Low-frequency quantum oscillations of the impedance of layered conductors at high magnetic field

O. V. Kirichenko* and I. V. Kozlov

B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, pr. Lenina 47, 61103 Kharkov, Ukraine

The propagation of electromagnetic waves in layered conductors is investigated by the method of the quantum kinetic equation. The quantum oscillations of the impedance for elastic scattering on impurities is calculated. An expression is obtained for the low-frequency oscillations of the impedance over a wide range of frequencies of the electromagnetic wave. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1480243]

By studying the propagation of waves in organic conductors placed in a high magnetic field **B**, one can investigate in detail the energy spectrum and relaxation properties of the charge carriers.[1] In conductors having a layered structure the electron energy spectrum has a quasi-two-dimensional character, and the electron energy $\varepsilon(\mathbf{p})$ depends weakly on the momentum projection $p_z = \mathbf{p} \cdot \mathbf{n}$ on the normal **n** to the layers. Layered conductors at low temperatures exhibit the clearest manifestations of the de Haas–van Alphen (dHvA) and Shubnikov–de Haas (SdH) quantum oscillation effects.[2–11] The study of the SdH oscillations of the dc resistivity of layered conductors is the subject of a great number of theoretical and experimental papers (see, e.g., the reviews[12] and the references cited therein). The experimental study of the propagation of electromagnetic waves in organic conductors has received much less attention,[13–17] even though the kinetic phenomena in alternating fields carry rich information about the electron systems in conducting media. Among the papers mentioned is a study of tetrathiafulvalene-based compounds of the form (BEDT–TTF)$_2$X (X stands for a set of various anions) in which the wave vector **k** and the static magnetic field were directed along the normal to the layers. Below we shall examine the propagation of electromagnetic waves in quasi-two-dimensional layered conductors in a geometry used in some of the studies cited, i.e., in which the Poynting vector and the magnetic field are parallel to **n**. In this case the alternating electromagnetic field is orthogonal to the vector of the quantizing magnetic field, and it is extremely important to take into account the quantum oscillations of the kernel of the scattering operator for the charge carriers. Here the amplitude of the SdH oscillations of the surface impedance is of a substantially different order of magnitude than in the approximation in which a magnetic-field-independent relaxation time τ is used in the quantum kinetic equation for the collision integral.[18] In contrast to the dHvA oscillations, the period of which is determined by the extremal-area (S_{extr}) cross sections of the Fermi surface, the SdH oscillations contain combination frequencies of the type

$$\nu = \frac{(n S_{\max} + n' S_{\min})c}{e\hbar},$$

where n and n' are any integers, c is the speed of light, e is the charge of the electron, and $\hbar$ is Planck's constant.

The distribution of the electric field of frequency ω in the conductor is easily found from Maxwell's equations

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2} \right) E_\alpha + \frac{4\pi i \omega}{c^2} j_\alpha = 2 E'_\alpha(0),$$

supplemented by the constitutive relation connecting the current density $\mathbf{j}(z,t)$ with the electric field. To determine the current density

$$\mathbf{j} = e \, \mathrm{Tr}(\hat{\mathbf{v}} \hat{f}) = \frac{2e^2 B}{c(2\pi\hbar)^2} \sum_{n,n'=0}^{\infty} \int dp_z \mathbf{v}_{n'n} \hat{f}_{nn'}, \quad (1)$$

it is necessary to find the density matrix $f_{nn'}$ with the use of the quantum kinetic equation;[19] here $\mathbf{v}_{n'n}$ is a matrix element of the velocity operator. Solely for the sake of brevity in the calculation we will use a rather simple dispersion relation for the charge carriers, in the form

$$\varepsilon_n(p_z) = \left(n + \frac{1}{2} \right) \hbar\Omega - A \cos\left(\frac{a p_z}{\hbar} \right), \quad n = 0,1,2,3..., \quad (2)$$

where a is the distance between layers, $\Omega = eB/mc$ is the cyclotron frequency, m is the effective mass of the conduction electrons, $A = \eta \varepsilon_f$, and the quasi-two-dimensionality parameter η will be assumed not too small,

