{n+p}$ by a Minkowski space $\mathbb{M}^{n+p}$ and the Ricci curvature, by the Ricci curvature of the induced Finsler metric on the submanifold $M^n \subset \mathbb{M}^{n+p}$, most likely, does not work. The reason for that is the fact that in Finsler geometry, the connection between the Ricci (or the flag) curvature of a submanifold and its shape is much weaker than that in Riemannian geometry. For example, we constructed a (locally) strictly saddle surface of positive flag curvature in a three-dimensional Minkowski space; moreover, by Burago-Ivanov, any two-dimensional Finsler metric admits a locally saddle isometric immersion in a four-dimensional Minkowski space.

One may impose some restrictions on the ambient Minkowski space: by the result of Borisenko, a submanifold of non-negative Ricci curvature in a *Randers* space which contains a line must be a cylinder; similar conclusion holds under certain assumptions on the index of relative nullity.

We consider submanifolds of non-negative Ricci curvature in arbitrary

Minkowski spaces which contain a line or whose relative nullity index is positive. For hypersurfaces, submanifolds of codimension two or of dimension two, we prove that such a submanifold is a cylinder, under certain conditions on the type (the inertia of the pencil of the second fundamental forms).

Complete classification of simple closed geodesics on regular tetrahedra in Lobachevsky space

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We described all simple closed geodesics on regular tetrahedra in three-dimensional Lobachevsky space.

Properties of closed geodesics on the regular tetrahedron in the hyperbolic space differ from one in Euclidean space. Moreover, the full classification of closed geodesics on regular tetrahedrons in Euclidean space follows from the the regular triangular tiling of the Euclidean plane [1]. In general it is impossible to make a triangular tiling of the Lobachevsky plane by regular triangles.

Theorem 1. On a regular tetrahedron in Lobachevsky space for any coprime integers (p, q), $0 \leq p < q$, there exists unique, up to the rigid motion of the tetrahedron, simple closed geodesic of type (p, q). The geodesics of type (p, q) exhaust all simple closed geodesics on a regular tetrahedron in Lobachevsky space.

The simple closed geodesic of type (p, q) has p points on each of two opposite edges of the tetrahedron, q points on each of another two opposite edges, and there are $(p+q)$ points on each edges of the third pair of opposite edges.

Theorem 2. Let $N(L, \alpha)$ be a number of simple closed geodesics of length not greater than L on a regular tetraedron with plane angles of the faces equal to α in Lobachevsky space. Then there exists a function $c(\alpha)$ such that

$$N(L, \alpha) = c(\alpha) L^2 + O(L \ln L),$$

where $O(L \ln L) \leq CL \ln L$ as $L \to +\infty$, $c(\alpha) > 0$ when $0 < \alpha < \frac{\pi}{3}$ and

$$\lim_{\alpha \to \frac{\pi}{3}} c(\alpha) = +\infty; \quad \lim_{\alpha \to 0} c(\alpha) = \frac{27}{32 (\ln 3)^2 \pi^2}.$$

1. D. B. Fuchs, E. Fuchs, *Closed geodesics on regular polyhedra*, Mosc. Math. J., **7**:2 (2007), 265–279.
2. V. Yu. Protasov, *Closed geodesics on the surface of a simplex*, Sbornik: Mathematics, **198**:2 (2007), 243-260.
3. I. Rivin, *Simple curves on surfaces*, Geometriae Dedicata, **87**:1-3 (2001), 345-360.
4. M. Mirzakhani, *Growth of the number of simple closed geodesics on hyperbolic surfaces*, Annals of Mathematics, **168** (2008), 97-125.

On the reverse isoperimetric problem for uniformly convex bodies

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In this talk we will present results on the reverse isoperimetric problem for uniformly convex bodies. A convex body K is said to be *uniformly convex* if it is convex and the principal curvatures along the boundary ∂K are uniformly bounded, in viscosity sense, by some positive constant λ either from above (λ-*concave* bodies), or from below (λ-*convex* bodies).

The *reverse isoperimetric problem* consists of finding a body of *least volume* among all bodies of given surface area. This is in contrast to the classical isoperimetric problem, when the volume is being maximized instead. For general sets with well-defined volume and surface area the reverse isoperimetric problem is boring. However, for uniformly convex bodies it has a non-trivial solution.

We will start with an overview of some earlier results on the reverse isoperimetric problem for λ-convex and λ-concave bodies (obtained jointly with Alexander Borisenko) and then, in the main part of the talk, we will focus on the most recent (and currently the most comprehensive) result — *the bratwurst theorem* — and its generalization.

The Bratwurst Theorem. *For any $n \geqslant 1$, the λ-sausage body, that is the convex hull of two balls of radius $1/\lambda$, is the unique volume minimizer among all λ-concave bodies of given surface area in $\mathbb{R}^{n+1}$.*

The main part of the talk is based on joint work with Roman Chernov (Jacobs University) and Kateryna Tatarko (University of Alberta).

Metrics of constant positive curvature with conic singularities

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Let S be a compact Riemann surface. We discuss conformal metrics of constant curvature 1 on S with finitely many conic singularities. The question is how many such metrics exist for given S with prescribed singularities and prescribed angles at the singularities. A survey of known results and some new results on this question will be given.

On m-convex hypersurfaces

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Let $\Gamma \subset \mathbb{R}^n$ be a C^2-hypersurface, $1 \leqslant p \leqslant n-1$. The p-curvature of Γ is the p-order elementary symmetric function of the principal curvatures, i.e., the p-trace of curvature matrix. The hypersurface $\Gamma \in R^n$ is m-convex if its p-curvatures, $p = 1, 2, \ldots, m$, are positive.

The notion of m-convexity is a generalization of the classic convexity. It appeared in the late 20th century as a result of a successful application of Gårding cones in the theory of fully nonlinear differential equations in partial derivatives, [1]. Namely, the solvability condition of the Dirichlet problem for m-Hessian equation in a bounded domain $\Omega \subset \mathbb{R}^n$ is expressed in terms of the $(m-1)$-convexity of $\partial\Omega$.

The systematic study of m-convex hypersurfaces is just at the beginning. The most complete overview of accumulated facts and methods is available in the paper [2]. In [3] we study the m-convexity of multidimensional paraboloids and hyperboloids.

This talk is about current studies, examples and applications of m-convex hypersurfaces.

The work was supported by the RFBR grant 18-01-00472.

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2. N.M. Ivochkina, N.V. Filimonenkova, *Differential geometry in the theory of Hessian operators*, arXiv:1904.04157 (2019).
3. N.V. Filimonenkova, P.A. Bakusov, *Analysis of m-convexity of multydimensional paraboloids and hyperboloids*, Matematicheskoe Prosveshchenie. Third series., **21** (2017), 64-86.

Around model flexors

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A non-flexible polyhedron P is called a model flexor, if its physical models behave like physical models of flexible polyhedra so that they admit deformations with almost unobservable / invisible changes in forms and sizes of faces but with essential changes in dihedral angles between faces [1]. The model flexibility may be simulated either by continuous deformations which preserve the combinatorial structure but slightly variate the sizes of faces, or by continuous isometric deformations which modify the combinatorial structure of polyhedra [2]-[5].

We will illustrate how both kinds of deformations may be used to simulate the model flexibility for some particular families of polyhedra (Alexandrov-Vladimirova star-like bipyramids, Milka birosettes, Jessen orthogonal icosahedron, Goldberg siamese dipyramids, Wunderlich antiprisms, etc).

1. A. Milka, *Linear bendings of right convex polyhedra*, Matematicheskaya fizika, analiz, geometriya, **1** (1994), 116-130.
2. A. Milka, *Linear bendings of star-like bipyramids*, European Journal of Combinatorics, **31** (2010), 1050-1064.
3. V. Gorkavyy, D. Kalinin, *On the model flexibility of the Jessen orthogonal icosahedron*, Contributions to Algebra and Geometry, **57** (2016), 607-622.
4. V. Gorkavyy, A. Milka, *Birosettes are model flexors*, Ukrainian Mathematical Journal, **70** (2018), 1022-1041.
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Automorphisms of the Kronrod-Reeb graph of Morse functions on the sphere

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Let M be a compact two-dimensional manifold and, $f \in C^{\infty}M\mathbb{R}$ is Morse's function and Γ_f its Kronrod-Reeb's graph. We denote the $O(f) = \{f \circ h \mid h \in D(M)\}$ orbit of f with respect to the natural right action of the group of diffeomorphisms $D(M)$ on $C^{\infty}M\mathbb{R}$, and $S(f) = \{h \in D(M) \mid f \circ h = f\}$ is the stabilizer of this function. It is easy to show that each $h \in S(f)$ induces an automorphism of the graph Γ_f. Let also $S'(f) = S(f) \cap D_{\text{id}}(M)$ – be a subgroup of $D_{\text{id}}(M)$, consisting of diffeomorphisms preserving f and isotopic to identical mappings and G_f be the group of automorphisms of the Kronrod-Reeb graph induced by diffeomorphisms belonging to $S'(f)$. This group is the key ingredient for calculating the homotopy type of the orbit $O(f)$.

In the previous article, the authors describe the structure of groups G_f for Morse functions on all orientational surfaces, except for sphere and torus. In this paper we study the case $M = S^2$. In this situation Γ_f is always a tree, and therefore all elements of the group G_f have a common fixed $\mathrm{Fix(G_f)}$ subtree, which can be even from one point. The main result is to calculate the groups G_f for all Morse functions $f : S^2 \to \mathbb{R}$ in which $\mathrm{Fix(G_f}$ is not the point.

