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Dynamics of bound soliton states in regularized dispersive equations

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The nonstationary dynamics of topological solitons (dislocations, domain walls, fluxons) and their bound states in one-dimensional systems with high dispersion are investigated. Dynamical features of a moving kink emitting radiation and breathers are studied analytically. Conditions of the breather excitation and its dynamical properties are specified. Processes of soliton complex formation are studied analytically and numerically in relation to the strength of the dispersion, soliton velocity, and distance between solitons. It is shown that moving bound soliton complexes with internal structure can be stabilized by an external force in a dissipative medium then their velocities depend in a step-like manner on a driving strength.

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05.45.Yv Solitons;
75.40.Gb Dynamic properties.

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1. Introduction

The soliton concept in applied science was formulated 35 years ago in the prominent review by Scott, Chu, and McLaughlin [1]. Since the soliton search developed into both well-established mathematical and physical theories. They cover a wide range of problems beginning from complete integrability of nonlinear equations [2,3] up to applications of the soliton concept for explanation of nonlinear phenomena in various fields of condensed matter physics [4–7]. Topological defects and inhomogeneties such as dislocations in crystals, domain walls and vortices in magnets, quanta of magnetic flux (fluxons) in long Josephson junctions, are a few examples of traditional physical objects which are described in terms of solitons in solid state physics.

Two pioneer works by Kosevich and Kovalev [8,9], devoted to nonlinear dynamics of one-dimensional crystals, initiated two novel directions in soliton investigations. In Ref. 8 the physical meaning of a self-localized excitation was introduced for the first time. In the long wave limit such an oscillating solitary wave corresponds to the breather which is interpreted as the soliton–antisoliton bound state. The authors proposed a regular asymptotic procedure to construct the self-localized oscillation [8,10]. In the short-wave limit Kosevich and Kovalev predicted the existence of a self-localized oscillation with a frequency above the upper edge of a linear excitation spectrum. Later a high localization limit of these high-frequency soliton states were studied and they called the intrinsic localized modes or discrete breathers which became a new concept in nonlinear lattice theory [11,12].

In the work [9], which concerned crowdion dynamics in an one-dimensional anharmonic crystal, Kosevich and Kovalev established, for the first time to our knowledge, existence of supersonic and radiationless motion of topological solitons in a highly dispersive nonlinear medium. The equations deduced in the work [9] generalize the Boussinesq equation for the case of the sine–Gordon (SG) and φ^4-models:

$$u_{tt} - u_{xx} + (\alpha - \gamma u_x) u_x u_{xx} - \beta u_{xxxx} + F(u) = 0, \quad (1)$$

where the external force equals to either $F(u) = \sin u$, or $F(u) = u^3 - u$, respectively. The equation (1) with the sine force and $\alpha = 0$

$$u_{tt} - u_{xx} - \gamma u_x^2 u_{xx} - \beta u_{xxxx} + \sin u = 0 \quad (2)$$

is known nowadays as the Kosevich–Kovalev equation [13,14]. For the special choice of parameter $\gamma = 3\beta/2$, Kosevich and Kovalev found an exact solution describing the 2π-kink moving with an arbitrary velocity [9]. One year later an integrable version of the equation was proposed by Konno, Kameyama, and Sanuki [15]:

$$u_{xt} - u_{xx} - \frac{3}{2}\beta u_x^2 u_{xx} - \beta u_{xxxx} + \sin u = 0, \qquad (3)$$

and this fact could explain formally the existence of the exact kink solution in Eq.(2). However significance of the fact of radiationless motion of topological solitons in highly dispersive media was realized after many years. In 1984 Peyrard and Kruskal showed numerically the existence of a stable moving 4π-soliton in the highly discrete SG model [16]:

$$\frac{\partial^2 u_n}{\partial \tau^2} + 2u_n - u_{n-1} - u_{n+1} + \frac{1}{d^2}\sin u_n = 0, \qquad (4)$$

where d is discreteness parameter. They tried to explain the formation of the bound state of two identical kinks by exploiting the fact of the presence of the Peierls potential in the lattice model. However, in work [17] it has been found that the radiationless motion of such a soliton complex can be described explicitly by the exact 4π-soliton solution in the framework of the dispersive SG equation with a fourth spatial derivative, i.e. Eq. (2) with $\gamma = 0$:

