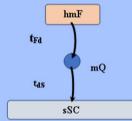
# Spin-dependent resonant tunneling through a magnetic quantum dot coupled to superconducting and ferromagnetic leads: F-mQD-S system Koshina E. A., Krivoruchko V. N.

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An actual problem is conclusive proof and undoubted distinguishing between topologically trivial subgap Andreev bound states (ABS) and topologically nontrivial magnetically polarized Majorana bound states. This motivated us to investigate nonequilibrium electrons tunneling through a ferromagnetic metal-magnetic quantum dot-s-wave superconductor (F-mOD-SC) nanostructure [1]. Special attention is devoted to analyzing the implications of the spin-splitting electron levels in the dot on the tunneling process and the system's conductance characteristics. Using Keldysh Green's function method, the expressions for tunnel current and probability of the Andreev reflection (AR) versus energy are derived and studied. In contrast to a system with non-magnetic QD, where the differential conductance exhibits a series of peaks with equal intervals, the conductance of the F-mOD-SC system demonstrates a more complicated pattern. This property originates from the splitting levels of the mQD by an effective (external and proximity-induced) magnetic field and, if the energy levels of the mOD are not equal spacing, the series of peaks will be much more complicated. We find that the system's resonant ARs conductance exhibits different kinds of peaks depending on a spin splitting of the mQD levels, the spin polarization magnitude of the Flead current, the gate voltage, and an external magnetic field magnitude.

# Contact structure

## The model



$$\begin{split} H &= H_F + H_{SC} + H_{mdot} + H_T \\ H_F &= \sum_{k\sigma} (E_{k\sigma} + \sigma M - eV) a_{k\sigma}^+ a_{k\sigma} \\ H_{mdot} &= \sum_{i,\sigma} (\varepsilon_i^0 + \sigma \mu_B H - ev_g) d_{i\sigma}^+ d_{i\sigma} \end{split}$$

$$H_T(\tau) = \sum_{k\sigma} \left\{ (t_{Fd} a_{k\sigma}^+ d_{i\sigma} + h.c.) + \left( t_{Sd} e^{ieV_S \tau} b_{k\sigma}^+ d_{i\sigma} + h.c. \right) \right\}$$

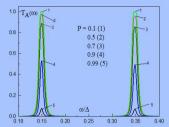
$$I_{tot} = I^{(A)} + I_{a\uparrow} + I_{a\downarrow},$$

Current through a quantum dot  $I_{tot}=I^{(A)}+I_{q\uparrow}+I_{q\downarrow},$  The tunnel current can be presented as a sum of three different

The tunnel current can be presented as a sum of three different contributions, were 
$$I^{(A)}$$
 arises from the AR processes and  $I_{q\uparrow}$  and  $I_{q\downarrow}$  are the quasiparticle currents with spin "up" ("down") 
$$I^{(A)} = \frac{2e}{\hbar} \int \frac{d\omega}{2\pi} \Gamma_{F\uparrow} \Gamma_{F\downarrow} \times \{|G_{\uparrow\downarrow}^r(\omega)|^2 [n_{F\uparrow}(\omega + eV_F - eV_S) - n_{F\downarrow}((\omega - eV_F + eV_S))] + |G_{\uparrow\downarrow}^r(\omega)|^2 [n_{F\downarrow}(\omega + eV_F - eV_S) - n_{F\uparrow}(\omega - eV_F + eV_S)]\}, G_{\uparrow\downarrow}^r(\omega) = G_{\uparrow\uparrow}^r(\omega) \frac{i}{2} \Gamma_S \frac{\Delta}{\sqrt{\omega^2 - \Delta^2}} A_{\downarrow\downarrow}^r(\omega)$$
$$G_{\uparrow\uparrow}^r(\omega) = \left[ \left( \sum_i \frac{1}{\omega - \epsilon_{i\uparrow}(h) + eV_F - eV_g} \right)^{-1} + \frac{i}{2} \Gamma_{F\uparrow} + \frac{i}{2} \Gamma_S \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} + \frac{\Gamma_S^2}{4} \Gamma_S \frac{\Delta^2}{\omega^2 - \Delta^2} A_{\downarrow\downarrow}^r(\omega) \right]^{-1}$$

$$\begin{split} A^r_{\downarrow\downarrow} &= \left[ (\sum_i \frac{1}{\omega - \epsilon_{i\downarrow}(h) + eV_F - eV_g})^{-1} + \frac{i}{2} \Gamma_{F\downarrow} + \frac{i}{2} \Gamma_S \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} \right]^{-1} \\ G^r_{\downarrow\uparrow}(\omega) &= G^r_{\downarrow\downarrow}(\omega) \frac{i}{2} \Gamma_S \frac{\Delta}{\sqrt{\omega^2 - \Delta^2}} A^r_{\uparrow\uparrow}(\omega) \\ G^r_{\downarrow\downarrow}(\omega) &= \left[ (\sum_i \frac{1}{\omega - \epsilon_{i\downarrow}(h) + eV_F - eV_g})^{-1} + \frac{i}{2} \Gamma_{F\downarrow} + \frac{i}{2} \Gamma_S \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} + \frac{\Gamma_S^2}{4} \Gamma_S \frac{\Delta^2}{\omega^2 - \Delta^2} A^r_{\uparrow\uparrow}(\omega) \right]^{-1} \\ A^r_{\uparrow\uparrow}(\omega) &= \left[ (\sum_i \frac{1}{\omega - \epsilon_{i\uparrow}(h) + eV_F - eV_g})^{-1} + \frac{i}{2} \Gamma_{F\uparrow} + \frac{i}{2} \Gamma_S \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} \right]^{-1} \end{split}$$