Theorem 1. *Let $f \in C^\infty(S^2, \mathbb{R})$ be Morse function on a sphere. Suppose that all elements of the group G_f have a common fixed edge E. Let $x \in E$ be an arbitrary point and A and B is the closure of the connected components $S^2 \setminus p^{-1}(x)$. Then A and B-double discs are invariant with respect to $S'_{id}(f)$, the restriction of $f|_A, f|_B$ are Morse functions and we have the following isomorphism:*

$$\phi : G_f \to G_{f_{|A}} \times G_{f_{|B}},$$

is determined by the formula $\phi(\gamma) = (\gamma_{|\Gamma_A};\ \gamma_{|\Gamma_B})$.

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About Aleksandrov's estimates

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We give a brief overview of the role of Aleksandrov's estimates and Pororelov's ideas in our research.

Conformally flat metrics of limited curvature and convex subsets of space of Lobachevsky

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Let $S^2 \subset R^3$ the two-dimensional single sphere in a three-dimensional Euclidean space scentry in origin of coordinates. For S^2 the conformal and flat metrics of a look is set $ds^2 = \exp(-2\sigma(x))dx^2$, $x \in S^2$. Curvature of a metrics of ds^2 it is calculated on a formula

$$K = \exp(2\sigma)\left(1 - \triangle\sigma\right),$$

where $\triangle\sigma$ - Laplace's operator concerning a sphere metrics. The metrics by means of logarithmic potential is restored through curvature

$$\sigma(z) = \int\limits_{S^2} \ln|z-t|^2 d_t K + \frac{1}{4\pi}\int\limits_{S^2} \sigma(t)dt + \ln(4) - 1. \qquad (1)$$

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First Betti numbers of orbits of Morse functions on surfaces

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Let $\mathcal{G}$ be a minimal class of groups satisfying the following conditions: 1) $1 \in \mathcal{G}$; 2) if $A, B \in \mathcal{G}$, then $A \times B \in \mathcal{G}$; 3) if $A \in \mathcal{G}$ and $n \geq 1$, then the wreath product $A \wr_n \mathbb{Z} \in \mathcal{G}$.

It is easy to see that every group $G \in \mathcal{G}$ can be written as a word in the alphabet $\mathcal{A} = \{1, \mathbb{Z}, (,), \times, \wr_2, \wr_3, \wr_4, \dots\}$. We will call such word a *presentation* of the group G in the alphabet $\mathcal{A}$.

Theorem. Let $G \in \mathcal{G}$, ω be an arbitrary presentation of G in the alphabet $\mathcal{A}$, and $\beta_1(\omega)$ be the number of symbols $\mathbb{Z}$ in the presentation ω. Then there are the following isomorphisms for the center and the commutator subgroup of G:

$$Z(G) \cong G/[G,G] \cong \mathbb{Z}^{\beta_1(\omega)}.$$

In particular, the number $\beta_1(\omega)$ depends only on the group G.

The groups from the class $\mathcal{G}$ appear as fundamental groups of orbits of Morse functions on surfaces. Let M be a compact surface and $\mathcal{D}$ be the group of C^∞-diffeomorphisms of M. There is a natural right action of the group $\mathcal{D}$ on the space of smooth functions $C^\infty(M, \mathbb{R})$. Let $\mathcal{O}(f) = \{f \circ h \mid h \in \mathcal{D}\}$ be the *orbit* of f under the above action. Let $\mathcal{O}_f(f)$ denote the path component of f in $\mathcal{O}(f)$.

Homotopy types of stabilizers and orbits of Morse functions were calculated in a series of papers by Sergiy Maksymenko [2] and Elena Kudryavtseva [1]. As a consequence of Theorem 1 we get the following.

Corollary. Let M be a connected compact oriented surface distinct from S^2 and T^2, f be a Morse function on M, $G = \pi_1\mathcal{O}_f(f) \in \mathcal{G}$, ω be an arbitrary presentation of G in the alphabet $\mathcal{A}$, and $\beta_1(\omega)$ be the number of symbols $\mathbb{Z}$ in the presentation ω. Then the first integral homology group $H_1(\mathcal{O}(f), \mathbb{Z})$ of $\mathcal{O}(f)$ is a free abelian group of rank $\beta_1(\omega)$:

$$H_1(\mathcal{O}(f), \mathbb{Z}) \simeq \mathbb{Z}^{\beta_1(\omega)}.$$

In particular, $\beta_1(\omega)$ is the first Betti number of $\mathcal{O}(f)$.

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Are log-concave measures better than log-concave with symmetry?

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In the recent years, a number of conjectures has appeared, concerning the improvement of the inequalities of isoperimetric type under additional assumptions of symmetry; this includes the B-conjecture, the Gardner-Zvavitch conjecture of 2007, the Log-Brunn-Minkowski conjecture of 2011, and some variants. The conjecture of Gardner and Zvavitch, also known as dimensional Brunn-Minkowski conjecture, states that even log-concave measures in $\subset R^n$ are in fact $\frac{1}{n}$- concave with respect to the addition of symmetric convex sets. In this talk we shall prove that the standard Gaussian measure enjoys $\frac{1}{2n}$ concavity with respect to centered convex sets. The improvements to the case of general log-concave measures shall be discussed as well: under certain assumption on the hessian of the potential, we show that an even log-concave measure is indeed better-than-log-concave with respect to the addition of symmetric convex sets. The methods of proof are variational, and the tools come from differential geometry and PDE. This is a joint work with A. Kolesnikov.

Diffeomorphisms preserving Morse-Bott functions

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Let M be a smooth compact manifold and P be either the real line or the circle. Notice also that there is a natural right action $\nu : \mathcal{C}^\infty(M,P) \times \mathcal{D}(M) \to \mathcal{C}^\infty(M,P)$, defined by $\nu(f,h) = f \circ h$ of the groups of diffemorphisms $\mathcal{D}(M)$ of M on the space $\mathcal{C}^\infty(M,P)$ of smooth maps $M \to P$. For $f \in \mathcal{C}^\infty(M,P)$ and a subset $X \subset M$ let

$$S(f) = \{h \in \mathcal{D}(M) \mid f \circ h = f\}, \qquad S(f,X) = S(f) \cap \mathcal{D}(M,X)$$

be the stabilizers of f with respect to the above action of $\mathcal{D}(M)$ and the induced action of $\mathcal{D}(M,X)$. Let also $S_{\mathrm{id}}(f)$ and $S_{\mathrm{id}}(f,X)$ be the identity path components of the corresponding stabilizers.

Theorem.[1] *Let $f : M \to P$ be a Morse-Bott map of smooth compact manifold M, so the set Σ_f of critical points of f is a disjoint union of smooth mutually disjoint closed submanifolds $C_1, \dots, C_k$. Let also $X \subset M \setminus \Sigma_f$ be a closed (possibly empty) subset. Then the maps*

$$\rho\colon S(f,X) \to \mathcal{D}(\Sigma_f), \qquad \rho(h) = h|_{\Sigma_f},$$

$$\rho_0\colon S_{\mathrm{id}}(f,X) \to \mathcal{D}_{id}(\Sigma_f) \equiv \prod_{i=1}^{k} \mathcal{D}_{id}(C_i), \qquad \rho_0(h) = (h|_{C_1}, \dots, h|_{C_k}),$$

are locally trivial fibrations over their images, and the map ρ_0 is surjective.

This result can be regarded as a variant of the well know result Cerf and Palais on local triviality of restrictions to critical submanifolds of Morse-Bott function f for f-preserving diffeomorphisms.

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Diffeomorphisms of the solid torus preserving a codimension one Morse-Bott foliation with one singular circle

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Let

$$T = D^2 \times S^1 = \{(x,y,w) \in \mathbb{R}^2 \times \mathbb{C} \mid x^2 + y^2 \le 1,\ |w| = 1\}$$

be a solid torus, $C_r = \{z \in D^2 \mid |z| = r\} \subset D^2$, $r \in [0,1]$ and

$$\mathcal{F}_{\mathcal{T}} = \{C_r \times S^1\}_{r \in [0,1]}$$

be a foliation on T into 2-tori parallel to the boundary and one singular circle $C_0 \times S^1$, which is the central circle of the torus T.

Denote by $\mathcal{D}(\mathcal{F}_{\mathcal{T}}, \partial T)$ the group of diffeomorphisms of T, which leave each leaf of the foliation $\mathcal{F}_{\mathcal{T}}$ invariant and fixed on T.

Theorem. The group $\mathcal{D}(\mathcal{F}_{\mathcal{T}}, \partial T)$ is contractible.

Solutions To Overdetermined Problems On Riemannian Manifolds

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We consider a Riemannian manifold of dimension greater or equal to two. An overdetermined problem can be defined as a second order partial differen- tial equation on a regular domain for which, both Dirichlet and Neumann boundary conditions are imposed. In the particular case when the boundary data are all con- stant functions, the corresponding problem is called Serrin's overdetermined problem. Moreover, a regular domain where Serrin's overdetermiend problem is solvable is called a Serrin domain.

In this talk, we prove the existence of a family of Serrin domains in any smooth and compact Riemannian manifolds. These domains are obtained by perturbing a small geodesic ball centered at a point of the manifold. Moreover, provided the man- ifold has a non degenerate critical point of the scalar curvature, the family made of boundaries of the Serrin domains constructed constitute a smooth foliation of a neighborhood of this critical point. In the last part of the talk, we also show that Serrin domains are Cheeger sets.