$$u_{tt} - u_{xx} - \beta u_{xxxx} + \sin u = 0 \qquad (5)$$

which is obtained as the long-wave limit of the Eq. (4). Almost simultaneously the topological bound soliton states were found numerically in the continuous nonlocal SG model describing long Josephson junctions [18]. These facts of existence of the multikink bound states in discrete and continuous systems were generalized as a universal phenomenon and led to the concept of the soliton complexes formed by strongly interacting kinks in highly dispersive media [19–21]. Physically such two-kinks states correspond, e.g., to a moving defect consisting of two neighboring dislocation half-planes, or to a narrow 360° magnetic domain wall, which arises even in the absence of magnetic field, or to a bound pair of fluxons in a long Joshephson junction.

There are some approaches to explain mechanisms of formation of the bound soliton states. The internal structure of the soliton complexes can be studied in detail in models that lead to piecewise linear equations with strong dispersion [20–22]. In this case the stationary states can be constructed as a superposition of two quasi-solitons possessing spatial periodic tails as asymptotics which cancel each other exactly for the composed complex by imposing some interference condition. Noting that these bound solitons occur in resonance with the linear spectrum waves they called embedded solitons [23,24]. The effect of the dispersion can be extracted already from the dispersion relations of corresponding linearized equations [25,26]. However a principal circumstance for a complex arising appears to be the influence of the strong dispersion as a factor leading to complication of internal structure of solitons beginning from a kink level [27]. A taking into account of the interaction of such flexible kinks allows to describe quantitatively conditions of a soliton complex formation [19,21,28].

A picture of the soliton complex formation becomes much more diverse when one considers internal dynamics of kinks, nonstationary motion of complexes, the conditions of their formation and stability depending on different physical factors including the influence of dissipative and external forces [29–31]. The present paper is devoted to investigation of this circle of tasks concentrating on a single kink propagation and especially on the bound soliton states of both types, soliton complexes and breathers, covering essentially nonlinear dynamics of the strongly dispersive SG model.

The paper is organized as follows. Section 2 introduces regularized dispersive equations and some their dynamical properties. Section 3 addresses the nonstationary dynamics of a single 2π-kink in all the range of the dispersive parameter. Section 4 devoted to analysis of the complex formation and its stability conditions. Section 5 deals with the breather dynamics. Section 6 addresses the influence of dissipation and external forces on stabilization of the soliton complexes with an internal structures. Last section summarized obtained results.

2. The regularized dispersive SG equations

To investigate analytically and numerically the nonstationary dynamics of kinks and their bound states we use the regularized dispersive SG equation with a fourth-order spatio-temporal derivative [19–21]:

$$u_{tt} - u_{xx} - \beta u_{xxtt} + \sin u = 0 \qquad (6)$$

where β is a dispersive parameter. This equation has advantage in comparison with Eq. (5) because it does not contain an artificial instability of states $u = 0, 2\pi, 4\pi \ldots$ with respect to a short-wave excitation. An idea of the regularization of dispersive equations belongs to Boussinesq who first proposed to use a mixed spatio-temporal derivative instead of the fourth spatial derivative for the shallow-water waves equations [4]. Such a replacement was justified in the lattice theory by Rosenau [32] for models with nonlinear interactions between atoms. The Boussinesq's idea was applied to the SG and double SG equations with higher dispersion in Refs. [19–21]. At present this approach is actively being used for analytical description of discreteness effects [33–35]. With respect to original discrete models an accuracy of this replacement can be easy to estimate. In the long-wave limit ($d >> 1$) after introducing a coordinate $x = n/d$ the second difference is replaced as $u_{n-1}+u_{n+1}-2u_n \approx u_{xx}+\beta u_{xxxx}$, where $\beta = 1/12d^2$. If one expresses a second derivative from Eq. (5) as $u_{xx} = u_{tt} - \beta u_{xxxx} + \sin u$ and inserts it in the fourth derivative and keeps terms which are linear with respect to β, one obtains the equation:

$$u_{tt} - u_{xx} + \sin u - \beta u_{xxtt} - \beta(\sin u)_{xx} = 0. \tag{7}$$

Hence to approximate Eq. (5) by Eq. (6) one needs to take into account the term with $-\beta(\sin u)_{xx}$. It would be expected that a form of static kinks would be different for all the Eqs. (4)–(7), depending on a value of β. Curiously, it appears that Eq. (5) does not possess a static 2π-kink solution satisfying boundary conditions $u(-\infty)=0$ and $u(\infty)=2\pi$ at all [21]. At the same time exact static kink and moving complex solutions exist simultaneously in Eq. (6). Therefore a lot of problems of kink and complex dynamics can be solved analytically in the framework of this equation. In particular the spectral problem for linear excitations of the static kink has been solved completely [14,29]. Thus Eq. (6) has an exact static kink solution for arbitrary β, which coincides with a kink of the usual SG equation:

$$u_{2\pi}(x) = 4\arctan\exp(x). \tag{8}$$

The kink solution of Eq. (7) can be find in an implicit form using the first integral:

$$\frac{du}{dx} = \frac{2\sin(u/2)}{1+\beta\cos u}\sqrt{1+\beta\cos^2(u/2)}. \tag{9}$$

It appears that even when the discreteness parameter $d=1$ and hence $\beta = 1/12$, the static kink solutions for the discrete equation (4) and continuous Eqs. (6) and (7) differ very slightly (see Fig.1). This justifies the use Eq.(6) instead Eqs. (5) and (7) to explain qualitatively a majority of effects which are inherent in the discrete model (4) but in reality arise due to the higher-order dispersion.

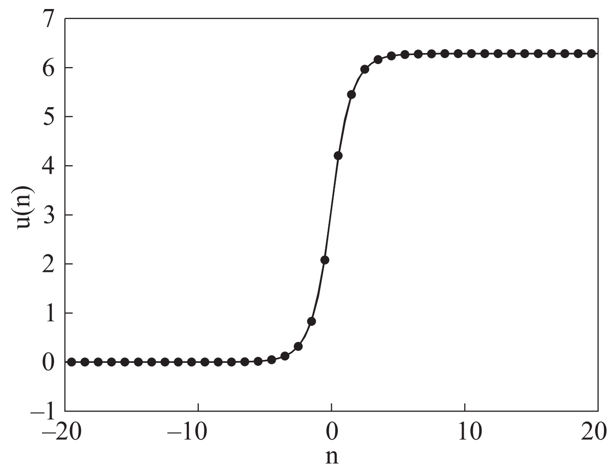

Fig. 1. Comparing static kink profiles for a discrete equation (4) and Eqs. (6) and (7) for $d=1$ ($\beta = 1/12$). Continuous solutions are undistiguishable.

The equation (6) can be derived from the Lagrangian:

$$L = \frac{1}{2}\int [u_t^2 - u_x^2 + \beta u_{xt}^2 - 2(1-\cos u)]dx. \tag{10}$$

Using the expression (10) it is easy to find the first integrals, total energy and momentum:

$$E = \frac{1}{2}\int [u_t^2 + \beta u_{xt}^2 + u_x^2 - 2(1-\cos u)]dx, \tag{11}$$

$$P = \int_{-\infty}^{\infty} u_x(u_t - \beta u_{xxt})dx. \tag{12}$$

Note that first two terms in the Eq. (11) give the kinetic energy therefore the higher-order dispersion in the regularized equations contributes to the kinetic energy whereas in the case of Eq. (5) it produces an additional contribution to the potential energy [21].

Spectrum of linear excitations for Eq. (6) can be found exactly for both cases of a homogenous ground state and in the presence of the static kink (8) [14,29]. The dispersion relation for continuous waves takes the form:

$$\omega(k) = \sqrt{(1+k^2)/(1+\beta k^2)}. \tag{13}$$

This spectrum has the peculiarity of being bounded in frequency not only from below but also from above. This property makes it similar to the spectrum of the initial discrete model (4). Moreover, it simply coincides with the spectrum of the SG model with a nonlocal interaction [18]. In the case of a kink there exists a discrete spectrum of internal modes of oscillations [29], the number of which becomes infinite when $\beta \to 1$ while the continuous spectrum degenerates to one frequency $\omega_0 = 1$.

