Broadband and resonant spectroscopy of thin film resonators from disordered superconductors

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Abstract and introduction

The technique of non-contact broadband transmission line and resonant flip-chip spectroscopy is utilized to probe resonances of mm-sized square resonators fabricated from strongly disordered molybdenum carbide films in the GHz frequency range[1]. The temperature dependence of the resonances was modelled by the surface impedance of the thin films via the complex conductivity of disordered superconductors[2], which reflects the Dynes superconducting density of states of these superconductors. The obtained Dynes broadening parameters from broadband spectroscopy relate reasonably well to those known from scanning tunnelling spectroscopy measurements. The resonant spectroscopy was performed on a high quality niobium resonator, with significant drop in resonance quality with nonlinear behavior when the probing resonator frequency tunes to the strongly temperature dependent resonance frequency of MoC film. The system was modelled by lumped element approximation of inductively coupled resonators with nonlinear inductance and yields qualitative agreement with the experimental data. The nonlinear response can be utilized in e.g. parametric amplifiers. The bare MoC thin films, due to their high disorder, can be utilized e.g. as tunable superconducting filters with small dimensions. Moreover, this method is also valid for e.g. NbN/NbTiN/MoN thin films, which are commonly used in superconducting electronics, where the kinetic inductance plays a crucial role in device performance (such as SNSPDs, or TWPAs).

Complex conductivity of disordered superconductors

- Disordered superconductors
  - smeared peaks in superconducting DOS
  - increased sheet resistance R_s
  - suppressed critical temperature T_c
  - Increased kinetic inductance L_k
- Approximative formula for L_k:

$$L_k = \frac{\hbar R_s}{\pi\Delta(T)\tanh\left(\frac{\Delta(T)}{2k_bT}\right)} \qquad (1)$$

- Mattis-Bardeen complex conductivity not accurate
- Dynes SDOS related to complex conductivity [2]
- $\sigma(T) = \sigma_1 + i\sigma_2$ calculated numerically from eq. [2] and ref.[2]; L_k is generalized to:

$$L_k = \frac{1}{2\pi f\sigma_2(T)} \qquad (2)$$

- For disordered samples, mismatch in Lk is increased up to 2x

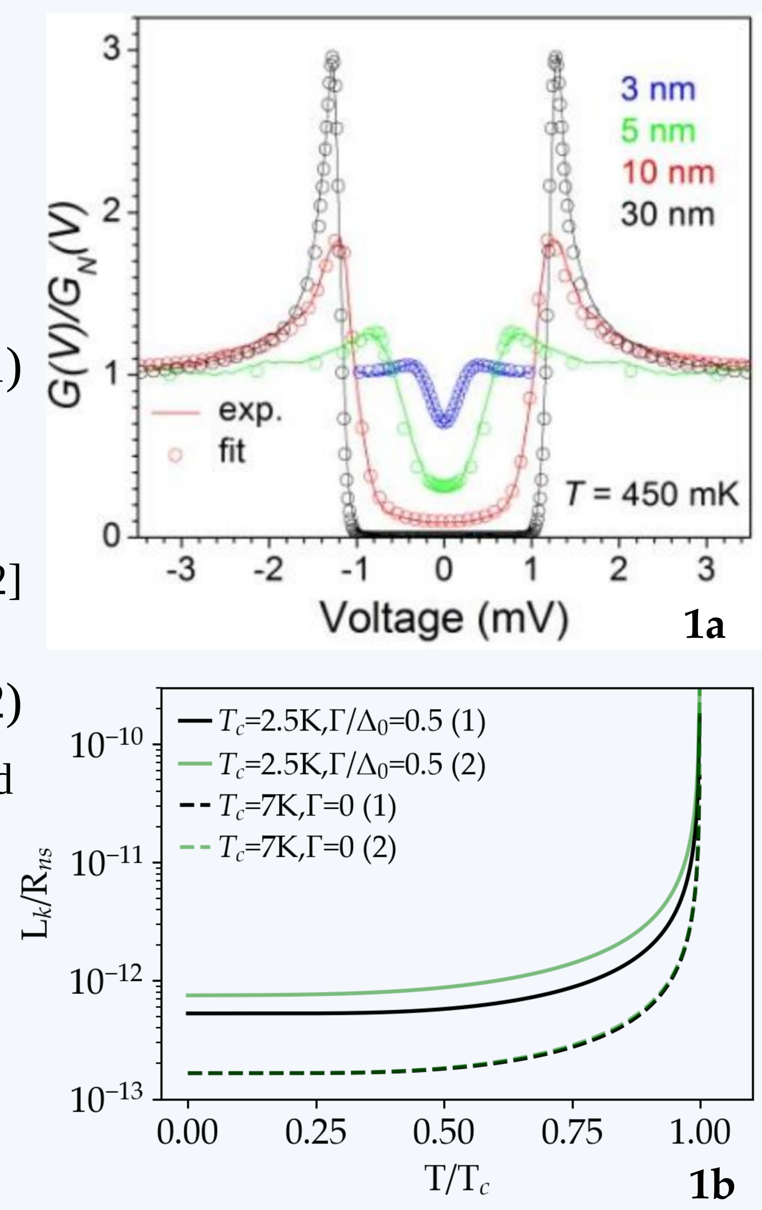

Figure 1: a) Typical tunneling spectra of thin MoC films at T=450mK; from [4]; b) Normalized kinetic inductance for low-disorder (dashed) and high-disorder (full) sample; note the difference between Lk from eq. (1) (green) and eq. (2) (black).

Temperature dependence of resonance frequency

- Lumped element model – capacitively coupled parallel RLC resonance circuit [3]:

$$f_n(T) \approx \frac{1}{\sqrt{(L+L_k)(C+C_c)}} = \frac{f_n^0}{\sqrt{1+g_nL_k(T)}} \qquad (3)$$

  - f_n^0, g_n - n[th] harmonic unperturbed resonance frequency and ratio of kinetic to geometric inductance (coupling strength)
  - Independent of geometry and coupling type
  - $L_k(T)$ from eq. (2), where $\sigma(f,T,\Delta_0,\Gamma)$ is the complex conductivity of Dynes superconductors [2] with Dynes' parameter of disorder Γ
  - By modelling the resonances, T_C^{spec} and Γ^{spec} were extracted

PKIR # (sample #)	R_s $(\Omega/sq.)$	L_k^{MB} $(\mathrm{pH}/sq.)$	L_k^{Γ} $(\mathrm{pH}/sq.)$	T_C^{DC} (K)	T_C^{spec} (K)	Γ^{spec}/Δ_0	Γ^{STS}/Δ_0
1	270	63	69	5.70	5.71	0.15	0.14
2	565	154	176	4.85	5.11	0.2	0.20
3	788	281	346	3.70	3.85	0.3	0.28
4	974	527	750	2.44	2.52	0.5	0.44
5	1022	563	800	2.40	2.14	0.5	0.50

Table 1: Sample parameters: R_s – sample sheet resistance; L_k^{MB}, L_k^{Γ}- kinetic inductance at T=0 for Mattis-Bardeen theory and model with Γ; T_C^{DC} critical temperature from DC; T_C^{spec}, Γ^{spec} and Γ^{STS} are estimated from the resonance temperature dependence (spec) and from scanning tunneling spectroscopy (STS) [4].

Broadband flip-chip spectroscopy

- Series of MoC PKIRs (planar kinetic inductance resonator) with thickness *t* = 5nm:
  - Disorder tuned by stoichiometry
- Flip-chip transmission line spectroscopy utilizing VNA (1 - 16GHz)
- multiple resonances observed in transmission spectra (fig. 1c) [1]
- High L_k - resonances of mm-sized PKIRs present at 2.5GHz