The volume preserving mean curvature flow in the sphere and the Lagrangian mean curvature flow in C^2

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I shall start with the second problem, giving account of a joint result with I. Castro and A. Lerma which studies the evolution, under mean curvature flow, of a Lagrangian surface in C^2 contained in a sphere S^3. We shall see that it is related with the volume preserving mean curvature flow of Hopf tori in S^3 and will introduce to the possible motions by the mean curvature flow of general tori in S^3 which are near the Clifford torus. This last part is a joint work (in progress) with M. C. Domingo-Juan.

Algebraic nonintegrability of magnetic billiards on the sphere and hyperbolic plane

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We consider billiard ball motion in a convex domain on a constant curvature surface influenced by the constant magnetic field. We examine the existence of integral of motion which is polynomial in velocities. We prove that if such an integral exists then the boundary curve of the domain determines an algebraic curve in C^3 which must be nonsingular. Using this fact we deduce that for any domain different from round disc for all but finitely many values of the magnitude of the magnetic field billiard motion does not have Polynomial in velocities integral of motion. Results were obtained with Misha Bialy, Tel Aviv University.

Hypersurfaces Of Spin^c Manifolds With Special Spinor Fields

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In this talk, we prove that every totally umbilical hypersurface $M \geq 4$ of a Riemannian Spin^c manifold carrying a parallel or a real Killing spinor is either a totally geodesic hypersurface or an extrinsic hypersphere. As applications, we prove that there are no extrinsic hyperspheres in complete manifolds with holonomy G_2 or Spin(7) and in some special Sasakian manifolds. This is joint work with Nadine Grosse (University of Freiburg, Germany).

Recent results on homogeneous geodesics and geodesic orbit spaces

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Let (M, g) be a Riemannian manifold and let $\gamma : \mathbb{R} \to M$ be a geodesic in (M, g). The geodesic γ is called *homogeneous* if $\gamma(\mathbb{R})$ is an orbit of an 1-parameter subgroup of $\mathrm{Isom}(M, g)$, the full isometry group of (M, g). A Riemannian manifold (M, g) is called a manifold with homogeneous geodesics or a geodesic orbit manifold if every geodesic γ of M is homogeneous. These definitions are naturally generalized to the case when all isometries are taken from a given Lie subgroup $G \subset \mathrm{Isom}(M, g)$, that acts transitively on M. In this case we get the notions of G-homogeneous geodesics and G-homogeneous geodesic orbit spaces. This terminology was introduced in [6] by O. Kowalski and L. Vanhecke, who initiated a systematic study of such spaces. We refer to [6], [1], [4], and [8] for expositions on general properties of geodesic orbit Riemannian manifolds and historical surveys.

This talk is devoted to recent results related to geodesic orbit spaces and homogeneous geodesics.

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Solutions of the Ermakov-Milne-Pinney Equation and Invariant Constant Mean Curvature Surfaces

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Nonlinear differential equations have been of an increasing interest in Differential Geometry and Physics during the last decades. One of the simplest examples is the today called *Ermakov-Milne-Pinney* (EMP, for short) equation, that is, the second order differential equation

$$x''(t) + q(t)x(t) = \frac{c}{x^3(t)}\,,$$

c being a constant, [3], [4].

On the other hand, *constant mean curvature* (CMC) surfaces have played a prominent role in Analysis and Differential Geometry. In particular, invariant CMC surfaces have rich symmetry which makes them ideal for modeling physical systems.

In this talk, we are going to show a correspondence between solutions of the EMP equation with constant coefficients and invariant CMC surfaces of both Riemannian and Lorentzian 3-space forms, [1]. Moreover, we are also going to prove that, after a suitable manipulation, the EMP equation with constant coefficients represents the Euler-Lagrange equation of the variational problem

$$\mathbf{\Theta}_\mu(\gamma) = \int_\gamma \sqrt{\kappa - \mu}\, ds$$

acting on an adequate space of curves, where κ denotes the curvature of the curve, [1]. This variational problem is an extension of a Blaschke's curvature energy and, as it turns out, it characterizes the profile curves of invariant CMC surfaces, [2].

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Infinitesimal conformal transformations of the second degree in the Riemannian space of the second approximation

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We consider a Riemannian space V_n, related to an arbitrary system of coordinates $\{x^1, x^2, \dots, x^n\}$. In the neighborhood of arbitrary fixed point $M_0(x_0^n)$, we construct the second approximation space $\widetilde{V}_n^2(y^h; \widetilde{g}_{ij}(y))$ by defining its metric tensor $\widetilde{g}_{ij}(y)$ as follows:

$$\widetilde{g}_{ij}(y) = \underset{\circ}{g}_{ij} + \frac{1}{3}\underset{\circ}{R}_{i\alpha\beta j} y^\alpha y^\beta,$$

where $\underset{\circ}{g}_{ij} = g_{ij}(M_0)$, $\underset{\circ}{R}_{i\alpha\beta j} = R_{i\alpha\beta j}(M_0)$.

In the space $\widetilde{V}_n^2$, we consider the following infinitesimal conformal transformations

$$y'^h = y^h + \widetilde{\xi}^h(y)\delta t \tag{1}$$

Definition 1. *The transformations* (1) *are called transformations of the second degree [2], if the displacement vector* $\xi^h(y)$ *is of the following form*

$$\widetilde{\xi}^h(y) = a^h + a^h_{\cdot l} y^l + a^h_{\cdot l_1 l_2} y^{l_1} y^{l_2},$$

where $a^h, a^h_{\cdot l}, a^h_{\cdot l_1 l_2}$ *are some constants.*

We treat the generalized Killing equations [1], namely $\nabla_{(i}\widetilde{\xi}_{j)} = \psi(y)\widetilde{g}_{ij}$, and get the following result.

Theorem 1. *Let* $\widetilde{V}_n^2$ *be a space of second approximation for the Riemannian space* (V_n, K) *of constant curvature* $K \neq 0$. *Then there exist the infinitesimal conformal transformations of the second degree with displacement vector* $\widetilde{\xi}^h(y)$ *of the form*

$$\widetilde{\xi}^h(y) = a^h + a^h_{\cdot l} y^l - \frac{K}{3} a_l y^l y^h,$$

where $a^h_{\cdot l}$ *satisfies the condition* $a^\alpha_{\cdot\,(i}\underset{\circ}{g}_{æ)\alpha} = 0$, *and* $a^h_{\cdot}$ *is an arbitrary constant.*

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One integral inequality for curvatures of closed curves in $\mathbb{R}^n$

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Let γ be an arbitrary smooth closed curve in $\mathbb{R}^n$, $n \geq 4$. Suppose that γ has nowhere vanishing curvatures k_1, k_2, ..., k_j for some $2 \leq j \leq n-1$. Then the following inequality holds true [1]-[3]:

$$\int\limits_{\gamma} \sqrt{k_{j-1}^2 + k_j^2 + k_{j+1}^2}\, ds > 2\pi. \qquad (1)$$

Consequently, a smooth closed curve in $\mathbb{R}^n$ with nowhere vanishing curvatures k_1, k_2, ..., k_{n-1} satisfy a series of $n-2$ integral inequalities.

The inequality (1) is proved to be *sharp* in the case of any odd j. Moreover, the sharpness is provided by curves of constant curvatures if n is even or by their slight modifications if n is odd [2].

As for the case of an even j, the problem of the sharpness of (1) still remains unsolved.

We start to explore the problem by considering the inequality

$$\int\limits_{\gamma} \sqrt{k_1^2 + k_2^2 + k_3^2}\, ds > 2\pi$$

for smooth closed curves with nowhere vanishing curvatures k_1, k_2 in $\mathbb{R}^4$.

The main result states that for curves with constant curvatures $k_1 > 0$, $k_2 > 0$, k_3 in $\mathbb{R}^4$, as well as for their specific modifications, the following sharp estimate holds true:

$$\int\limits_{\gamma} \sqrt{k_1^2 + k_2^2 + k_3^2}\, ds \geq 2\sqrt{5}\pi.$$

It is conjectured that the same estimate holds for smooth closed curves with arbitrary curvatures $k_1 > 0$, $k_2 > 0$, k_3 in $\mathbb{R}^4$.

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On illumination of the boundary of a convex body in $\mathbb{E}^n$, $n = 4, 5, 6$

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Let H_n be the minimal number of smaller homothetic copies of an n-dimensional convex body required to cover the whole body. Equivalently, H_n can be defined via illumination of the boundary of a convex body by external light sources. It is a simple observation that for an n-cube (or n-parallelotope) exactly 2^n smaller copies are needed, so $H_n \geq 2^n$. The Levi-Hadwiger-Gohber-Markus's conjecture is that $H_n = 2^n$ with equality attained only for n-parallelotopes. One can refer to [1] for a recent survey.

The best known upper bound in three-dimensional case is $H_3 \leq 16$ and is due to Papadoperakis [3]. The method is based on the reduction of the illumination problem for a *general* convex body to that of covering *specific* sets of relatively simple structure by certain rectangular parallelotopes. We use Papadoperakis' approach to improve by a factor of approximately three the best previously known upper bounds on H_n for $n = 4, 5, 6$. In particular, we show $H_4 \leq 96$ where the previous bound was $H_4 \leq 296$ (obtained in [2]).