# A probability of Andreev Reflection $T_A(\omega)$ vs the energy for F-mQD-S structure



$$\begin{split} & Fig.1. \ The \ mQD \ has \ two \ states \ \varepsilon_1{}^o = 0.25\Delta \ \varepsilon_2{}^o = -0.25\Delta, \\ & P = 0.1\text{-}0.99, V_g = 0, \ and \quad E_{Ze} = 0.05\Delta, \ \Gamma_{F0} = \Gamma_S = 0.01. \end{split}$$

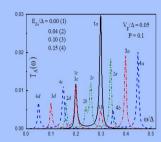


Fig.2. The mQD has two states  $\epsilon_1^{~0}=0.25\Delta$   $\epsilon_2^{~0}=-0.0$  P= 0.1,  $V_g=0$  0.05 $\Delta$  and  $E_{Ze}=0.0$ - 0.15 $\Delta$ ,  $\Gamma_{F0}=\Gamma_S$ 

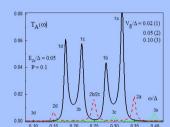


Fig.3. The mQD has two states  $\epsilon_1^0 = 0.25 \Delta \epsilon_2^0 = -0.25 \Delta$ , P = 0.1,  $V_g = 0.02 \Delta$ -0.1 $\Delta$  a  $E_{Ze} = 0.05 \Delta$ ,  $\Gamma_{F0} = \Gamma_S = 0.01$ .

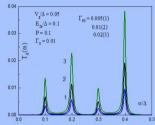


Fig.4. The mQD has two states  $\epsilon_1^o=0.25\Delta\,\epsilon_2^o=-0.25\Delta,$  P= 0.1, V<sub>g</sub> = 0.05 $\Delta$  a  $E_{Ze}=0.1\Delta,$   $\Gamma_S=0.0$   $\Gamma_{F0}$  =0.005-0.02

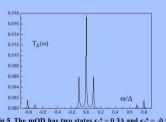


Fig.5. The mQD has two states  $\epsilon_1^0 = 0.3\Delta$  and  $\epsilon_2^0 = -0.5\Delta$ P= 0.1,  $V_{\sigma} = 0.25\Delta$ , and  $E_{Z_{\sigma}} = 0.05\Delta$ ,  $\Gamma_{F0} = \Gamma_S = 0.01$ .

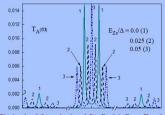


Fig.6.  $\epsilon_1^{\ 0} = 0.3\Delta$  and  $\epsilon_2^{\ 0} = -0.5\Delta$ , P = 0.1,  $\Gamma_{F0} = \Gamma_S = V_g = 0.25\Delta$ , and  $E_{Ze} = 0(1), 0.25(2), 0.05\Delta(3)$ .

The resonant Andreev reflection probability  $T_4(\omega)$  vs the energy. The dependence on the spin polarization of the F-lead current P the spin-splitting of the mQD levels under the effect of the effective (proximity induced and external) Zeeman energy  $E_{\rm Ze}$  (Fig. 2), on the gate voltage  $V_{\rm g}({\rm Fig.~3})$  and on linewidth  $\Gamma_{\rm F0}$  (Fig. 4).

Fig. 5 demonstrates that under the effect of a gate voltage and proximity-induced spin splitting Zeeman energy the dot's zero-base conductance peak imitates (can be interpreted as) MZM. Fig. 6 demonstrates a series of additional peaks appeared due to energy levels spin-splitting in the mQD by the external magmetic field. I.e., for distinguishing between trivial and topological zero-bias conductance peculiarities arising from the coalescence of ABS, the magnetic field effect on the peaks' position is important.

### **Conclusions**

In this report, motivated by recent proposals for Majorana fermions the tunneling transport peculiarities of a realization, we considered ferromagnetic metal-magnetic quantum dot-superconductor hybrid structure. Special attention is devoted to analyzing the implications of the spin splitting electrons levels in the dot on the tunneling process and system's conductance characteristics. The sensitivity of the F-mQD-SC nanostructures conductance behavior on such external influence as (i) the spin splitting energy levels in the dot by an applied magnetic field, (ii) the current spin polarization of the Flead, (iii) the gate voltage etc. have been studied in detail. For this simple model system, we found a specific distinguishing between trivial and topological conductance characteristics arising from the coalescence of Andreev bound states. We suggest the results obtained can provide helpful clarification for understanding recent experiments in superconductor ferromagnet hybrid nanostructures with topologically protected excitations.

[1] V. N. Krivoruchko and E. A. Koshina. Low Temp. Phys. 49, 1015 (2023). https://doi.org/10.1063/10.0020593