$$\frac{\hbar\Omega}{\varepsilon_f} \ll \eta \ll 1,$$

so that there are sufficient Landau levels on the Fermi surface $\varepsilon(\mathbf{p}) = \varepsilon_f$ that one may use a quasiclassical approximation for calculating the impedance. We limit consideration to the case of the normal skin effect, when the relation between the current density $j_i = e \, \mathrm{Tr}(\hat{v}_i \hat{f})$ and the electric field **E** can be treated, to sufficient accuracy, as local:

$$j_i(\mathbf{r},t) = \sigma_{ij} E_j(\mathbf{r},t).$$

The approximation of a local relation is completely admissible when the drift of the conduction electrons along the wave vector **k** over the mean free time τ of the charge carriers is much less than the skin depth δ, i.e.,

$$\eta v \tau \ll \delta. \quad (3)$$

Thanks to the symmetry of the spectrum it is sufficient to consider only the components of the conductivity tensor in the plane of the layers. For circular polarization of the wave, $E^\pm = E_x \pm iE_y$, the conductivity tensor becomes diagonal.

In order for the quantum oscillations to be substantial, the mean free time of the charge carriers must be much greater than the period of gyration of the electron along its orbit ($\Omega\tau \gg 1$). We shall take into account only the elastic scattering on impurities, assuming that the range of the scattering potential is much less than the de Broglie wavelength of the electrons. This allows us to calculate the conductivity tensor without having to assume that the interaction potential of the electron with the impurity is small. The calculation is done by the method of the quantum kinetic equation, following Refs. 18–20.

The electron gas is described by a density matrix which satisfies the quantum kinetic equation. We write the latter in the form proposed in Ref. 19:

$$-i\omega\hat{f}_1 + \frac{i}{\hbar}[\hat{\varepsilon};\hat{f}_1] + \frac{i}{\hbar}n_{imp}\mathrm{Tr}_\alpha[\hat{V};\hat{F}_0(\hat{f}_1)]$$

$$= \frac{i}{\hbar}\left[\frac{e\mathbf{E}\cdot\hat{\mathbf{v}}}{i\omega};\hat{f}^{(0)}\right] - \frac{i}{\hbar}n_{imp}\mathrm{Tr}_\alpha[\hat{V};\hat{F}_1]; \qquad (4)$$

$$\frac{\partial\hat{F}_0(\hat{f})}{\partial t} + \frac{i}{\hbar}[\hat{\varepsilon}+\hat{V};\hat{F}_0(\hat{f})] = -\frac{i}{\hbar}[\hat{V};\hat{f}];$$

$$\frac{\partial\hat{F}_1}{\partial t} + \frac{i}{\hbar}[\hat{\varepsilon}+\hat{V};\hat{F}_1] = \frac{i}{\hbar}\left[\frac{e\mathbf{E}\cdot\hat{\mathbf{v}}}{i\omega};\hat{F}_0(\hat{f}^{(0)})\right],$$

where $\hat{f}^{(0)}$ is the Fermi–Dirac distribution function, $\hat{f}_1$ is the correction linear in the field to the density matrix, $\hat{V}$ is the impurity operator, $\hat{F} = \hat{F}_0 + \hat{F}_1$ is the binary correlation operator of the electron and one impurity, n_{imp} is the impurity concentration, the trace Tr_α is taken over the states of the impurity, and α is the set of quantum numbers characterizing the state of the impurity; from now on, the subscript α will be dropped from all notations except Tr_α. The system of equations (4) is a chain of Bogolyubov equations broken off at the two-impurity correlation operator. The impurity is assumed to be uniformly distributed and infinitely heavy. We use the gauge

$$\mathbf{A} = (0,Bx,0) + \frac{c\mathbf{E}}{i\omega}, \quad \varphi = 0.$$

The energy of the electron in the field of the wave has the form $\hat{H}_1 = -(e\mathbf{E}\cdot\hat{\mathbf{v}})/(i\omega)$ and contains the velocity operator, which we write in the $|n,P_y,P_z\rangle$ representation, which is the natural one for the given gauge. P_y determines the coordinate of the center of the electron orbit $x_0 = (cP_y)/(eB)$, and P_z is the same as the kinematic momentum component p_z. Unlike the coordinate operator, which enters the Hamiltonian through the use of the gauge $\mathbf{A} = (0,Bx,0)$, $\varphi = -\mathbf{E}\cdot\mathbf{r}$, the matrix elements of the velocity operator do not depend on P_y:

$$v_x \pm iv_y = v^\pm = v_0^\pm - \frac{eE^\pm}{i\omega m}, \qquad (5)$$

where

$$v_{0nn'}^+ = -\frac{i\hbar}{m}\left(\frac{2eB\hbar}{c}n'\right)^{1/2}\delta_{n+1,n'},$$

$$v_{0nn'}^- = \frac{i\hbar}{m}\left(\frac{2eB\hbar}{c}n\right)^{1/2}\delta_{n-1,n'}.$$

Thus, taking the electric field into account through the use of a vector potential allows one to avoid an additional summation in the expressions for the density matrix and substantially simplifies the calculations.

Equations (4) can be written in the form

$$-i\omega\hat{X} + \frac{i}{\hbar}[\hat{\varepsilon}+\hat{V};\hat{X}] = \hat{Y}, \qquad (6)$$

where $\hat{X}$ is the operator which we are seeking, and $\hat{Y}$ is the right-hand side of the equation. It was shown in Ref. 19 that Eq. (6) has a solution of the form

$$\hat{X} = \frac{\hbar}{2\pi}\int dz\hat{G}^+\left(z + \frac{\omega}{2} + \hat{V}\right)\hat{Y}\hat{G}^-\left(z - \frac{\omega}{2} + \hat{V}\right). \qquad (7)$$

The Green's function $\hat{G}^\pm$ satisfies the relation

$$\hat{G}^\pm(z-\hat{V}) = \hat{G}^\pm(z) + \hat{G}^\pm(z)\hat{T}^\pm(z)\hat{G}^\pm(z), \qquad (8)$$

where $T_{\nu\mu} = \langle\varphi_\nu|\hat{V}|\psi_\mu\rangle$ is the T matrix, φ_ν is the eigenfunction of the Hamiltonian without the impurity, and ψ_ν is the wave function of the electron in the presence of the impurity.

We now calculate the Green's function for a layered conductor. Following Ref. 20, one can show that for a short-range impurity at

$$x,y \ll \frac{r_L}{n_f}, \quad z \ll a, \qquad (9)$$

where $n_f = \varepsilon_f/(\hbar\Omega)$ and r_L is the Larmor radius, the Green's function can be written in the form

$$G^\pm(r,r',E) = \sum_\nu \frac{\phi_\nu(r)\phi_\nu^*(r')}{E - \varepsilon_\nu \pm i\delta}$$

$$= \Phi(r,r')[G_{cl}(r-r',E) + G_q^\pm], \qquad (10)$$

where

$$\Phi(r,r') = \exp\left[\frac{i\hbar c}{2eB}(x+x')(y-y')\right];$$

G_{cl} is the real part of the Green's function in the absence of magnetic field, and the dependence of G_q on $(r-r')$ can be neglected. In the case of a quasi-two-dimensional spectrum, G_{cl} is rather complicated for explicit calculation. Unlike the case of the quadratic spectrum, G_{cl} depends on E and also depends on $(r-r')$ in a complicated way. However, the explicit form will not be needed in the calculations that follow. For calculating G_q we use the Poisson summation formula, as a result of which the expression for G_q takes the form

$$G_q^{\pm}(\varepsilon) = \mp \frac{im}{2\hbar^2 a}$$

$$\times \left[1 + 2\sum_{k=1}^{\infty} (-1)^k \exp\left(\pm \frac{2\pi i k\varepsilon}{\hbar\Omega} \right) J_0\left(\frac{2\pi kA}{\hbar\Omega} \right) \right],$$

(11)

where J_0 is the Bessel function. The series appearing in Eq. (11) is conditionally convergent, but it can be shown that when the Dingle broadening of the Landau levels is taken into account, a small factor $C_D^k = \exp(-k/\Omega\tau)$ will appear in the oscillatory part of the Green's function, and then the series becomes absolutely convergent.