In the 4-dimensional case we also obtain a *precise* solution of two related covering problems. Namely, the smallest number of rectangular parallelotopes with sides parallel to the coordinate axes and the sum of dimensions strictly less than 1 (or ≤ 1) that is needed to cover the union of all 2-dimensional faces of the 4-dimensional unit cube is 89 (or 88, respectively).

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Manifolds and surfaces with locally Euclidean metrics

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The metric of a Riemannian manifold M^n is called locally Euclidean (l.E.) if any its point has a neighbourhood isometric to a ball in R^n with the standart Euclidean metric. The world of l.E. metrics is rather vast, f.e. the metric of any polyhedron with removed vertices is l.E. one.

In the theory of l.E. metrics there are many questions to be studied. F.e.

1) How to check is a given metric

$$ds^2 = g_{ij} du^i du^j \qquad (1)$$

l.E. if the coefficients g_{ij} are not sufficiently smooth?

2) How to find an existing isometry to a ball in R^n if it is known that the metric (1) is l.E. one?

3) What are properties of an isometry to $R * n$ from a l.E. M^n (what is class of smoothness about, is it an immersion, or embedding and so on).

4) The questions of isometric immersions and embeddings of a domain with l.E. metric in a Eulidean space of a greater dimension.

5) Properties of submanifolds with l.E. metrics.

6) Bendings of polyhedra as surfaces with l.E. metrics.

7) Monge-Ampére types differential equations with zero right sides (local and global propeties).

We can't give answers to all these questions nevertheless there are some at least partial results many of which are presented in [1], [2], [3], [4].

The work of author is supported by Grant of Scientific Schools, project No. 6222.2018.1.

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Grünbaum's inequality for projections and sections

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Let $K \subset \mathbb{R}^n$ be a convex body. The centroid of K is given by

$$g(K) := \frac{1}{\mathrm{vol}_n(K)} \int\limits_K x\,dx \in \mathbb{R}^n.$$

We will assume throughout that the centroid of K is at the origin. Grünbaum [1] proved that, for every $\xi \in S^{n-1}$,

$$\mathrm{vol}_n(K \cap \xi^+) \geq \left(\frac{n}{n+1}\right)^n \mathrm{vol}_n(K).$$

Here, $\xi^+ := \{x \in \mathbb{R}^n \,:\, \langle x, \xi \rangle \geq 0\}$. Note that there is equality when K is a cone and ξ is chosen to be the inward pointing unit normal at the cone's base.

I will discuss recent extensions of Grünbaum's inequality to orthogonal projections [3] and sections [2] of convex bodies. For every k-dimensional subspace E and every $\xi \in S^{n-1} \cap E$, we proved the following:

$$\begin{aligned} &\mathrm{vol}_k((K|E) \cap \xi^+) \geq \left(\frac{k}{n+1}\right)^k \mathrm{vol}_k(K|E) \\ \text{and}\qquad &\mathrm{vol}_k(K \cap E \cap \xi^+) \geq \left(\frac{k}{n+1}\right)^k \mathrm{vol}_k(K \cap E). \end{aligned}$$

Here, $\cdot|E$ denotes the orthogonal projection onto E.

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Helical tractrices and pseudo-spherical submanifolds in $\mathbb{R}^n$

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The classical theory of pseudo-spherical submanifolds and their Bianchi-Bäcklund transformations deals with n-dimensional submanifolds in $(2n-1)$-dimensional spaces of constant curvatures, see [1]-[2]. In [3] Yu. Aminov and A. Sym settled the problem asking for generalizations of the classical theory to the case of pseudo-spherical submanifolds with arbitrary codimension. A survey of results concerning this problem may be found in [4].

Particularly, we are interested in searching for submanifolds with arbitrary dimension and codimension, which may be viewed as analogues / generalizations of the classical Beltrami and Dini surfaces in $\mathbb{R}^3$.

The pseudo-spherical submanifolds in $\mathbb{R}^n$ that admit Bianchi transformations degenerated to curves and hence inherit features of the Beltrami surface, were completely described in [5]. As well, the pseudo-spherical submanifolds in $\mathbb{R}^n$ that admit Bäcklund transformations degenerated to straight lines and hence inherit features of the Dini surfaces, were completely described in [6].

For constructing submanifolds in $\mathbb{R}^n$ that admit Bäcklund transformations degenerated to curves different from straight lines, we propose to use *helical tractrices*. By definition, a smooth oriented curve γ in $\mathbb{R}^n$ is called a helical tractrix if the endpoints of unit segments tangent to γ form a curve of constant curvatures in $\mathbb{R}^n$.

It is conjectured that submanifolds in $\mathbb{R}^n$ obtained by particular skew rotations of helical tractrices have constant negative sectional curvature and inherit basic features of the Dini surfaces concerning their Bäcklund transformations. We provide arguments partially confirming this conjecture.

1. Yu.A. Aminov, *Geometry of submanifolds*. CRC Press, London, 2001.
2. K. Tenenblat, *Transformations of manifolds and applications to differential equations*. Longman, Harlow, 1998.
3. Yu. Aminov, A. Sym, *On Bianchi and Backlund transformations of surfaces in E^4*, Math. Phys., Anal., Geom., **3** (2000), 505-512.

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5. V. Gorkavyy, O. Nevmerzhitska, *Pseudo-spherical submanifolds with a degenerate Bianchi transformation*, Res. in Math., **60** (2011), 103-116.
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A Steiner formula in the L_p Brunn Minkowski theory

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The classical Steiner formula is one of the central parts of Brunn Minkowski theory. It expresses the volume of the parallel body $K + tB_2^n$ of a convex body K with Euclidean ball B_2^n as a polynomial in parameter t, where the intrinsic volumes arise as the coefficients of this polynomial.

The L_p Brunn Minkowski theory is an extension of the classical Brunn Minkowski theory which was initiated by Lutwak in [1] and rapidly evolved over the past years. It centers around the study of affine invariants associated with convex bodies. One of the main objects in the L_p Brunn Minkowski theory is the L_p affine surface area

$$as_p(K) = \int\limits_{\partial K} \frac{H_{n-1}(x)^{\frac{p}{n+p}}}{\langle x, \nu(x)\rangle^{\frac{n(p-1)}{n+p}}} d\mathcal{H}^{n-1}(x),$$

where $\nu(x)$ denotes the outer unit normal at $x \in \partial K$, the boundary of K, $H_{n-1}(x)$ is the Gauss curvature at x and $\mathcal{H}^{n-1}$ is the standard surface area measure on ∂K.

In this talk we present an analogue of the classical Steiner formula for the L_p affine surface area of a Minkowski outer parallel body for any real parameter p. This new Steiner type formula includes the classical Steiner formula and the Steiner formula from the dual L_p Brunn Minkowski theory as special cases.We introduce the coefficients in our new Steiner type formula which we call L_p quermassintegrals and also observe some of their properties.

1. E. Lutwak, *The Brunn-Minkowski-Firey theory II. Affine and geominimal surface areas*, Adv. Math. **118** (2) (1996), 244-294.

Isomorphisms of Sobolev spaces on Riemannian manifolds and quasiconformal mappings

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We prove that a measurable mapping of domains on Riemannian manifolds induces, by the composition rule, an isomorphism of Sobolev spaces with the first weak derivatives, whose summability index is equal to (different from) the topological dimension of the manifold, if and only if it coincides almost everywhere with some quasiconformal (quasiisometric) mapping.

Projective classification of points of a submanifold $F^3 \subset E^6$.

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Affine classification of points of submanifolds $F^n \subset E^{n+m}$ in the Euclidean space was given by A.Borisenko [1]. It was proved that the necessary condition for the number of affine classes to be finite is $m\left(\frac{n(n+1)}{2} - m\right) \leq n^2 - 1$. It follows that in case $F^3 \subset E^6$ there are infinitely many classes of affinely equivalent points. We prove that one can distinguish a *finite number* of classes of points in this case too, but with respect to wider (projective) group of transformations. The classification is based on the notion of indicatrix of normal curvature and the projective classification of Steiner surfaces [2].

Normal curvature of a submanifold $(F^n, g) \subset E^{n+m}$ at $q \in F^n$ in direction $X \in T_qF^n$ with respect to $\xi \in T_q^{\perp}F^n$ is a number $k_\xi(q,X) = \left.\frac{B_\xi(X,X)}{g(X,X)}\right|_q$, where $B_\xi(X,X)$ is a second fundamental form of the submanifold with respect to unit normal ξ. If X varies all over the unit sphere $S^n \subset T_qF^n$, then $k_\xi(q,X) : S^{n-1} \to T_qF^n$ defines the affine mapping for each fixed ξ. The image of the mapping $k_\xi(q)$ is called by *indicatrix of the normal curvature* at $q \in F^n$ with respect to $\xi \in T_q^{\perp}F^n$. The indicatrix can be considered as the *affine projection* of a projective immersion $ind : RP^{n-1} \to RP^m$ given by

$$ind(X^1 : X^2 : \ldots : X^n) = (B_1(X,X) : \ldots : B_m(X,X) : g(X,X)),$$

where $B_1, \dots, B_n$ are the second fundamental forms with respect to some normal frame.

Definition 1 *The points of a regular submanifold $F^n \subset E^{n+m}$ are said to be projectively equivalent if their normal curvature indicatrices are the same up to projective transformations $GL(n) \times GL(m+1)$ of $RP^{n-1} \times RP^m$ acting over their projective images.*

Denote by $\nu = \dim(span(B_1, \dots, B_n)$ the point-wise codimension and define extended point-wise codimension by $\mu = \dim(span(B_1, \dots, B_n, g))$. Define a *point-wise indicatrix index* as a pair (ν, μ).