At last it is remarkable that Eqs. (5) and (6) have exact solutions describing a moving 4π-soliton complexes. For Eq. (6) the moving bound state of strongly coupled kinks has a form:

$$u_{4\pi}(x,t) = 8\arctan\left[\exp\left(\frac{x-V_0 t}{l_0}\right)\right]. \tag{14}$$

The velocity V_0 of such a complex, its effective width l_0 and its energy E_0 are specified functions of the parameter β:

$$V_0(\beta) = \sqrt{1+\frac{\beta}{3}} - \sqrt{\frac{\beta}{3}}, \qquad l_0 = (3\beta V_0^2)^{1/4},$$

$$E_0 = 32\left(l_0^{-1} - \frac{l_0}{9}\right). \tag{15}$$

In next two sections we discuss dynamical properties of a single kink and specify conditions of the soliton complex formation.

3. Dynamics of a kink in the dispersive SG model

Internal oscillations of the static kink of the regularized Eq. (6) have been studied theoretically and main features of its nonstationary motion have been revealed numerically [14,29–31]. Here we present analytical approaches to the kink dynamics. One of them consists in application of a perturbation theory for the case of a weak dispersion (small β). In this limit dynamical properties of a kink would be expected to be similar to those in the usual SG equation. The latter has a moving kink $u_{2\pi}(z)$ obtained from the expression (8) by the Lorentz transformation of coordinates: $z=(x-Vt)/\sqrt{1-V^2}$. Therefore one can seek a solution of Eq. (6) in the form:

$$u(x,t)=u_{2\pi}(z)+u_1(z,\tau) \tag{16}$$

where $\tau=(t-Vx)/\sqrt{1-V^2}$ and a small addition function u_1 to the kink form obeys the linearized equation:

$$\left(\frac{\partial^2}{\partial\tau^2}+L\right)u_1 \equiv u_{1\tau\tau}-u_{1zz}+\left(1-\frac{2}{\cosh^2 z}\right)u_1 = =\beta(u_{2\pi}(z)+u_1)_{xxtt}. \tag{17}$$

In the first approximation one has to neglect the term u_1 in the right-hand side of Eq. (17). Then it is easy to find a partial solution of the equation:

$$\Delta u(z)=\alpha\left(3\frac{\sinh z}{\cosh^2 z}-\frac{z}{\cosh z}\right),\quad \alpha=\frac{\beta V^2}{(1-V^2)^2}. \tag{18}$$

A general solution can be written as $u_1(z,\tau)= =\Delta u(z)+v(z,\tau)$ where $v(z,t)$ is a solution of a homogeneous part of Eq. (17) (without the right-hand side). This solve a evolution problem of the SG kink in the dispersive system for the case of small α. Really suppose that at the initial moment $u(z,0)=u_{2\pi}(z)$ and $u_{1\tau}(z,0)=0$. It means that $v(z,0)=-\Delta u(z)$ and $v_\tau(z,0)=0$. Owing to the knowledge of eigenfunctions of operator L, one can solve completely the initial problem for the function $v(z,t)$:

$$v(z,t)=\alpha\sqrt{\frac{\pi}{8}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{1+k^2}}\cos(\sqrt{1+k^2}\,\tau)(\cos kz\tanh z-k\sin kz)\times \times\frac{1}{\cosh\frac{\pi}{2}k}\left(\frac{\pi}{2}k\tanh\frac{\pi}{2}k-1-9k^2\right)dk. \tag{19}$$

This addition to the kink form describes decaying oscillations of the effective kink width which correspond to the SG quasimode. The addition to the stationary reverse kink width $\kappa(\tau)$ is easily found from Eq. (19):

$$\kappa(\tau)=u_{1z}(z,\tau)\big|_{z=0}=\alpha\sqrt{\frac{\pi}{8}}\int_{-\infty}^{\infty}\frac{1-k^2}{\sqrt{1+k^2}}\times \times\frac{\cos(\sqrt{1+k^2}\,\tau)}{\cosh\frac{\pi}{2}k}\left(\frac{\pi}{2}k\tanh\frac{\pi}{2}k-1-9k^2\right)dk \tag{20}$$

and its temporal behavior (Fig. 2) repeats entirely the kink velocity modulation found numerically [29]. The power spectrum of the oscillation reveals Rice's frequency value $\omega_R=2\sqrt{3}/\pi$ [36].