As was shown in Ref. 20, the electron wave function in the field of the impurity satisfies Dyson's equation

$$\psi_\nu(r) = \varphi_\nu(r) + \int G(r,r',E)V(r')\psi_\nu(r')d^3r'.$$

When the Green's function in the form (11) is substituted in, the wave function $\psi_\nu(r)$ in the region specified by inequalities (9) can be represented in the form

$$\psi_\nu(r) = \frac{\varphi_\nu(R_{\mathrm{imp}})}{1 - (2\pi\hbar^2/m)f_{\mathrm{imp}}G_q^{\pm}(E)} \psi_0(r),$$

(12)

where $\psi_0(r)$ satisfies the equation

$$\psi_0(r) = 1 + \int G_{\mathrm{cl}}(r,r',E)V(r')\psi_0(r')d^3r',$$

R_{imp} is the coordinate of the impurity; the total scattering amplitude f_{imp} is given by the expression

$$f_{\mathrm{imp}} = \frac{m}{2\pi\hbar^2} \int V(r)\psi(r)d^3r;$$

(13)

$G_{\mathrm{cl}}(r,E)$ is a smooth function of E and varies substantially over energy intervals $\Delta E \sim \varepsilon_f$. However, for calculating the galvanomagnetic coefficients, only the energy region near the Fermi level is important, where

$$\Delta E = E - \varepsilon_f \sim \max(\hbar\Omega, \hbar\omega) \ll \varepsilon_f.$$

Thus the dependence on E of $G_{\mathrm{cl}}(r,E)$ and $\psi_0(r)$ can be neglected, taking $E \simeq \varepsilon_f$. The expression for the T matrix in the case of a quasi-two-dimensional spectrum can be written in the form

$$T_{\mu\nu}^{\pm}(E) = t^{\pm}(E)\varphi_\mu^*(R_0)\varphi_\nu(R_0),$$

$$t^{\pm}(E) = \frac{(2\pi\hbar^2/m)f_{\mathrm{imp}}}{1 - (2\pi\hbar^2/m)f_{\mathrm{imp}}G_q^{\pm}(E)},$$

(14)

which agrees with formula (7) of Ref. 20 for the case of a quadratic dispersion relation. Essentially, the energy dependence enters only in the kernel of the T matrix.

In calculating the oscillatory part of the conductivity tensor, only the off-diagonal elements of the velocity operator are important. The contribution from the diagonal part of $\hat{v}$ in the formula for the current,

$$\mathbf{j}_{\mathrm{diag}} = e\mathrm{Tr}(\hat{\mathbf{v}}_{\mathrm{diag}}\hat{f}) = -\frac{e^2\mathbf{E}}{i\omega m}\mathrm{Tr}(\hat{f}) = -\frac{e^2\mathbf{E}}{i\omega m}n_e,$$

is expressed in terms of the electron density n_e and cannot oscillate as the magnetic field varies. In the expression for the density matrix the diagonal part of $\hat{v}_\pm$ can also be neglected. In fact, $\hat{\mathbf{v}}$ enter the expression for $\hat{f}$ only in the form

of the combination $e\mathbf{E}\cdot\hat{\mathbf{v}}/i\omega$, but because the diagonal components of the velocity are linear in the field $\mathbf{E}$, the corresponding correction to $\hat{f}$ will be quadratic in the field. Since we are neglecting the diagonal components of $\hat{\mathbf{v}}$ in the equation for the current, we will be interested only in the off-diagonal part of the density matrix $\hat{f}$. As was shown in Ref. 19, for the off-diagonal components of the density matrix the collision integral $in_{\mathrm{imp}}\mathrm{Tr}_\alpha[V_\alpha;F_0(f_1)]_{nm}/\hbar$ appearing in the system of equations (4) reduces to multiplication of f_1 by the quantity

$$\frac{1}{\tau_{nm}} = \frac{i}{\hbar}n_{\mathrm{imp}}[t^+(\varepsilon_m + \hbar\omega) - t^-(\varepsilon_n - \hbar\omega)].$$