Theorem 1 *There are 10 projective classes of points of a submanifold $F^3 \subset E^6$ in accordance to the values of point-wise index (ν, μ) and the type of normal curvature indicatrix, namely*

Index	*Type of indicatrix*
(3,4)	• *Rome surface* • *Cross-cap* • *Cross-cup* • *T-surface*
(3,3)	*a part of plane which do not pass trough the origin*
(2,3)	*a part of plane which pass trough the origin*
(2,2)	*a straight line segment which do not pass trough the origin*
(1,2)	*a straight line segment which pass trough the origin*
(1,1)	*a point which do not coincide with the origin*
(0,1)	*a point which coincide with the origin*

The non-degenerate indicatrices of index (3,4) belong to the class of Steiner surfaces. The plots (as well as parametric and other equations) of Stainer surfaces can be found on A. Coffman web-page [3].

The author thanks professor A.Borisenko for valuable discussions, remarks and suggestions.

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3. http://users.pfw.edu/CoffmanA/steinersurface.html

On Gruenbaum-type inequalities and their applications

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Let K be a convex body in $\mathbb{R}^n$. According to an old result of Gruenbaum, if K is cut by a hyperplane passing through its centroid, then the volumes of the two resulting pieces cannot be too small (they are larger than $1/e$ times the volume of K). We will discuss recent generalizations of Gruenbaum's result to projections and sections of convex bodies, as well as their applications.

Part II. DIFFERENTIAL EQUATIONS AND ANALYSIS

On the Correlation Functions of the Characteristic Polynomials of the Non-Hermitian Random Matrices with Independent Entries

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The talk is concerned with the non-Hermitian random matrices which entries are independent identically distributed complex random variables. One of the most interesting questions about these matrices is the local statistics of their eigenvalues. It was established in [1] that the k-point correlation function converges in vague topology to that for Complex Ginibre Ensemble if the first four moments of the entries are the same as in the Gaussian case. There is a hope to reduce the restrictions on the third and the fourth moments using the supersymmetry approach (SUSY).

In the present talk we consider the correlation functions of the characteristic polynomials. Although formally not in the realm of local eigenvalue statistics, the correlation functions of the characteristic polynomials are similar to the spectral correlation functions from the SUSY point of view; and are also of independent interest.

The main result is that the correlation functions of the characteristic polynomials behave like those for Complex Ginibre Ensemble up to a factor depending only on the forth absolute moment of the common probability law of the matrix entries.

1. T. Tao, V. Vu., *Random matrices: universality of local spectral statistics of non-Hermitian matrices*, Ann. Probab., **43(2)** (2015), 782-874.

Theoretically investigating the loss of stability and supercritical behaviour of a thin-walled shell

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The loss of stability of the original form of equilibrium under the influence of external forces is the main cause of the destruction of shell structures made of modern structural materials. The destruction at the same time occurs suddenly and almost instantly, which, as a rule, is accompanied by large human victims. To date, developed a large number of various options for the theory of shells,

At the moment when the shell loses stability, along with the main form of equilibrium of the middle surface, another form of equilibrium appears, infinitely close to the main one. It is the infinitesimal deviations of the middle surface of the shell from the basic form of equilibrium that correspond to the real critical load causing a loss of stability of the shell.

In the classical formulation of shell stability problems, the radial displacements and, which contradicts the stability equations, are assigned to the critical load in accordance with the critical load. Nevertheless, to date, it is believed that the load found as a result of this decision is critical. She received the name of the upper critical load.

When studying the stability of shells, we use dynamic and static stability criteria [1, 2]. The stability equations for shells are derived from the nonlinear equations of the theory of shells using the static Euler criterion. The equations of stability of the shells and the corresponding boundary conditions determine the deviations of the middle surface of the shell from the basic form of equilibrium at the moment of loss of stability, linking among themselves infinitesimal displacements, deformations and force factors. There is a paradoxical situation. The equations of the mechanics of a deformable solid, which provide the highest accuracy in the mechanics of shells, are derived by discarding small quantities of a higher order of smallness.

The paper proposes a method for constructing a physically consistent theory of the stability of shells, which allows for the determination of real critical forces to use theoretical material on the stability of shells, accumulated over a century of development of the theory of stability. Using this method, the calculated formulas are obtained for an isotropic cylindrical shell of medium length when loaded with axial forces, lateral and all-round pressure.

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On reachability and controllability problems for the heat equation controlled by the Neumann boundary condition on a half-axis

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Consider the control system on a half-axis

$$w_t = w_{xx}, \qquad x \in (0,+\infty),\ t \in (0,T), \tag{1}$$

$$w_x(0,\cdot) = u, \qquad t \in (0,T), \tag{2}$$

$$w(\cdot,0) = w^0, \qquad x \in (0,+\infty), \tag{3}$$

where $T > 0$ is given, $u \in L^\infty(0,T)$ is a control, the state $w(\cdot,t)$, $t \in (0,T)$, and the initial state w^0 belong to the space $H^1(0,+\infty)$ of the Sobolev type. We consider the steering condition $w(\cdot,T) = w^T$, $x > 0$ for this system, where $w^T \in H^1(0,+\infty)$.

A state $w^T \in H^1(0,+\infty)$ is called *reachable* from a state $w^0 \in H^1(0,+\infty)$ in a given time T if there exists a control $u \in L^\infty(0,T)$ such that there exists a unique solution w to system (1)–(3) and $w(\cdot,T) = w^T$.

A state $w^T \in H^1(0,+\infty)$ is called *approximately reachable* from a state $w^0 \in H^1(0,+\infty)$ in a given time T if there exists a sequence $\{u_n\}_{n=1}^\infty \subset L^\infty(0,T)$ such that there exists a unique solution w_n to system (1)–(3) with $u = u_n$ and $\|w^T - w_n(\cdot,T)\| \to 0$ as $n \to \infty$.

For $w^T \in H^1(0,+\infty)$ necessary and sufficient conditions of reachability from the origin are obtained in a given time Tin the case $|u| \le L$ on $(0,T)$, where $L > 0$ is a given constant. Under these conditions, using the

Markov power moment problem, it is constructed a sequence $\{u_n\}_{n=1}^{\infty}$ of bang-off-bang controls, $u_n(t) \in \{-1, 0, 1\}$, $t \in (0, T)$, solving the approximate reachability problem from the origin.

A state $w^0 \in H^1(0, +\infty)$ is called *approximately controllable* in a given time T if for any $w^T \in H^1(0, +\infty)$ and for any $\varepsilon > 0$ there exists a control $u_\varepsilon \in L^\infty(0, T)$ such that for the solution w_ε to system (1)–(3) with $u = u_\varepsilon$ we have $\|w^T - w_\varepsilon(\cdot, T)\| < \varepsilon$.

It is shown that each state $w^0 \in H^1(0, +\infty)$ is approximately controllable in a given time T. The controls solving the approximate controllability problems are constructed explicitly.

These results are illustrated by examples.

Note, that the same problems were considered in [1] for the heat equation controlled by the Dirichlet boundary condition on a half-axis.

1. L. Fardigola, K. Khalina, *Reachability and Controllability Problems for the Heat Equation on a Half-Axis*, J. Math. Phys., An., Geom., **15** (2019), 57-78.

One-point initial problem for a nonhomogeneous linear differential-difference equation in a Banach space

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Let A be a bounded linear operator with a spectral radius $r(A)$ on a complex Banach space X and $h \in \mathbb{C},\ h \neq 0$. The operator Bruwier series is called a following formal operator series

$$F(z) = \sum_{n=0}^{\infty} (n!)^{-1}(z + nh)^n A^n, \quad z \in \mathbb{C}.$$

Theorem 1. *Let $r(A)|h| < 1/e$. Then the operator Bruwier series $F(z)$ is an entire operator-function of an exponential type less than $|h|^{-1}$. Moreover, the operator family $U(z) = F(z)(F(0))^{-1},\ z \in \mathbb{C}$ forms uniformly continuous group with the infinitesimal generator $T = AF(h)(F(0))^{-1}$, which satisfies the operator equation $T = Ae^{hT}$. Consequently, $F(z) = e^{zT}F(0),\ z \in \mathbb{C}$.*

Now, we consider an initial problem

$$u'(z) = Au(z+h) + f(z),\ u(0) = u_0, \tag{1}$$

where $f : \mathbb{C} \to X$ be an entire vector-valued function.

Theorem 2. *Let $|h|r(A)e^{\sigma|h|+1} < 1$ and $f(z)$ be an entire vector-valued function of exponential type σ. Then for any $u_0 \in X$ there exists a unique solution of the initial problem (1) in the class of entire vector-valued functions of exponential type less than $\sigma + \frac{1}{|h|}$,*

$$u(z) = U(z)\left(u_0 - \sum_{m=0}^{\infty} A^m \int_0^{mh} \frac{(mh-\zeta)^m}{m!} f(\zeta)d\zeta\right) +$$

$$+\sum_{n=0}^{\infty} A^n \int_0^{z+nh} \frac{(z+nh-\zeta)^n}{n!} f(\zeta)d\zeta.$$

We also present some results about existence and uniqueness of a solution of an one-point problem for the implicit equation $Bu'(z) = Au(z+h) + f(z)$ with closed operators A and B.

