Stationary Josephson effect in a weak link between nonunitary triplet superconductors

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A stationary Josephson effect in a weak link between misoriented nonunitary triplet superconductors is investigated theoretically. The non-self-consistent quasiclassical Eilenberger equation for this system is solved analytically. As an application of this analytical calculation, the current-phase diagrams are plotted for the junction between two nonunitary bipolar $f$-wave superconducting banks. A spontaneous current parallel to the interface between superconductors is observed. Also, the effect of misorientation between crystals on the Josephson and spontaneous currents is studied. Such experimental investigations of the current-phase diagrams can be used to test the pairing symmetry in the above-mentioned superconductors. © 2005 American Institute of Physics. [DOI: 10.1063/1.1943531]

1. INTRODUCTION

In recent years, triplet superconductivity has become a popular subject for research in the field of superconductivity. Particularly, the nonunitary spin triplet state in which Cooper pairs may carry a finite averaged intrinsic spin moment has attracted much attention in the last decade. A triplet state in the momentum space $k$ can be described by the order parameter $\Delta(k) = i(d(k) \cdot \sigma)\hat{\sigma}_j$ in a $2 \times 2$ matrix form in which the $\hat{\sigma}_j$ are $2 \times 2$ Pauli matrices ($\sigma = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$). The three-dimensional complex vector $d(k)$ (gap vector) describes the triplet pairing state. In the nonunitary state, the product $\Delta(k)^\dagger\Delta(k) = d(k) \cdot d^*(k) + i(d(k) \times d^*(k)) \cdot \sigma$ is not a multiple of the unit matrix. Thus in a nonunitary state the time reversal symmetry is necessarily broken spontaneously and a spontaneous moment $m(k) = i(d(k) \times d^*(k))$ appears at each point $k$ of the momentum space. In this case the macroscopically averaged moment $\langle m(k) \rangle$ integrated on the Fermi surface does not vanish. The value $\langle m(k) \rangle$ is related to the net spin average by $\text{Tr}[\Delta(k)^\dagger\hat{\sigma}_j\Delta(k)]$. It is clear that the total spin average over the Fermi surface can be nonzero. As an application, the nonunitary bipolar state of $f$-wave pairing symmetry has been considered for the $B$ phase of superconductivity in the compound UPt$_3$ which has been created at low temperatures $T$ and small values of the magnetic field $H$.  

In the present paper, the ballistic Josephson weak link via an interface between two superconducting bulks with different orientations of the crystallographic axes is investigated. This type of weak-link structure can be used for the demonstration of the pairing symmetry in the superconducting phase. Consequently, we generalize the formalism of Ref. 8 for the weak link between triplet superconducting bulks with a nonunitary order parameter. In the Ref. 8 the Josephson effect at a point contact between unitary $f$-wave superconductors was studied. Also, the effect of misorientation on the charge transport was investigated, and a spontaneous current tangential to the interface between the $f$-wave superconductors was observed.

In this paper the nonunitary bipolar $f$-wave model of the order parameter is considered. It is shown that the current-phase diagrams are totally different from the current-phase diagrams of the junction between the unitary triplet (axial and planar) $f$-wave superconductors. Roughly speaking, these different characters can be used to distinguish between nonunitary bipolar $f$-wave superconductivity and the other types of superconductivity. In the weak-link structure between the nonunitary $f$-wave superconductors, the spontaneous current parallel to the interface has been observed as a fingerprint for unconventional superconductivity and spontaneous time reversal symmetry breaking. The effect of misorientation on the spontaneous and Josephson currents is investigated. It is possible to find a value of the phase difference at which the Josephson current is zero but a spontaneous current, which is produced by the interface and is tangential to the interface, is present. In some configurations and at the zero phase difference, the Josephson current is not generally zero but has a finite value. This finite value corresponds to a spontaneous phase difference which is related to the misorientation between the gap vectors $d$.  

The arrangement of the rest of this paper is as follows. In Sec. 2 we describe the configuration that we have investigated. For a non-self-consistent model of the order parameter, the quasiclassical Eilenberger equations are solved and suitable Green functions are obtained analytically. In Sec. 3 the formulas obtained for the Green functions are used for the calculation of the current densities at the interface. An analysis of numerical results will be presented in Sec. 4 together with some conclusions in Sec. 5.
FIG. 1. Scheme of a flat interface between two superconducting bulks which are misoriented as much as $\alpha$.