(15)

The right-hand side of the equation for $\hat{F}_0$ in system (4) contains the commutator with the impurity operator. Therefore, in the explicit expression for $\hat{F}_0$, the operator $\hat{V}$ will appear together with the T matrix, but it can be canceled out by reduction with Dyson's equation:

$$\hat{T}^{\pm} = \hat{V} + \hat{V}\hat{G}^{\pm}\hat{T}^{\pm} = \hat{V} + \hat{T}^{\pm}\hat{G}^{\pm}\hat{V},$$

(16)

where now the expression for $\hat{F}_0$ can be written as

$$\hat{F}_0 = \frac{i}{2\pi} \int dz[-\hat{G}^+(z)\hat{T}^+(z)\hat{f}^{(0)}\hat{G}^-(z)$$

$$+ \hat{G}^+(z)\hat{f}^{(0)}\hat{T}^-(z)\hat{G}^-(z)$$

$$- \hat{G}^+(z)\hat{T}^+(z)\hat{f}^{(0)}\hat{G}^-(z)\hat{T}^-(z)\hat{G}^-(z)$$

$$+ \hat{G}^+(z)\hat{T}^+(z)\hat{G}^+(z)\hat{f}^{(0)}\hat{T}^-(z)\hat{G}^-(z)].$$

(17)

We note that $\hat{F}_0(\hat{f}^{(0)})$ has the meaning of an impurity correction to the Fermi–Dirac function:

$$\hat{f}^{(0)}(\hat{\varepsilon} + \hat{V}) = \hat{f}^{(0)}(\hat{\varepsilon}) + \hat{F}_0(\hat{f}^{(0)}).$$

The calculation of $\hat{f}_1$ can be reduced to the evaluation of $\hat{F}_1$. For this we write $\hat{F}_1$ and $\hat{f}_1$ in the form of a sum:

$$\hat{F}_1 = \hat{F}_a + \hat{F}_b, \quad \hat{f}_1 = \hat{f}_a + \hat{f}_b,$$

for which the system of equations (4) will take the form

$$-i\omega\hat{F}_a + \frac{i}{\hbar}[\hat{\varepsilon};\hat{F}_a] = \frac{i}{\hbar}\left[\frac{e\mathbf{E}\cdot\hat{\mathbf{v}}}{i\omega};\hat{F}_0 \right],$$

$$-i\omega\hat{F}_b + \frac{i}{\hbar}[\hat{\varepsilon};\hat{F}_b] = -\frac{i}{\hbar}[\hat{V};\hat{F}_1],$$

$$-i\omega\hat{f}_a + \frac{i}{\hbar}[\hat{\varepsilon};\hat{f}_a] + \hat{\tau}^{-1}\hat{f}_a = \frac{i}{\hbar}\left[\frac{e\mathbf{E}\cdot\hat{\mathbf{v}}}{i\omega};\hat{f}^{(0)} \right],$$

$$-i\omega\hat{f}_b + \frac{i}{\hbar}[\hat{\varepsilon};\hat{f}_b] + \hat{\tau}^{-1}\hat{f}_b = -\frac{i}{\hbar}n_{\mathrm{imp}}\mathrm{Tr}_\alpha[\hat{V};\hat{F}_1].$$

(18)

It is not hard to see that

$$f_{nm}^b = \frac{-i\omega + i(\varepsilon_n - \varepsilon_m)/\hbar}{-i\omega + i(\varepsilon_n - \varepsilon_m)/\hbar + 1/\tau_{nm}}n_{\mathrm{imp}}\mathrm{Tr}_\alpha F_{nm}^b.$$

(19)

It can be shown that $\hat{F}_a$ corresponds to a shift of the energy level due to the presence of the impurity and can be neglected. Indeed, the corresponding contribution to the conductivity