Some approximate solutions of the Bryan-Pidduck equation

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The Boltzmann equation for the model of rough spheres (or the Bryan-Pidduck equation) has the form [1]:

$$D(f) = Q(f,f); \tag{1}$$

$$D(f) \equiv \frac{\partial f}{\partial t} + \left(V, \frac{\partial f}{\partial x}\right), \tag{2}$$

$$Q(f,f) \equiv \frac{d^2}{2} \int_{R^3} dV_1 \int_{R^3} d\omega_1 \int_{\Sigma} d\alpha B(V - V_1, \alpha)$$

$$\cdot \Big[f(t, V_1^*, x, \omega_1^*) f(t, V^*, x, \omega^*) - f(t, V, x, \omega) f(t, V_1, x, \omega_1) \Big]. \tag{3}$$

The solution to this equation will be look for in the next form:

$$f(t,V,x,\omega)=\sum_{i=1}^{\infty}\varphi_i(t,x)M_i(V,\omega), \tag{4}$$

where $M_i(V,\omega)$ are the exact solutions of the Bryan-Pidduck equation

$$D(M_i)=Q(M_i,M_i)=0$$

and the coefficient functions $\varphi_i(t,x)$ are non-negative functions, smooth on $\mathbb{R}^4$.

As a value of the deviation between the parts of equation (1) we will consider a uniform-integral error of the form:

$$\Delta=\sup_{(t,x)\in\mathbb{R}^4}\int_{\mathbb{R}^3}dV\int_{\mathbb{R}^3}d\omega\Big|D(f)-Q(f,f)\Big|. \tag{5}$$

In the paper [2], a several cases of coefficient functions $\varphi_i(t,x)$ were obtained for which the deviation (5) can be done arbitrarily small. This is possible thanks to a special selection of hydrodynamic flow parameters.

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Hardy Spaces of Fuchsian Groups in the Upper Half-plane

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We establish exact conditions for non triviality of all subspaces of the Hardy space (with respect to the Lebesgue measure) in the upper half plane, that consist of the character automorphic functions with respect to the action of a discrete subgroup of $SL_2(\mathbb{R})$. Such spaces are the natural objects in the context of the spectral theory of almost periodic differential operators. It is parallel to the celebrated Widom - Pommerenke characterization for Hardy spaces (with respect to the harmonic measure) with the following modification: the Green function of the group is substituted with the Martin function and also the Martin measure must be a pure point one.

The time-optimal control problem and a vector moment min-problem

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Our talk deals with the time-optimal control problem [1].

As was shown in [2], a controllability problem for a linear system can be reduced to the abstract moment L-problem [3]. It turned out that an analytic solution of the linear time-optimal problem for systems with a one-dimensional control can be obtained by reducing to the Markov moment min-problem proposed in [4]-[6].

In the talk, a vector moment min-problem is introduced, which is used for solving the linear time-optimal problem for systems with a multi-dimensional control.

1. L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze, & E. Mishchenko *The mathematical theory of optimal processes*, Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow 1961, 391 pp. (Russian); translated by Interscience, New York, 1962.
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The envelope for family of ellipsoids in the controllability function method

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Consider the system

$$\dot{x} = A_0 x + b_0 u, \tag{1}$$

where $x \in \mathbb{R}^n$ is a state, u is a scalar control satisfying the constraint $|u| \leq 1$. Here the superdiagonal elements of the matrix A_0 are equal to 1 and the other elements are equal to zero; the last element of the vector b_0 is equal to 1 and the other elements are equal to zero.

The approach presented in the talk is based on the controllability function method proposed by V.I. Korobov in 1979. In [1], a control $u(x)$ solving the feedback synthesis problem for system (1) was given. This control satisfies the conditions: 1) $|u(x)| \leq 1$; 2) the trajectory $x(t)$ of the closed system $\dot{x} = A_0 x + b_0 u(x)$ starting from an arbitrary initial point $x(0) \in \mathbb{R}^n$ ends at the origin in a finite time.

In the controllability function method, the angle between the motion direction and the decrease direction of the controllability function is not less than the corresponding angle in the dynamic programming method and is not greater than the angle in the Lyapunov function method [1, p. 10]. The main advantage of the controllability function method is the finiteness of the motion time.

For example, in the case $n = 2$, control system (1) takes the form

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ u \end{pmatrix}, \tag{2}$$

i.e., we consider the feedback synthesis problem for the motion of a cart. Let $a_1 < -4.5$. The controllability function $\Theta = \Theta(x_1, x_2)$ is defined for $x \neq 0$ as the unique positive solution of the equation

$$\frac{(4 + a_1)\Theta^4}{a_1(3 + a_1)} - a_1 x_1^2 + 4\Theta x_1 x_2 + \Theta^2 x_2^2 = 0. \tag{3}$$

Let

$$u(x) = \frac{a_1 x_1}{\Theta^2(x_1, x_2)} - \frac{3x_1}{\Theta(x_1, x_2)}. \tag{4}$$

Then the trajectory of the closed system starting from an initial point $x(0) = x_0 \in \mathbb{R}^2$ ends at the point $x(T) = 0$ in the finite time $T = T(x_0) = \Theta(x_0)$.

We analyze the envelope for one-parametric family (3) at $\Theta = 1$ for system (2). It is close to the curve describing all points from which we may steer to the origin due to the Pontryagin maximum principle [2] for the time $t = 1$.

1. V.I. Korobov *The method of controllability function*, R&C Dynamics, M.-Izhevsk, 2007 (Russian).
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Rigorous theory of 1d turbulence

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My talk is a review of the results on the turbulence in the 1d viscous Burgers equation on a circle, obtained in theses of my former PhD students Andrey Biryuk and Alexandre Boritchev, based on my earlier work on other nonlinear PDEs. The results were next developed in their papers, in my LN on this topic, and were put to the final form, which I will present, in a MS of my joint book with A.Boritchev. Namely, I will talk about the Burgers equations on a circle, perturbed by a random force which is smooth in x and white in time t, and explain that Sobolev norms of its solutions admit upper and lower estimates, which are asymptotically sharp as the viscosity goes to zero. This assertions allows to derive for solutions of the equation results, which are rigorous analogies of the main predictions of the Kolmogorov theory of turbulence. They were non-rigorously obtained by physicists Aurell-Frisch-Lutsko-Vergassola in 1992.

On some applications of atomic functions and their generalizations

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On some applications of atomic functions and
their generalizations

The function is called atomic if it is a solution with a compact support of the linear functional differential equation with constant coefficients and linear transformations of the argument [1]. Generalized Fup-functions, which were constructed in [2], naturally generalize atomic functions.

Consider the function $f(x) \in L_2(R)$ such that

1) $suppf(x) = [-1, 1]$;
2) $f(x) \geq 0$ for any $x \in [-1, 1]$;
3) $f(-x) = f(x)$;
4) $\int\limits_{-\infty}^{\infty} f(x)dx = 1$.

Denote by $F(t)$ the Fourier transform of this function.

The function

$$f_{N,m}(x) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} e^{itx} \left(\frac{\sin(t/N)}{t/N} \right)^{m+1} F(t/N)dt,$$

where $N > 0$ and $m = 2, 3, 4, \ldots$ is called a generalized Fup-function.

It was shown in [1, 3] that spaces of linear combinations of shifts of atomic functions and generalized Fup-functions have good approximation properties. In terms of the Kolmogorov width, these spaces are asymptotically extremal for approximation of classes of periodic differentiable functions. This means that atomic functions and generalized Fup-functions are as good constructive tool as classic trigonometric polynomials.

In this talk, we introduce the system of non-stationary smooth wavelets with a compact support constructed using generalized Fup-functions and also present the results of a comparison of this system with a trigonometric functions using the example of lossy image compression.

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Couplings of symmetric operators with possibly unequal and infinite deficiency indices

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We extend the known results on couplings of symmetric operators A_j, $j \in \{1,2\}$, in the sense of A.V. Shtraus to the case of operators A_j with arbitrary (possibly unequal and infinite) deficiency indices. In particular, we generalize to this case the coupling method based on the theory of boundary triplets for symmetric operators. This enables us to obtain the abstract Titchmarsh formula, which gives the representation of the Weyl function of the coupling in terms of Weyl functions of boundary triplets for A_1^* and A_2^*. In applications to differential operators on $\mathbb{R}$ this formula turns into the classical Titchmarsh formula, which gives a representation of the characteristic matrix $\Omega(\cdot)$ in terms of Titchmarsh-Weyl functions on semiaxes $\mathbb{R}_+$ and $\mathbb{R}_-$. Moreover, by using the coupling method we parameterize all Naimark exit space extensions $\widetilde{A} = \widetilde{A}^*$ of the second kind of a densely defined symmetric operator A with finite possibly unequal deficiency indices.

Solvability of a boundary value problem for a fourth-order mixed type equation

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In a rectangular domain, we study the boundary value problem for a fourth-order mixed differential equation of mixed type containing a wave operator and the product of the inverse and direct heat conduction operators.

$$Lu \equiv \begin{cases} u_{xx} - u_{tt} = f_1(x,t), & \in \ \Omega^+, \\ u_{xxxx} - u_{tt} = f_2(x,t), & \in \ \Omega^-, \end{cases} \tag{1}$$

where $f_1(x,t)$, $f_2(x,t)$ - are the specified functions. In the $\Omega = \{(x,t): \ 0 < x < p, \ \ -T_1 < t < T_2\}$, area where $\Omega^+ = \Omega \cap (t > 0)$, $\Omega^- = \Omega \cap (t < 0)$, the boundary problem is investigated. Reference to equation (1).