Thus the perturbation theory predicts that the initial SG kink has to evolve into a steady moving profile $u_K(z)=u_{2\pi}(z)+\Delta u(z)$. However it is known [21] that the equation for stationary waves

$$u_{zz}+\alpha u_{zzzz}-\sin u=0, \tag{21}$$

does not possess an exact solution for a moving 2π-kink although one can find formally first terms in asymptotic series for such a solution, which coincide with Eq. (18). The paradox is solved by noting that the solution $u_K(z)$ can be expressed as superposition of two π-kinks in a form

$$u_K(z)=2\arctan\exp\left[\left(1-\frac{\alpha}{2}\right)z+i\sqrt{3\alpha}\right]+ +2\arctan\exp\left[\left(1-\frac{\alpha}{2}\right)z-i\sqrt{3\alpha}\right] \tag{22}$$

which prompts the ansatz for an adiabatic approach to the 2π-kink dynamics. The nonstationary evolution of the kink at small α reduces to decaying collective oscillations of the effective kink width and velocity with a consequent growth of a kink steepness and a slow energy loss due to the radiation emission. With increasing the dispersive parameter the notable oscillating kink tail appears and this phenomenon can be described by the following ansatz:

$$u_{Kr}(z,t)=u_K(z)+a[1-\tanh(z)]\sin(k_0(z-vt)), \tag{23}$$

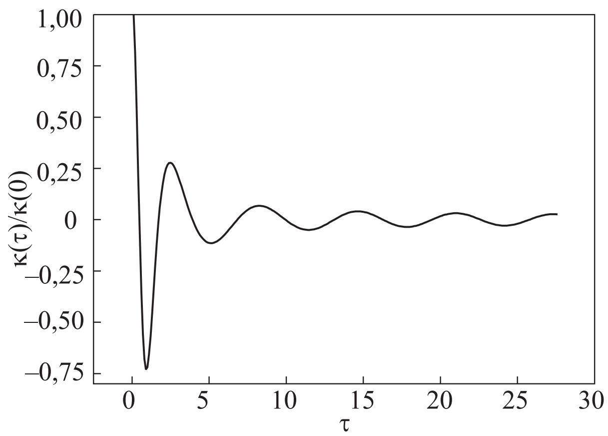

Fig. 2. Decaying oscillations of the reverse effective kink width during motion in the case of small β.

where the second term corresponds to radiation on the wake of the kink. We have carried out a numerical modeling of the dynamics of kinks and soliton complexes (details of the numerical scheme can be found in Ref. 31). Results of the simulations for small β are in a good relation with expression (22) and (23) and confirm entirely theoretical predictions. For large enough parameter β and the initial velocity $V_{\rm in}$ a moving kink emits the breather as shown in Fig. 3.

4. Kinks interaction and formation of soliton complexes

An analytical approach to the description of the soliton-complex formation in dispersive equations was proposed in [21,29]. It is based on the use of the collective variable ansatz which is constructed by taking into account the translational and internal degrees of freedom of a soliton as well as interactions between solitons and solitons with radiation. Now using results of previous section we can specify the form of ansatz:

$$u_{wb}(x,t) = u_K(\xi + R) + u_K(\xi - R) + f_b(\xi,t)(1-\tanh(\xi)). \tag{24}$$

Here first two terms are kinks superposition and the last term describes a small-amplitude breather $f_b(\xi,t) = a\sin(\Omega t - k(\xi-\xi_0))/\cosh(\varepsilon(\xi-\xi_0))$ or radiation emitting. It turns out that the condition of the complex formation of the closely sited solitons can be found from the energy expression of the pair of strongly interacting solitons without taking into account the breather or radiation [19,21,28]. Now we use this approximation for the description of the regularized SG system. So we suppose that the complex dynamics can be considered in the framework of the soliton ansatz

$$u_{kk}(x,t) = 4\arctan(\exp(\xi + R)) + 4\arctan(\exp(\xi - R))) \tag{25}$$

which is prompted by the form of a generalization of the exact solution in Eq. (14). Here $\xi = \kappa(x - X(t))$ and $X(t)$, $\kappa(t)$, and $R(t)$ are functions of time. Functions $\kappa(t)$ and $X(t)$ describe the changing of the effective width of solitons and their translational motion, respectively. The function corresponds to the changing separation between solitons, which is defined obviously as $L = 2R/\kappa$. Let the distance between solitons be small. Inserting the ansatz into Eqs. (11) and (12), we find the effective Lagrangian for two interacting solitons in the strongly dispersive medium:

$$L = 16\left\{\frac{\kappa_t^2}{\kappa^3}\left[\frac{\pi^2}{12} - R^2\left(\frac{\pi^2}{36}-\frac{2}{3}\right)\right] - \frac{\kappa_t}{\kappa^2}RR_t + \kappa(X_t^2-1)\times\right.$$
$$\times\left(1-\frac{R^2}{3}\right) - \frac{1}{3\kappa}\left(1+\frac{R^2}{5}\right) + \frac{\beta}{2}\left\{\frac{\kappa_t^2}{\kappa}\left[\left(\frac{\pi^2}{18}+\frac{2}{3}\right)-\right.\right.$$
$$\left.\left.\left. - R^2\left(\frac{7\pi^2}{90}-\frac{2}{3}\right)\right] - 2\kappa_t RR_t + \frac{2}{3}\kappa^3 X_t^2\left(1-\frac{7R^2}{5}\right)\right\}\right\}. \tag{26}$$

Analysis of Eq. (26) shows that the soliton complex is stable for high values of its velocity and a small distance between composite kinks due to the effective kinks attraction. For value of velocity much larger than the velocity of stationary motion the complex also dissociates in the manner shown in Fig. 4.

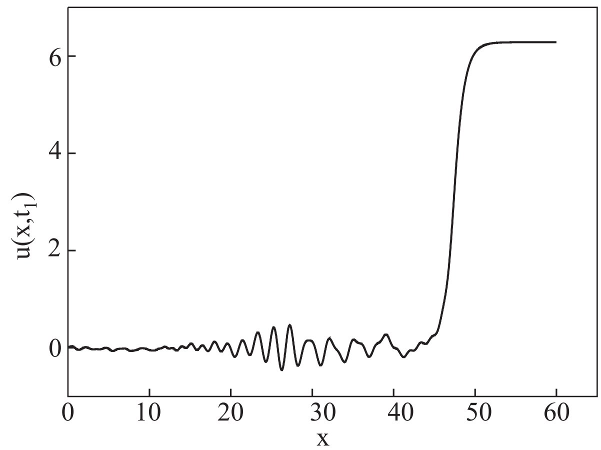

Fig. 3. A fast kink evolution with generating a breather on its wake for $\beta = 1/6$ and $V_{\rm in} = 0.86$.

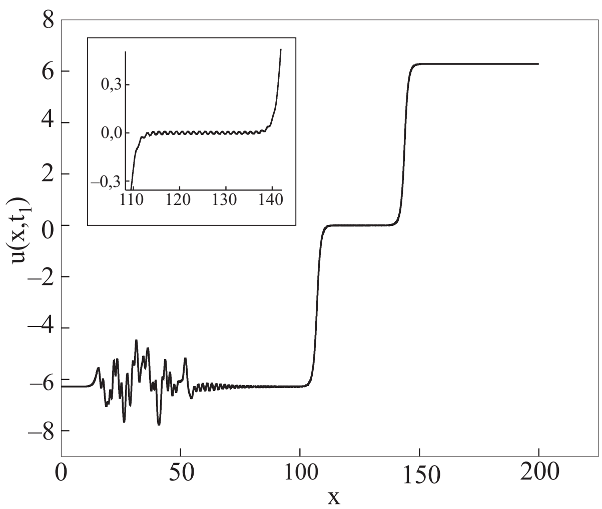

Fig. 4. Decay of a soliton complex for $\beta = 1$, $V_{\rm in} = 0.9$ and $t_1 = 500$. The first kink moves with a constant velocity $V_1 = 0.152$. Behind the second kink are breather modes. The inset shows the spatial modulation of the field between kinks on an expanded scale.