362   Low Temp. Phys. **28** (5), May 2002

O. V. Kirichenko and I. V. Kozlov

$$\sigma_{F_a} \sim \mathrm{Tr}[\hat{v};\hat{F}_0]\hat{v} \sim \int d\bar{P}_z \sum_n (\bar{F}^0_{n+1,n+1}-F^0_{n,n})n$$

$$\sim \int d\bar{P}_z \sum_n \bar{F}^0_{n,n} \sim \mathrm{Tr}(\hat{f}^{(0)}(\hat{\varepsilon}+\hat{V})-\hat{f}^{(0)}(\hat{\varepsilon}))$$

is proportional to the difference of the electron densities in the presence and absence of the impurity, i.e., it does not depend on the magnetic field **B**. Thus we obtain

$$f^1_{nm}=\frac{\dfrac{i}{\hbar}\left[\dfrac{e\mathbf{E}\cdot\hat{\mathbf{v}}}{-i\omega};f^{(0)}\right]_{nm}}{-i\omega+\dfrac{i}{\hbar}(\varepsilon_n-\varepsilon_m)+\dfrac{1}{\tau_{nm}}}$$

$$+\frac{-i\omega+\dfrac{i}{\hbar}(\varepsilon_n-\varepsilon_m)}{-i\omega+\dfrac{i}{\hbar}(\varepsilon_n-\varepsilon_m)+\dfrac{1}{\tau_{nm}}}n_{\mathrm{imp}}\mathrm{Tr}_\alpha F^b_{nm}. \qquad (20)$$

Since only some of the terms appearing in $\hat{F}^b$ are important, we write it in the following form:

$$\hat{F}_b=\frac{i}{2\pi}\int dz_1 \left\{ \hat{G}^+\left(z_1+\frac{\omega}{2}\right)\left[\frac{e\mathbf{E}\cdot\hat{\mathbf{v}}}{i\omega};\hat{F}_0(\hat{f}^{(0)})\right]\right.$$

$$\times\hat{G}^-\left(z_1-\frac{\omega}{2}\right)+\hat{G}^+\left(z_1+\frac{\omega}{2}\right)\hat{T}^+\left(z_1+\frac{\omega}{2}\right)$$

$$\times\hat{G}^+\left(z_1+\frac{\omega}{2}\right)\left[\frac{e\mathbf{E}\cdot\hat{\mathbf{v}}}{i\omega};\hat{F}_0(\hat{f}^{(0)})\right]\hat{G}^-\left(z_1-\frac{\omega}{2}\right)$$

$$+\hat{G}^+\left(z_1+\frac{\omega}{2}\right)\left[\frac{e\mathbf{E}\cdot\hat{\mathbf{v}}}{i\omega};\hat{F}_0(\hat{f}^{(0)})\right]$$

$$\times\hat{G}^-\left(z_1-\frac{\omega}{2}\right)\hat{T}^-\left(z_1-\frac{\omega}{2}\right)\hat{G}^-\left(z_1-\frac{\omega}{2}\right)$$

$$+\hat{G}^+\left(z_1+\frac{\omega}{2}\right)\hat{T}^+\left(z_1+\frac{\omega}{2}\right)\hat{G}^+\left(z_1+\frac{\omega}{2}\right)$$

$$\times\left[\frac{e\mathbf{E}\cdot\hat{\mathbf{v}}}{i\omega};\hat{F}_0(\hat{f}^{(0)})\right]\hat{G}^-\left(z_1-\frac{\omega}{2}\right)$$

$$\left.\times\hat{T}^-\left(z_1-\frac{\omega}{2}\right)\hat{G}^-\left(z_1-\frac{\omega}{2}\right)\right\}. \qquad (21)$$

The first term in (21) corresponds to a shift of the energy level due to the presence of the impurity, and it can be neglected. The next term contains an expression of the form $\hat{T}\hat{v}\hat{T}$, which vanishes upon summation over P_y in (20) as a consequence of the orthogonality of the Hermite polynomials. For the same reason, only the part with the commutator $[e\mathbf{E}\cdot\hat{\mathbf{v}}/i\omega;\hat{F}_0]$ is important in the rest of the terms.

The expression for $\hat{f}_1$ simplified in this way must be substituted into the equation for the current (1). We note that the T matrix is nondiagonal in P_y and P_z, and each appearance of it in formula (21) leads to the necessity of summing over these quantum numbers. Thus, to simplify the calculations that follow it would be desirable to reduce expression (21) to a form in which the T matrix enters each term only once. This can be done by employing the following arguments.