Problem 1. Find a function $u(x,t)$, such that:

1) is continuous in $\overline{\Omega}$, together with its derivatives given in the boundary conditions;

2) is a regular solution of equation (1) in $\Omega^+ \cup \Omega^-$;

3) satisfies the boundary conditions:

$$u(0,t) = u(p,t) = 0, \qquad -T_1 \le t \le T_2,$$

$$u_{xx}(0,t) = u_{xx}(p,t) = 0, \ \ -T_1 \le t \le 0,$$

$$u(x,T_2) = 0, \qquad 0 \le x \le p,$$

$$u(x,-T_1) = 0, \qquad 0 \le x \le p,$$

4) satisfies the gluing condition

$$u_t(x,+0) = u_t(x,-0), \qquad 0 < x < p.$$

Definition 1 A function $u(x,t) \in V(\Omega)$, where

$$V(\Omega) = \left\{ u(x,t) : u \in C\left(\overline{\Omega}\right), u_{xx}, u_{tt} \in C(\Omega^+), \right. \\ \left. u_{xx} \in C\left(\overline{\Omega^-}\right), u_{xxxx}, u_{tt} \in C(\Omega^-), \ u_t \in C(\Omega) \right\},$$

is called a regular solution of problem 1, for, $f(x,t) \in C(\Omega)$ if it satisfies equation (1) in Ω.

Definition 2 A function $u(x,t) \in L_2(\Omega)$ is called a strong solution of problem 1 for$f(x,t) \in L_2(\Omega)$if there is a sequence $\{u_k\}$, $k = 1, 2, ...$ regular solutions such that $\|u_k - u\|_{L_2(\Omega)} \to 0$, $\|Lu_k - f\|_{L_2(\Omega)} \to 0$ for $k \to \infty$.

Theorem Let the numbers p and T_2 be such that for $n = 1, 2...$ $N_n(T) \neq 0$, where

$$N_n(T) \equiv \left| \left(1 - e^{-2\frac{n^2\pi^2}{p^2}T_1}\right) \cdot \cos\tfrac{n\pi}{p}T_2 + \tfrac{n\pi}{p}\left(1 + e^{-2\frac{n^2\pi^2}{p^2}T_1}\right) \cdot \sin\tfrac{n\pi}{p}T_2 \right|,$$

then if there is a regular solution to problem 1, then it is unique.

On the persistence probability for Kac polynomials, random matrices and random walks

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A natural quantity that characterizes a given real stochastic process $X(t)$ is its persistence probability, i.e. the probability that a process doesn't change the sign up to time t. For most of the systems this probability decays as a power law in the limit of large time, and corresponding power is called *persistence constant* of a process. Problem of calculating persistence constants for various models have experienced some recent attention due to applications in finances, theoretical physics as well as to other questions in pure probability.

Our main goal is to find persistence constant for a family of Kac polynomials that can be defined as

$$\theta = -\lim_{n\to\infty} \frac{\log \mathbb{P}\left[K_{2n}(x) \neq 0,\ x \in [0,1]\right]}{\log n},$$

where the polynomial K_{2n} is given by

$$K_N(z) = \sum_{k=0}^{N} a_k z^k, \quad a_k \text{ are } N(0,1) \text{ i.i.d. random variables.}$$

It was argued by P. Forrester that eigenvalues of rank-one truncated random orthogonal matrices and random roots of Kac polynomial have identical statistical properties. By studying corresponding RMT problem we show

Theorem 1 *Let M_{2n} be a top left minor of size $2n \times 2n$ of orthogonal matrix chosen uniformly at random (with respect to Haar measure) from orthogonal group $O(2n+\ell)$. Then in the limit of large n and fixed ℓ*

$$-\lim_{n\to\infty} \frac{\log \mathbb{P}\left[M_{2n} \textit{ has no real eigenvalues}\right]}{2\log n} = \theta(\ell), \ \textit{with } \theta(1) = \frac{3}{16}.$$

Contrary to the above indirect calculation of persistent constant for Kac polynomials we use yet another stochastic model tightly connected with Kac polynomials. It was shown in a seminal paper by A. Dembo,

B. Poonen, Q.-M. Shao, O. Zeitouni that Gaussian Stationary Process (GSP) X_t with mean zero and covariance $\mathbb{E}[X_t X_s] = \operatorname{sech}\frac{t-s}{2}$ has the same persistence constant

$$\theta = -\lim_{T\to\infty}\frac{1}{T}\log\mathbb{P}\left[X_t > 0, t\in[0,T]\right].$$

By using Pfaffian structure of zero crossings of X_t and connection to the problem of finding expected exit time from a strip for a RW we prove

Theorem 2 $\mathbb{P}\left[X_t > 0, t\in[0,T]\right] = C_1 \mathrm{e}^{-\frac{3T}{16}}\left(1+\overline{o}\left(T^{-1}\right)\right), \quad T\to\infty.$

The talk is based on results obtained in collaboration with M.Gebert (UC Davis, USA) and G. Schehr (Paris-Sud University, France).

Nested ellipsoids

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Let us consider the feedback synthesis for the motion of a cart

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}x_2\\ u\end{pmatrix}. \tag{1}$$

Here $t\geq 0$, $(x_1,x_2)\in\mathbb{R}^2$ is a state; u is a scalar control, $|u|\leq 1$. The controllability function method was advanced by V.I. Korobov in 1979. In [1] a control $u(x)$ solving the feedback synthesis problem for system (1) was given.

Definition 1. The feedback synthesis problem lies in finding a control of the form $u=u(x),\ x\in\mathbb{R}^2$ such that: 1) $|u(x)|\leq 1$;
2) the trajectory $x(t)$ of the closed-loop system

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}x_2\\ u(x)\end{pmatrix}. \tag{2}$$

starting at an arbitrary initial point $x_0\in\mathbb{R}^2$, ends at the origin at some finite time $T(x_0)$.

Let us underline some difficulties related to solving this problem.
1. Since there exist infinitely many trajectories passing through the origin

(note that the time of motion is finite), the right-hand side of equation (2) cannot satisfy the Lipschitz condition in a neighborhood of the origin according to the existence and uniqueness theorem for smooth differential equations.
2. The control must satisfy the preassigned constraint.

Theorem 1. [1] *Let $\nu \geq 1$. The controllability function $\Theta = \Theta(x_1, x_2)$ is defined for $x \neq 0$ as a unique positive solution of the equation*

$$2\Theta^4 = x_1^2(\nu+2)^3(\nu+3) + 2x_1x_2\Theta(\nu+2)^2(\nu+3) + 2x_2^2\Theta^2(\nu+2)^2, \quad (3)$$

Let

$$u(x_1, x_2) = -\frac{x_1(2+\nu)(3+\nu)}{2\Theta^2} - \frac{x_2(2+\nu)}{\Theta} \quad (4)$$

Then the trajectory of the closed-loop system starting from an initial point $x(0) = x_0 \in \mathbb{R}^2$ ends at the point $x(T) = 0$ at a finite time $T = T(x_0)$ such that $T(x_0) = \Theta(x_0)$.

We analyze one-parametric family (3) at $\Theta = 1$ for the system (1). Proved numerically that when value of parameter $\nu \geq 1$ increases the family of ellipsoids are nested.

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On global behavior of Orlicz-Sobolev classes in terms of prime ends

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An *end* of a domain D is an equivalence class of chains of cross-cuts of D. We say that an end K is a *prime end* if K contains a chain of cross-cuts $\{\sigma_m\}$, such that $\lim\limits_{m\to\infty} M(\Gamma(C, \sigma_m, D)) = 0$ for some continuum C in D, where M is the modulus of the family $\Gamma(C, \sigma_m, D)$. We say that the boundary of a domain D in $\mathbb{R}^n$ is *locally quasiconformal* if every point $x_0 \in \partial D$ has a neighborhood U that admit a conformal mapping φ onto the unit ball $\mathbb{B}^n \subset \mathbb{R}^n$ such that $\varphi(\partial D \cap U)$ is the intersection of $\mathbb{B}^n$ and a coordinate hyperplane. We say that a bounded domain D in $\mathbb{R}^n$ is *regular* if D can be mapped quasiconformally onto a bounded domain with a locally quasiconformal boundary. If $\overline{D}_P$ is the completion of a regular domain D by its prime ends and g_0 is a quasiconformal mapping of a domain D_0 with locally quasiconformal boundary onto D, then this mapping naturally determines the metric $\rho_0(p_1, p_2) = \left|\widetilde{g_0}^{-1}(p_1) - \widetilde{g_0}^{-1}(p_2)\right|$, where $\widetilde{g_0}$ is the extension of g_0 onto $\overline{D_0}$. Let $\varphi : [0, \infty) \to [0, \infty)$ be a nondecreasing function, $x = (x_1, \ldots, x_n)$, $f(x) = (f_1(x), \ldots, f_n(x))$, $|\nabla f(x)| = \sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left(\frac{\partial f_i}{\partial x_j}\right)^2}$. Now we write $f \in W^{1,\varphi}_{loc}(D)$, if $f_i \in W^{1,1}_{loc}$ for each $i = 1, \ldots, n$, and $\int\limits_G \varphi\left(|\nabla f(x)|\right)\, dm(x) < \infty$ for every domain $G \subset D$ with a compact closure $\overline{G} \subset D$. Let D be a domain in $\mathbb{R}^n$, $n \geq 2$, and $f : D \to \mathbb{R}^n$ be a continuous mapping. A mapping $f : D \to \mathbb{R}^n$ is said to be *discrete* if the preimage $f^{-1}(y)$ of every point $y \in \mathbb{R}^n$ consists of isolated points, and *open* if the image of every open set $U \subset D$ is open in $\mathbb{R}^n$. A mapping f is closed if the image of every closed set $U \subset D$ is closed in $f(D)$. Set $l\left(f'(x)\right) := \min\limits_{|h|=1} |f'(x)h|$, $J(x,f) := \det f'(x)$, $K_{I,\alpha}(x, f) = \frac{|J(x,f)|}{l(f'(x))^{\alpha}}$ if $J(x, f) \neq 0$, $K_{I,\alpha}(x, f) = 1$ if $f'(x) = 0$ and $K_{I,\alpha}(x, f) = \infty$ otherwise. For a given number $\alpha > 1$, domains D, $D' \subset \mathbb{R}^n$, a non-degenerate continuum $A \subset D$, a number $\delta > 0$ and a Lebesgue measurable function $Q(x)$ denote by $\mathfrak{F}^{A,\delta}_{\varphi,Q,\alpha}(D, D')$ the family of all homeomorphisms $f \in W^{1,\varphi}_{\rm loc}$ of D onto D' such that $K_{I,\alpha}(x, f) \leqslant Q(x)$ and $\operatorname{diam} f(A) := \sup\limits_{x,y \in f(A)} |x-y| \geqslant \delta$. The following statement holds.