2. FORMALISM AND BASIC EQUATIONS

We consider a model of a flat interface $y=0$ between two misoriented nonunitary $f$-wave superconducting half-spaces (Fig. 1) as a ballistic Josephson junction. In the quasiclassical ballistic approach, in order to calculate the current, we use “transport-like” quasiclassical Eilenberger equations\(^9\) for the energy-integrated Green functions $\tilde{g}(\tilde{\psi}_F, \mathbf{r}, \mathbf{e}_m)$

$$v_F \cdot \nabla \tilde{g} + [\mathbf{e}_m \tilde{\sigma}_3 + i \tilde{\Delta}, \tilde{g}] = 0,$$  

(1)

and the normalization condition $\tilde{g} \tilde{g} = 1$, where $\mathbf{e}_m = \pi T(2m+1)$, with $m = 1, 2, \ldots$, are discrete Matsubara energies, $T$ is the temperature, $v_F$ is the Fermi velocity, and $\tilde{\sigma}_i = \sigma_i \otimes \mathbf{1}$, in which the $\sigma_j$ ($j = 1, 2, 3$) are Pauli matrices. The Matsubara propagator $\tilde{g}$ can be written in the form:

$$\tilde{g} = \begin{pmatrix} g_1 + g_1 \cdot \tilde{\sigma}  & (g_1 + g_2 \cdot \tilde{\sigma}) i \tilde{\sigma}_2 \\ i \tilde{\sigma}_2 (g_1 + g_3 \cdot \tilde{\sigma})  & g_4 - \tilde{\sigma}_2 g_4 \cdot \tilde{\sigma}_2 \\ \end{pmatrix},$$  

(2)

where the matrix structure of the off-diagonal self-energy $\tilde{\Delta}$ in the Nambu space is

$$\tilde{\Delta} = \begin{pmatrix} 0 & \mathbf{d} \cdot \tilde{\sigma} \tilde{\sigma}_2 \\ i \tilde{\sigma}_2 \mathbf{d}^* \cdot \tilde{\sigma} & 0 \end{pmatrix}.$$  

(3)

The nonunitary states, for which $\mathbf{d} \times \mathbf{d}^* \neq 0$ are investigated. Fundamentally, the gap vector (order parameter) $\mathbf{d}$ has to be determined numerically from the self-consistency equation,\(^1\) while in the present paper, we use a non-self-consistent model for the gap vector which is much more suitable for analytical calculations.\(^10\) Solutions to Eq. (1) must satisfy the following conditions for the Green functions and the gap vector $\mathbf{d}$ in the bulks of the superconductors far from the interface:

$$\frac{1}{\Omega_n} \int [i \tilde{\sigma}_2 (i \mathbf{d}^*_n + \mathbf{d}^*_n \times \mathbf{A}_n) \cdot \tilde{\sigma} - \tilde{\sigma}_2 (1 + \mathbf{A}_n \cdot \tilde{\sigma}) \tilde{\sigma}_2],$$  

(4)

where

$$\mathbf{A}_n = \frac{i \mathbf{d}_n \times \mathbf{d}^*_n}{e_m^2 + \mathbf{d}_n \cdot \mathbf{d}^*_n + \sqrt{(e_m^2 + \mathbf{d}_n \cdot \mathbf{d}^*_n)^2 + (\mathbf{d}_n \times \mathbf{d}^*_n)^2}}$$  

(5)

and

$$\Omega_n = \sqrt{2[(e_m^2 + \mathbf{d}_n \cdot \mathbf{d}^*_n)^2 + (\mathbf{d}_n \times \mathbf{d}^*_n)^2]}.$$  

(6)

$$\mathbf{d}(\pm \varphi) = \mathbf{d}_{1,2}(T, \tilde{\psi}_F) \exp \left( \mp \frac{i \varphi}{2} \right),$$  

(7)

where $\varphi$ is the external phase difference between the order parameters of the bulks and $n = 1, 2$ labels the left and right half-spaces, respectively. It is clear that poles of the Green function in the energy space are at

$$\Omega_n = 0.$$  

(8)

Consequently,

$$(-E^2 + \mathbf{d}_n \cdot \mathbf{d}^*_n)^2 + (\mathbf{d}_n + \mathbf{d}^*_n)^2 = 0$$  