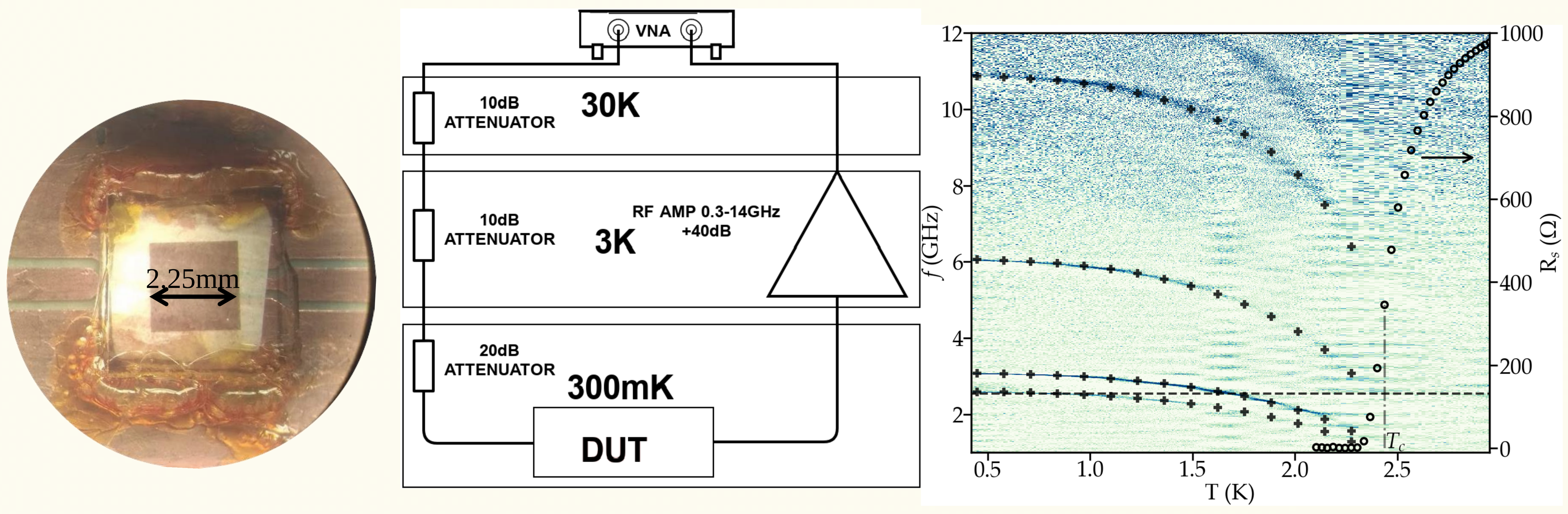

Figure 4: a) A flip-chip setup. PKIR suspended over CPW and glued by non-conducting Ge varnish; b) Scheme of the microwave setup in sorption refrigerator; c) Resonance spectra of PKIR #4, and corresponding fits, according to eq. (3). Measured transmission spectra – color-coded (minimum – blue)

Broadband model

- Transmission spectra are modeled by series of shunted RLC resonators with inductive L_C or capacitive coupling C_C to the transmission line (fig. 2b);
- Qualitative agreement with the measured spectra for MoN PKIR with Rs=720Ω is shown in fig. 2a and 2c

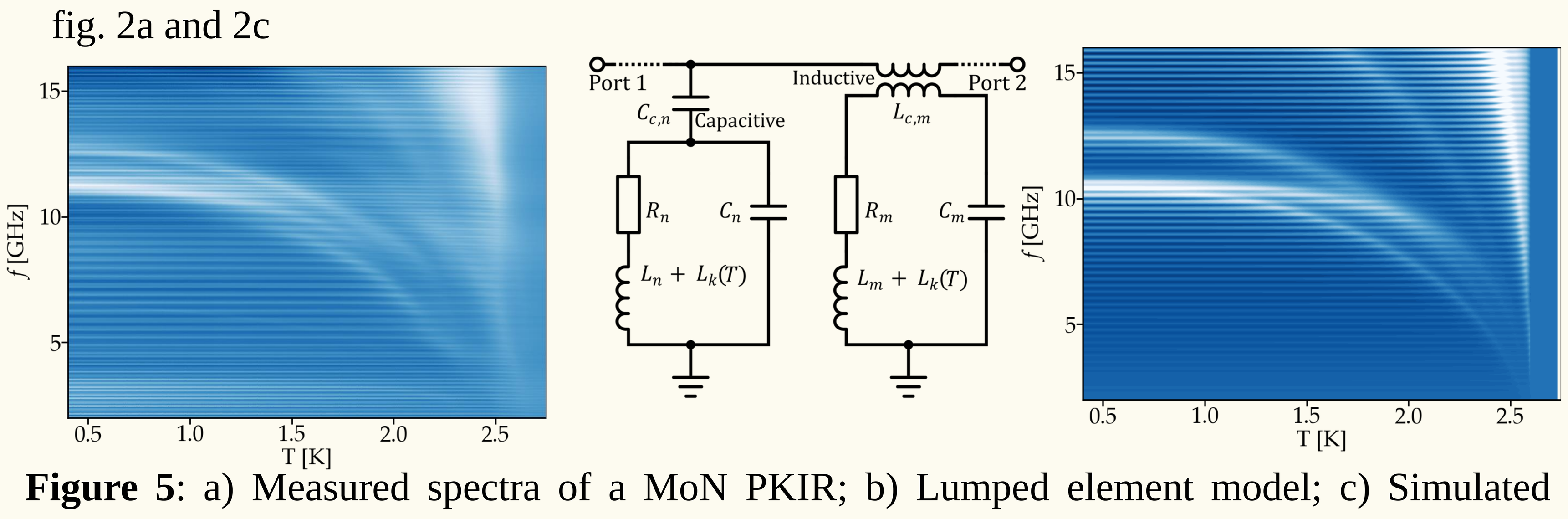

Figure 5: a) Measured spectra of a MoN PKIR; b) Lumped element model; c) Simulated spectra – 5 lumped element resonances with temperature dependent kinetic inductance.