1. As we know, the scattering tensor obeys the optical theorem, which in our case is conveniently written in the form

$$\hat{T}^+(a)(\hat{G}^+(a)-\hat{G}^-(b))\hat{T}^-(b)=\hat{T}^+(a)-\hat{T}^-(b);$$

$$-2\pi i\hat{T}^+(a)\delta(\hat{\varepsilon}-a)\hat{T}^-(a)=\hat{T}^+(a)-\hat{T}^-(a), \qquad (22)$$

which can easily be obtained from the Born expansion of the T matrix or by substituting the Green's function and T matrix in explicit form.

2. It follows from the explicit form of the T matrix that it obeys the relation

$$\hat{T}^\pm(a)=\frac{t^\pm(a)}{t^+(b)}\hat{T}^+(b), \qquad (23)$$

which together with the optical theorem gives an efficient way of simplifying the tensor expressions.

After all the transformations have been done, the expression for the density matrix becomes extremely awkward, and we will not write it out. After some calculations, the conductivity tensor can be written in the form $\sigma^\pm=\sigma_a^\pm+\sigma_b^\pm$, where

$$\sigma_a^\pm=\pm\frac{ie^2}{2\pi^2\hbar^2\omega}\sum_n \int dP_z \frac{n[f^{(0)}(\varepsilon_n\pm\hbar\Omega)-f^{(0)}(\varepsilon_n)]}{\omega\mp\Omega-\dfrac{n_{\mathrm{imp}}}{\hbar}[t^+(\varepsilon_n+\hbar\omega)-t^-(\varepsilon_n-\hbar\omega)]}, \qquad (24)$$

$$\sigma_b^\pm=\pm\frac{ie^2 n_{\mathrm{imp}}}{2\pi^2\hbar^2\omega(\omega\mp\Omega)}\sum_n \int dP_z \frac{n}{\omega\mp\Omega-\dfrac{n_{\mathrm{imp}}}{\hbar}[t^+(\varepsilon_n+\hbar\omega)-t^-(\varepsilon_n-\hbar\omega)]}\left[-\frac{i}{2\pi}(\omega\pm\Omega)\right.$$

$$\left.\times\int dz\frac{t^+(z)-t^-(z)}{\varepsilon_n+\omega\mp\Omega-z+i\delta}\frac{f^{(0)}(z)-f^{(0)}(\varepsilon_n)}{z-\varepsilon_n-i\delta}+\frac{i}{2\pi}(\omega\pm\Omega)\int dz\frac{t^+(z)-t^-(z)}{\varepsilon_n-\omega\pm\Omega-z-i\delta}\frac{f^{(0)}(z)-f^{(0)}(\varepsilon_n)}{z-\varepsilon_n+i\delta}\right], \qquad (25)$$

where n_e is the electron density, and $\sigma_a^{\pm}$ and $\sigma_b^{\pm}$ correspond to $\hat{f}_a$ and $\hat{f}_b$ in the density matrix. For brevity in the writing of these formulas we have omitted certain terms which are important for the classical part of $\sigma^{\pm}$ but do not contribute to the oscillatory part of the conductivity. The given expression can be used to calculate the conductivity for any values of ω, although elementary estimates of the penetration depth of the electromagnetic wave into the conductor show that for the layered conductors under study, the resonance region ($\omega - \Omega \sim 1/\tau$) corresponds to the case of the anomalous skin effect and cannot be treated in the local-coupling approximation. In the absence of resonance and for $\omega \neq 0$ the main contribution to the quantum oscillations of the conductivity tensor is given by the correction linear in $1/\tau$. The second-order correction in $1/\tau$ becomes important only for the static case $\omega \ll \Omega$, and therefore in the expressions quadratic in the inverse relaxation time one can set $\Omega - \omega \approx \Omega$. We note that for taking the trace Tr in expressions (24) and (25), the density of states $\nu(\varepsilon)$ appearing in the trace in the formula for the current (1) is expressed in terms of the Green's function (11) as