(9)

and

$$E = \pm \sqrt{\mathbf{d}_n \cdot \mathbf{d}^*_n \pm i \mathbf{d}_n \times \mathbf{d}^*_n}$$  

(10)

in which $E$ is the energy value of the poles. Equation (1) has to be supplemented by the continuity conditions at the interface between superconductors. For all quasiparticle trajectories, the Green functions satisfy the boundary conditions both in the right and left bulks as well as at the interface. The system of equations (1) and the self-consistency equation for the gap vector $\mathbf{d}$ (Ref. 1) can be solved only numerically. For unconventional superconductors such solution requires information about the interaction between the electrons in the Cooper pairs and the nature of unconventional superconductivity in novel compounds which in most cases is unknown. Also, it has been shown that the absolute value of a self-consistent order parameter is suppressed near the interface and at the distances of the order of the coherence length, while its dependence on the direction in the momentum space almost remains unaltered.\(^11\) This suppression of the order parameter changes the amplitude value of the current, but does not influence the current-phase dependence drastically. For example, it has been verified in Ref. 12 for the junction between unconventional $d$-wave superconductors, in Ref. 11 for the case of unitary “$f$-wave” superconductors, and in Ref. 13 for pinholes in $^3$He, that there is good qualitative agreement between self-consistent and non-self-consistent results for not very large angles of misorientation. It has also been observed that the results of the non-self-consistent model in Ref. 14 are similar to experiment.\(^15\) Consequently, despite the fact that this solution cannot be applied directly to a quantitative analysis of a real experiment, only a qualitative comparison of calculated and experimental current-phase relations is possible. In our calculations, a simple model of the constant order parameter up to the interface is considered, and the pair-breaking and scattering on the interface are ignored. We believe that under these strong assumptions our results describe the real situation qualitatively. In the framework of such a model, analytical expressions for the current can be obtained for a certain form of the order parameter.

3. ANALYTICAL RESULTS

The solution of Eq. (1) allows us to calculate the current densities. The expression for the current is
\[ j(\mathbf{r}) = 2i\pi eTN(0) \sum_m \langle \mathbf{v}_F \mathbf{g}_1(\mathbf{\xi}_F, r, \mathbf{e}_m) \rangle, \]

where \( \langle \ldots \rangle \) stands for averaging over the directions of an electron momentum on the Fermi surface \( \mathbf{\xi}_F \) and \( N(0) \) is the electron density of states at the Fermi level of energy. We assume that the order parameter is constant in space and in each half-space it equals its value \( \langle \ldots \rangle \) far from the interface in the left or right bulks. For such a model, the current-phase dependence of a Josephson junction can be calculated analytically. It enables us to analyze the main features of current-phase dependence for any model of the nonunitary order parameter. The Eilenberger equations \( (1) \) for Green functions \( \mathbf{g} \), which are supplemented by the condition of continuity of solutions across the interface, \( y = 0 \), and the boundary conditions at the bulks, are solved for a non-self-consistent model of the order parameter analytically. In the ballistic case the system of equations for functions \( \mathbf{g}_1 \) and \( \mathbf{g}_2 \) can be decomposed into independent blocks of equations.

The set of equations which enables us to find the Green function \( \mathbf{g}_1 \) is:

\[ v_F \mathbf{k} \mathbf{g}_1 = i(\mathbf{d} \cdot \mathbf{g}_2 - \mathbf{d}^* \cdot \mathbf{g}_2); \]  
\[ v_F \mathbf{k} \mathbf{g}_2 = -2(\mathbf{d} \cdot \mathbf{g}_1 + \mathbf{d}^* \cdot \mathbf{g}_2); \]  
\[ v_F \mathbf{k} \mathbf{g}_2 = -2\varepsilon_m \mathbf{g}_2 + 2i\gamma \mathbf{d} \cdot \mathbf{g}_2; \]  
\[ v_F \mathbf{k} \mathbf{g}_2 = 2\varepsilon_m \mathbf{g}_2 - 2i\gamma \mathbf{d} \cdot \mathbf{g}_2, \]

where \( \mathbf{g}_1 = \mathbf{g}_1 - \mathbf{g}_2 \). Equations \( (12)-(14) \) can be solved by integrating over the ballistic trajectories of electrons in the right and left half-spaces. The general solution satisfying the boundary conditions \( (4) \) at infinity is