5. Breather properties in the regularized dispersive SG equation

The form of static breather can be found analytically as an asymptotic series using the Kosevich–Kovalev scheme of construction of the self-oscillation solution [8]:

$$u(x,t) = A(x)\sin\omega t + B(x)\sin 3\omega t + C(x)\sin 5\omega t + \dots \tag{27}$$

For a main harmonics with a frequency ω which is close to the linear spectrum lower edge, i.e. for $\varepsilon = \sqrt{1-\omega^2} << 1$, one obtains the following effective equation:

$$\psi_{tt} - \psi_{xx} + \psi - \beta\psi_{xxtt} - \frac{1}{8}|\psi|^2\psi = 0. \tag{28}$$

Here a complex function $\psi(x,t)$ determines the solution $u(x,t) = \mathrm{Re}(\psi(x,t))$ in the first approximation with respect to the small parameter ε. Seeking the solution of Eq. (28) in the form $\psi = f(x)\exp(i\omega t)$ we derive the nonlinear ordinary equation:

$$(1-\beta\omega^2)f_{xx} - (1-\omega^2)f + \frac{1}{8}f^3 = 0, \tag{29}$$

which gives a coordinate dependence of the harmonic amplitude as a usual soliton profile:

$$f = \frac{4\varepsilon}{\cosh\kappa x}, \qquad \kappa^2 = \frac{1-\omega^2}{1-\beta\omega^2}. \tag{30}$$

However one can see a new feature of the breather, which consists in vanishing the effective width dependence on the amplitudeε in the limit $\beta \to 1$. In fact it appears that in this case the amplitude of breather is not already a constant but a slowly time-oscillating function. This results in the main frequency splitting and a complex breather behavior showing in Fig. 5. Such a behavior is similar to dynamical properties of breathers in discrete ant nonlocal SG models [37,38]. At last have we found that a single breather motion is accompanied by a small breather bursting process and emitting radiation as shown in Fig. 6. As one has seen in previous sections the excitation of breather modes plays a crucial role in the kink and soliton complex dynamics in the case of a strong dispersion.

6. Stabilization of soliton complexes by driving forces in dissipative media

Finally, we have investigated the influence of external forces and dissipation on the dynamics of soliton complexes. For this purpose we add the dissipation term λu_t and a driving force f_0 to the right-hand side of Eq. (6)

$$u_{tt} + \lambda u_t - u_{xx} - \beta u_{xxtt} + \sin u = f_0. \tag{31}$$

The term f_0 in the right-hand side corresponds, for example, to the bias current in a long Josephson junction. The result of a numerical modeling are presented in Fig. 7 for 4π-complex profiles and in Fig. 8 for their step-like velocity dependences on the driving force strength (one can compare this result with the velocity-force dependence for a single 2π-kink in the discrete SG model [39]). Parameters are chosen as follows: $\lambda = 0.1$ and six sequential values of f_0 from –0.1 to –0.35. It turns out that the driving force under conditions of dissipation permits stabilization not only of the soliton complex but also of its «excited» states with internal structures. For waves of stationary profile, the derivatives $u_t$and $u_x$are proportional to each other, and both have the form of closely spaced double peaks. These derivatives are directly related to experimentally measurable quantities, in particular, the voltage $U \sim u_t$ and magnetic field $H \sim u_x$ in the case of a long Josephson junction, and in a crystal with dislocations the derivative $u_x$determines the elastic deformation of the medium. In conclusion we note that the possibility of observing multisoliton excitations in long

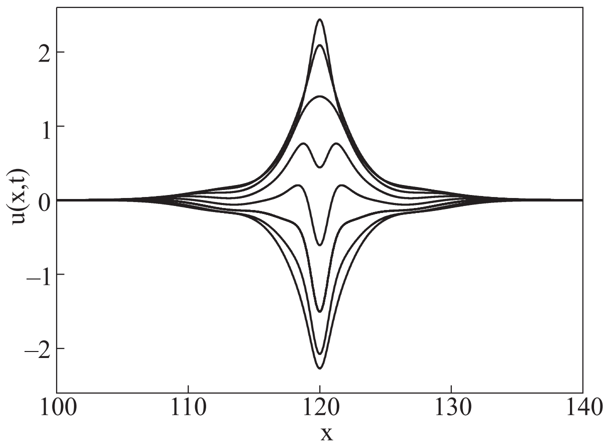

Fig. 5. A half-period evolution of a static breather at $\beta = 0.9$.