Theorem. *Let $n \geqslant 3$, $\alpha > 1$, let D be a regular domain in $\mathbb{R}^n$, and let D' be a domain in $\mathbb{R}^n$ that has a locally quasiconformal boundary. Suppose that the boundary of the domain D' is strongly accessible with respect to α-modulus and $Q \in L^1_{\rm loc}(\mathbb{R}^n)$. Suppose also that $\varphi : (0, \infty) \to [0, \infty)$ is a non-decreasing function satisfying the Calderon condition $\int\limits_1^{\infty}\left(\frac{t}{\varphi(t)}\right)^{\frac{1}{n-2}} dt < \infty$. If $Q \in FMO(\overline{D})$ for every $x_0 \in \overline{D}$, then $f \in \mathfrak{F}^{A,\delta}_{\varphi,Q,\alpha}(D, D')$ has a continuous extension of $\overline{f} \colon \overline{D}_P \to \overline{D'}_P$, while the family $\mathfrak{F}^{A,\delta}_{\varphi,Q,\alpha}(\overline{D}_P, \overline{D'}_P)$ of all extended mappings is equicontinuous in $\overline{D}_P$.*

Surfaces of maximal singularity for homogeneous control systems

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We consider the class of nonlinear systems that are linear with respect to controls and homogeneous at the origin. At a point different from the origin such a system may be non-homogeneous, so its homogeneous approximation is of interest. The homogeneous approximation problem attracts a great attention during several decades [1]–[6]. Roughly speaking, the approximation property means that, after some change of variables in the initial system, the trajectories of the initial system and of the approximating system with the same control are close.

We describe the set of points where a homogeneous approximation coincides with the initial system; the question was proposed by I. Zelenko [9]. For a regular system, this set is a neighborhood of the origin, but for nonregular systems the picture is much more complicated.As the main tool, we use the free algebra approach proposed in our previous papers [7, 8].

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Pointwise potential estimates of solutions to high-order quasilinear elliptic equations

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Let Ω be a bounded open set of $\mathbb{R}^n$, $n \geqslant 3$, $f \in L^1(\Omega)$. We consider quasilinear $2m$-order ($m \geqslant 2$) partial differential equations of the form:

$$\sum_{|\alpha|\leqslant m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, \nabla_m u) = f(x),\ x \in \Omega,$$

where $\nabla_m u = \{D^\alpha u : |\alpha| \leqslant m\}$.

We assume that the coefficients $\{A_\alpha\}_{|\alpha|\leqslant m}$ are Carathéodory functions such that for some constants $K > 0$, $p > 1$, for any $\xi = \{\xi_\alpha \in \mathbb{R} : |\alpha| \leqslant m\}$ and for a.e. $x \in \Omega$, the following inequalities hold:

$$\sum_{|\alpha|\leqslant m} |A_\alpha(x,\xi)| \leqslant K \sum_{1\leqslant|\alpha|\leqslant m} |\xi_\alpha|^{p-1}, \quad \sum_{|\alpha|=m} A_\alpha(x,\xi)\xi_\alpha \geqslant K^{-1} \sum_{|\alpha|=m} |\xi_\alpha|^p.$$

Let $n = mp$, $u \in W^{m,p}(\Omega)$ be an arbitrary generalized (in the sense of distributions) solution of the given equation, x_0 be a Lebesgue point of the function u, and let $B_{2R}(x_0)$ be an open ball with center x_0 and radius $2R < 2$. Then the main result of our report is the following estimate:

$$\begin{aligned} |u(x_0)| \leqslant C\Bigg(R^{-n} \int\limits_{B_R(x_0)} |u|^p\, dx\Bigg)^{1/p} \\ + C \sum_{1\leqslant|\alpha|\leqslant m-1} \Bigg(\int\limits_{B_R(x_0)} |D^\alpha u|^{n/|\alpha|}\, dx\Bigg)^{|\alpha|/n} + C\mathbf{W}^f_{m,p}(x_0; 2R), \end{aligned} \tag{6}$$

where $\mathbf{W}^f_{m,p}(x_0; 2R) = \int\limits_0^{2R} \Big(\int\limits_{B_r(x_0)} |f| dx\Big)^{1/(p-1)} r^{-1} dr$ is the Wolff-type potential of f, and the constant $C > 0$ depends only on n, m and K.

The proof of inequality (6) is based on the development of Kilpeläinen–Malý method proposed in [1] for the p-Laplace equation $-\Delta_p u = f$ and modified in [2] for $2m$-order equations with m-(p, q)

growth and coercivity conditions in the case $n \geqslant q > mp$. We use estimate (6) at an arbitrary Lebesgue point $x_0 \in \Omega$ of the solution u to prove its local boundedness, and then continuity in Ω, provided that $\lim_{\rho\to 0} \sup_{x\in\Omega} \mathbf{W}^f_{m,p}(x;\rho) = 0$.

Acknowledgements. The research of the first author was supported by grants of Ministry of Education and Science of Ukraine (project numbers are 0118U003138, 0119U100421).

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Laplacian eigenfunctions with a level set having infinitely many connected components

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We construct a Riemannian metric on the 2-dimensional torus, such that for infinitely many eigenvalues of the Laplace-Beltrami operator, the corresponding eigenfunction has a level set with infinitely many connected components (i.e., a linear combination of two eigenfunctions may have infinitely many nodal domains).

The talk is based on a joint work with Lev Buhovsky and Alexander Logunov.

Propagation of singularities for solutions of quasilinear parabolic equations with absorption term

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In the cylindrical domain $Q = (0,T) \times \Omega$, $0 < T < \infty$, where $\Omega \subset R^n$ is a bounded domain such that $\partial\Omega \in C^2$ the following problem for a quasilinear parabolic equation is considered:

$$
\begin{aligned}
&(|u|^{q-1}u)_t - \Delta_p(u) = -b(t,x)|u|^{\lambda-1}u, \quad \lambda > p \geqslant q > 0,\\
&u = \infty \qquad \text{on } (0,T)\times\partial\Omega, \\
&u = \infty \qquad \text{on } \{0\}\times\Omega,
\end{aligned}
\tag{7}
$$

Here $b(t,x)$ (the absorption potential) is a continuous function in $[0,T]\times\overline{\Omega}$ such that the following conditions holds:

$$
b(t,x) > 0 \quad \text{in } [0,T)\times\overline{\Omega}, \quad b(t,x) = 0 \quad \text{on } \{T\}\times\Omega, \tag{8}
$$

$$
a_1(t)g_1(d(x)) \le b(t,x) \le a_2(t)g_2(d(x)) \quad \forall\,(t,x)\in[0,T)\times\Omega, \tag{9}
$$

where $g_1(s) \le g_2(s)$ are arbitrary nondecreasing positive functions for all $s > 0$.

In the paper [1] the precise estimate of a profile of an arbitrary large solution of problem (7) has been obtained under the mentioned conditions (8)–(9) and the additional condition $p > q$. In the case when $p = q$ the analogous results were obtained in [2, 3].

In the paper [4] the linear case was studied ($p = q = 1$) and the estimate of solution were obtained, this result was extended for nonlinear case in [3].

Investigation in [4] was carried out by comparison with auto-similar solutions, and in [1, 2, 3] a method of energy estimates was used.

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On the truncated two-dimensional moment problem

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We discuss the truncated two-dimensional moment problem with rectangular data. Namely, we consider the following problem:
to find a non-negative measure $\mu(\delta)$, $\delta \in \mathfrak{B}(\mathbb{R}^2)$, such that

$$\int_{\mathbb{R}^2} x_1^m x_2^n d\mu = s_{m,n}, \qquad 0 \le m \le M, \quad 0 \le n \le N,$$

where $\{s_{m,n}\}_{0\le m\le M,\ 0\le n\le N}$ is a prescribed sequence of real numbers; $M, N \in \mathbb{Z}_+$.

For some cases of small size truncations explicit numerical conditions for the solvability of the moment problem are given. In all these cases some solutions of the moment problem can be constructed.

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