Resonant spectroscopy

- MoC sample #4 suspended over a high-Q Nb coplanar resonator with f_0 = 2.539GHz and $Q_0 \approx 4\times10^6$ at T = 0.35K
- At T=0.35K, $f_{r,PKIR}(T) = f_0$ - system of two coupled resonators
- Strong power dependence measured, originating in the nonlinear L_k of MoC film
- Modeled as a RLC resonator model with non-linear inductance $L_k = L_k^0(T)\left(1+\left(\frac{I_{PKIR}}{I^*}\right)^2\right)$

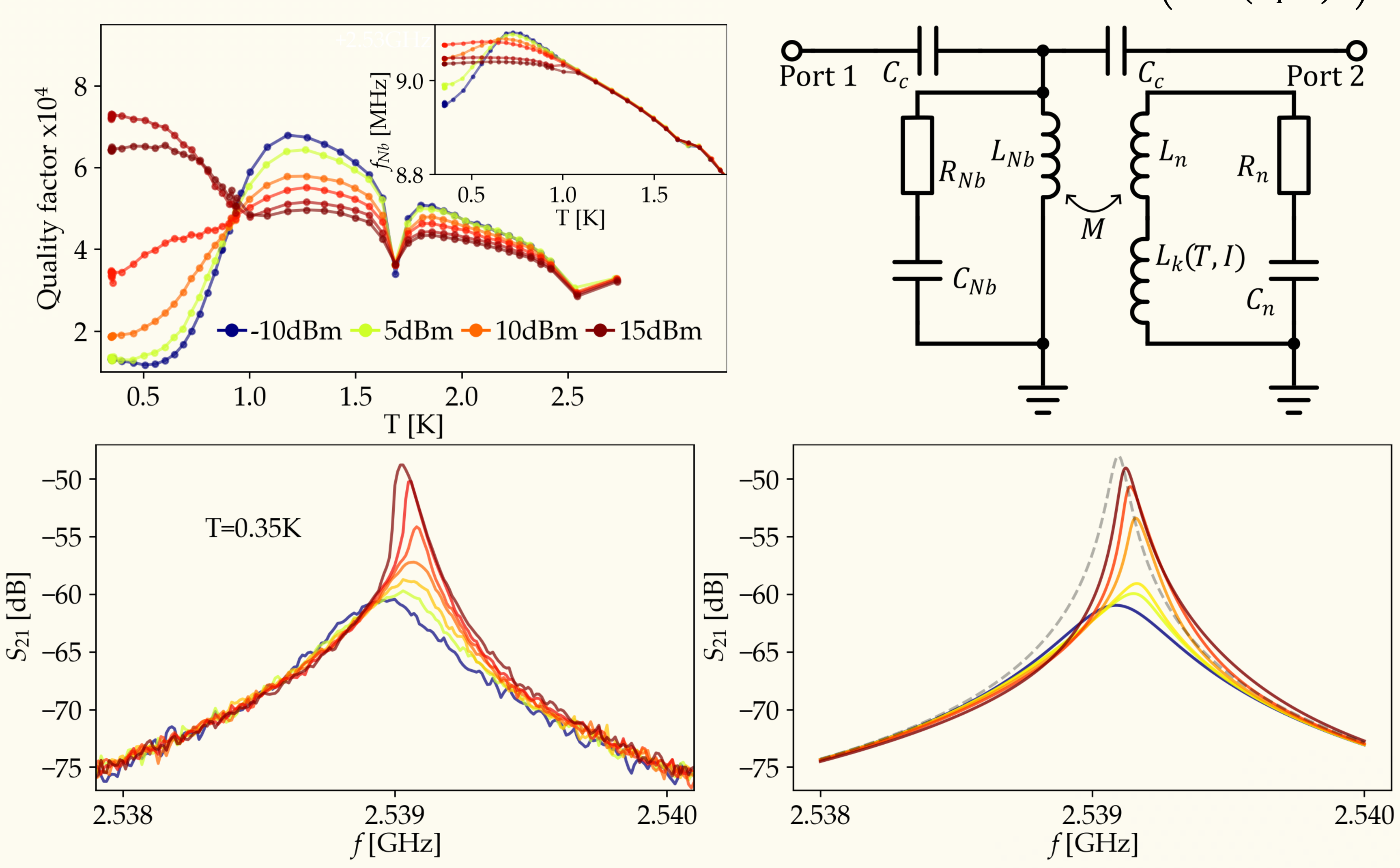

Figure 3: : a) f_0 and *Q*-factor of the Nb resonator coupled ot the PKIR; b) Model of the m-th PKIR mode coupled to n[th] Nb mode; c) Measured spectra at T = 0.35K for different input powers; d) Transmission spectra of the model. Dashed line - PKIR resonance when f_0 detuned

Conclusion

Temperature dependence of the resonances measured in GHz flip-chip scpectroscopy, governed by kinetic inductance L_k, were fitted with complex conductivity of Dynes superconductors [2]. The obtained parameters T_C and Γ are in agreement with STS and transport measurements (Table 1). In the resonator measurement, strong non-linear power dependence was observed, which was modelled as coupled resonators with quadratic nonlinear inductance.

References

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