\[ g_1^{(n)} = \frac{\varepsilon_m}{\Omega_n} + a_n \exp(-2s\Omega_n t); \]  
\[ g_2^{(n)} = -2\varepsilon_m \mathbf{A}_n + C_n \exp(-2s\Omega_n t); \]  
\[ g_2^{(n)} = \frac{id_n \mathbf{d} \cdot \mathbf{A}_n}{\Omega_n} - \frac{2i\eta_d \mathbf{d} \cdot \mathbf{d}^* \mathbf{C}_n}{2s\eta \Omega_n - 2\eta_m} \exp(-2s\Omega_n t); \]  
\[ g_3^{(n)} = \frac{id_n \mathbf{d}^* \cdot \mathbf{d}^* \mathbf{A}_n}{\Omega_n} + \frac{2i\eta_d \mathbf{d} \cdot \mathbf{d}^* \mathbf{C}_n}{2s\eta \Omega_n + 2\eta_m} \exp(-2s\Omega_n t), \]

where \( t \) is the time of flight along the trajectory, \( \text{sgn}(t) = \text{sgn}(y) = s \) and \( \eta = \text{sgn}(x) \). By matching the solutions \( (16)-(19) \) at \( y = 0 \), we find the constants \( a_n \) and \( C_n \). Indices \( n = 1,2 \) label the left and right half-spaces, respectively. The function \( g_1(0) = g_1^{(1)}(-0) = g_1^{(2)}(+0) \) which is a diagonal term of the Green matrix and determines the current density at the interface, \( y = 0 \), is as follows:

\[ g_1(0) = \frac{\eta_3(\mathbf{d}_1 \cdot \mathbf{d}_2(\eta \Omega_1 + \varepsilon)^2 - \mathbf{d}_1(\eta \Omega_2 - \varepsilon)^2 + B)}{[\mathbf{d}_2(\eta \Omega_1 + \varepsilon) + \mathbf{d}_1(\eta \Omega_2 - \varepsilon)]^2}, \]

where \( B = i\mathbf{d}_1 \cdot \mathbf{d}_2(A + A_2(\eta \Omega_2 - \varepsilon)(\eta \Omega_1 + \varepsilon)) \). We consider a rotation \( \mathbf{\hat{R}} \) only in the right superconductor (see Fig. 1), i.e., \( \mathbf{d}_2(\mathbf{\hat{R}}^{-1} \mathbf{\hat{k}}) = \mathbf{\hat{R}} \mathbf{d}_1(\mathbf{\hat{R}}^{-1} \mathbf{\hat{k}}) \); \( \mathbf{\hat{k}} \) is the unit vector in the momentum space. The crystallographic \( c \) axis in the left half-space is selected parallel to the partition between the superconductors (along the \( z \) axis in Fig. 1). To illustrate the results obtained by computing the formula \( (20) \), we plot the current-phase diagrams for two different geometries. These geometries correspond to the different orientations of the crystals in the right and left sides of the interface (Fig. 1).

(i) The basal \( ab \) plane in the right side has been rotated around the \( c \) axis by \( \gamma \mathbf{e}_1 \parallel \mathbf{e}_2 \).

(ii) The \( c \) axis in the right side has been rotated around the \( b \) axis by \( \gamma \mathbf{e}_1 \parallel \mathbf{e}_2 \).

Further calculations require a certain model of the gap vector (order parameter) \( \mathbf{d} \).

4. ANALYSIS OF NUMERICAL RESULTS

In the present paper, the nonunitary \( f \)-wave gap vector in the \( B \) phase (low temperature \( T \) and low field \( H \)) of superconductivity in the compound UPT\(_3\) has been considered. This nonunitary bipolar state which explains the weak spin-orbit coupling in UPT\(_3\) is

\[ \mathbf{d}(T, v_F) = \Delta_0(T)k_y(\mathbf{\hat{k}}(k_x^2 - k_y^2) + \gamma^2 ik_x k_y). \]

The coordinate axes \( \mathbf{\hat{x}}, \mathbf{\hat{y}}, \mathbf{\hat{z}} \) are chosen along the crystallographic axes \( \mathbf{\hat{a}}, \mathbf{\hat{b}}, \mathbf{\hat{c}} \) in the left side of Fig. 1. The function \( \Delta_0 = \Delta_0(T) \) describes the dependence of the gap vector on the temperature \( T \) (our numerical calculations are done at the low value of temperature \( T/T_c = 0.1 \)). Using this model of the order parameter \( (21) \) and solution to the Eilenberger equations \( (20) \), we have calculated the current density at the interface numerically. These numerical results are listed below.