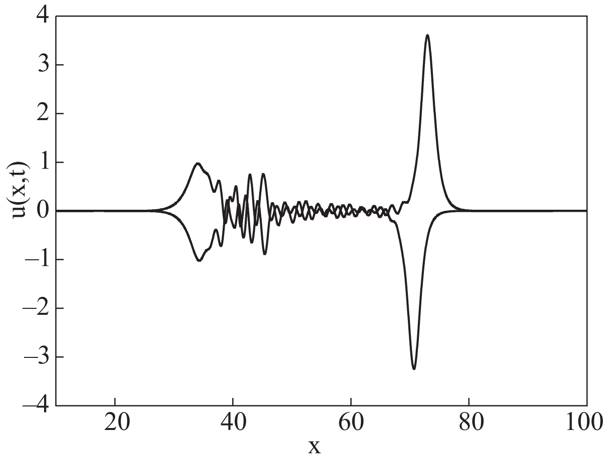

Fig. 6. Two moving breather profiles divided of a half-period in time at $\beta = 0.9$.

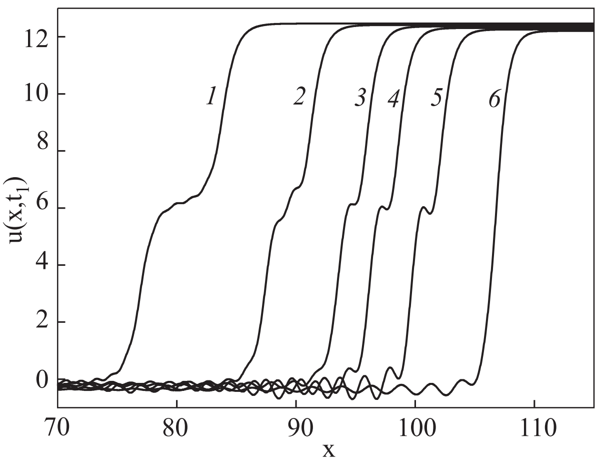

Fig. 7. Propagation of stable soliton complexes with an internal structure under the influence of external forces and in the presence of dissipation. The coefficient $\lambda = 0.1$ and the curves with numbers n = 1, . . . ,6 corresponds to $f_0 = -0.05(1+n)$ and to the same instant of time $t_1 = 100$.

Josephson junctions was demonstrated quite some time ago [40].

7. Summary

Thus we have studied the nonstationary dynamics and interactions of topological solitons (kinks) in one-dimensional systems with a strong dispersion. Analytical approach has been proposed for investigation of dynamical features of a single kink motion which accompanied by emitting radiation and small-amplitude breathers. Collective coordinate ansatz has been also proposed to study processes of soliton complex formation in relation to the strength of the dispersion, soliton velocity, and distance between solitons. The breather solution has been constructed in a small amplitude-limit and its internal oscillation and propagation in the dispersive medium have been investigated in detail. It has been shown that theoretical results are in good relation with numerical simulations and quantitatively explain them. It is demonstrated that stable bound soliton states with complex internal structure can propagate in a dissipative medium owing to their stabilization by external forces.

The results obtained can be used for explanation and description of new effects in the dynamics of topological solitons in highly dispersive media — in particular, dislocations in nonideal lattices, fluxons in Josephson junction systems, and magnetic domain walls in anisotropic magnets.

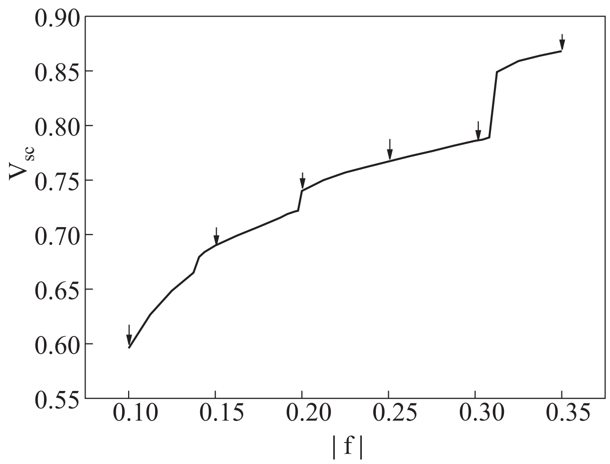

Fig. 8. A step-like dependence of the complex velocity on a driving force strength. Arrows show velocity values corresponding to complex profiles presented in Fig. 7.

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