1. The nonunitary property of Green’s matrix diagonal term consists of two parts. An explicit part, which is contained in the mathematical expression \( B \) in Eq. \( (20) \), and an implicit part in the \( \Omega_{1,2} \) and \( \mathbf{d}_{1,2} \) terms. These \( \Omega_{1,2} \) and \( \mathbf{d}_{1,2} \) terms are different from their unitary counterparts. In the mathematical expression for \( \Omega_{1,2} \) the nonunitary mathematical terms \( \mathbf{A}_{1,2} \) are presented. The explicit part will be present only in the presence of misorientation between gap vectors, \( B = i\mathbf{d}_1 \cdot \mathbf{d}_2(A_1 + A_2)(\eta \Omega_2 - \varepsilon)(\eta \Omega_1 + \varepsilon) \), but the implicit part will be present always. So, in the absence of misorientation \( (\mathbf{d}_1 \parallel \mathbf{d}_2) \), although the implicit part of nonunitary exists the explicit part is absent. This means that in the absence of misorientation, current-phase diagrams for planar unitary and nonunitary bipolar systems are the same, but the maximum values are slightly different.

2. For geometry (i) one of the current components parallel to the interface, \( j_z \), is zero, as in the unitary case, and the other parallel component \( j_x \) has a finite value (see Fig. 4). This last is a difference between the unitary and nonunitary cases. Because in the junction between unitary \( f \)-wave superconducting bulks all parallel components of the current \( (j_x \) and \( j_z) \) for geometry (i) are absent.

3. In Figs. 2 and 3 the Josephson current \( j_x \) is plotted for a certain nonunitary \( f \)-wave model in different geometries. Figures 2 and 3 are plotted for the geometries (i) and (ii), respectively. They are completely unusual and totally different from their unitary counterparts which were obtained in Ref. 8.

4. In Fig. 2 for geometry (i), it is observed that by increasing the misorientation, some small oscillations appear in the current-phase diagrams as a result of the nonunitary
property of the order parameter. Also, the Josephson current at the zero external phase difference $\varphi = 0$ is not zero but has a finite value. The Josephson current will be zero at the some finite values of the phase difference.

5. In Fig. 3 for geometry (ii), it is observed that by increasing the misorientation, new zeros appear in the current-phase diagrams, and the maximum value of the current will be change nonmonotonically. In contrast to the case for geometry (i) (Fig. 2), the Josephson currents at the phase differences $\varphi = 0$, $\varphi = \pi$, and $\varphi = 2\pi$ are exactly zero.

6. The current-phase diagram for geometry (i) and the $x$ component (Fig. 4) is totally unusual. By increasing the misorientation, the maximum value of the current increases. The components of current parallel to the interface for geometry (ii) are plotted in Figs. 5 and 6. All the terms are zero at the phase differences $\varphi = 0$, $\varphi = \pi$, and $\varphi = 2\pi$. The maximum value of the current-phase diagrams is not a monotonic function of the misorientation.

5. CONCLUSIONS

Thus we have studied theoretically the supercurrents in a ballistic Josephson junction in the model of an ideal transparent interface between two misoriented UPt$_3$ crystals with nonunitary bipolar $f$-wave superconducting bulks which are subject to a phase difference $\varphi$. Our analysis has shown that misorientation between the gap vectors creates a current parallel to the interface and that different misorientations between gap vectors influence the spontaneous parallel and normal Josephson currents. These have been shown separately in Ref. 8 for the currents in point contacts between two bulk unitary axial superconductors and between two bulk planar $f$-wave superconductors. We have also shown that the misorientation of the superconductors leads to a spontaneous
phase difference that corresponds to zero Josephson current and to the minimum of the weak-link energy in the presence of a finite spontaneous current. This phase difference depends on the misorientation angle. The tangential spontaneous current is not generally equal to zero in the absence of the Josephson current. The difference between unitary planar and nonunitary bipolar states can be used to distinguish between them. This experiment can be used to test the pairing symmetry and recognize the different phases of $\text{UPt}_3$.

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