$$\frac{eB}{c(2\pi\hbar)^2}\sum_n \int dP_z ... = \frac{1}{(2\pi\hbar)^3}\int \nu(\varepsilon)d\varepsilon ...$$

$$= \int \frac{d\varepsilon}{2\pi i}[G_q^- - G_q^+] ... \qquad (26)$$

To simplify the remaining calculations somewhat, let us assume that the oscillations of the scattering tensor are small, i.e.,

$$\frac{f_{\text{imp}}}{a}\left(\hbar \frac{\Omega}{A}\right)^{1/2} C_D \ll l, \qquad (27)$$

and we will keep only the leading terms in the expansion in powers of $\hbar\Omega/\varepsilon_f$ and $1/\Omega\tau$, assuming that $1/\Omega\tau \gg \hbar\Omega/\varepsilon_f$. After (11), (14), and (26) have been substituted into (24) and (25), the expression for the conductivity will contain products of series:

$$\sum_k^{\infty} (-1)^k \exp\left(\frac{2\pi ik\varepsilon}{\hbar\Omega}\right)J_0\left(\frac{2\pi kA}{\hbar\Omega}\right)$$

$$\times \sum_l^{\infty} (-1)^l \exp\left[\frac{2\pi il}{\hbar\Omega}(\varepsilon + \Delta)\right]J_0\left(\frac{2\pi lA}{\hbar\Omega}\right)$$

$$= \sum_{k,l}^{\infty} (-1)^{k+l} \exp\left(\frac{2\pi il\Delta}{\hbar\Omega}\right)\exp\left[\frac{2\pi i\varepsilon}{\hbar\Omega}(k+l)\right]$$

$$\times J_0\left(\frac{2\pi kA}{\hbar\Omega}\right)J_0\left(\frac{2\pi lA}{\hbar\Omega}\right), \quad \Delta = 0, \pm\hbar\omega, \qquad (28)$$

the required absolute convergence of which, as we have said, is ensured by the Dingle broadening of the Landau levels. The terms of the series with $k, l \neq 0$ in Eq. (28) contain the Bessel function, which gives an additional small factor of $\sqrt{\hbar\Omega/A}$. Thus the main contribution to the high-frequency oscillations of the conductivity will come from the part of the sum (28) with $k \neq l = 0$ and $l \neq k = 0$. In addition, we drop the products with $k + l = 0$, the phase of which does not depend on ε (they cause oscillations at the difference frequencies). As will be seen from the calculations below, this will

cause the phase of the oscillations of the corresponding part of the conductivity tensor to be independent of ε_f, with the result that its amplitude will not be suppressed by the usual temperature smearing but it will be hit twice by the Dingle factor C_D. Thus, in the absence of resonance ($\omega - \Omega \gg 1/\tau$) the quantum correction to the conductivity tensor due to the presence of an impurity has the form

$$\sigma_q^{\pm} = \frac{2e^2 n_e}{m\tau} \frac{1}{(\omega \mp \Omega)^2}$$

$$\times \left\{ \left(1 \pm \frac{3i}{\Omega\tau}\right)\sum_{k=1}^{\infty} \exp\left(\frac{2\pi ik\omega}{\Omega}\right)J_0^2\left(\frac{2\pi kA}{\hbar\Omega}\right)C_D^2 \right.$$

$$+ \left(1 \pm \frac{3}{2}\frac{i}{\Omega\tau}\right)\sum_{k=1}^{\infty} \frac{(-1)^k i\Omega}{\pi k\omega}\cos\left(\frac{2\pi k\varepsilon_f}{\hbar\Omega}\right)$$

$$\left. \times \left(\exp\left(2\pi ik\frac{\omega}{\Omega}\right) - 1\right)J_0\left(\frac{2\pi kA}{\hbar\Omega}\right)C_D C_t \right\}, \qquad (29)$$

where $C_t = [2\pi^2 k_B T/(\hbar\Omega)]/[\sinh(2\pi^2 k_B T/(\hbar\Omega))]$ is a factor causing temperature smearing,

$$\frac{1}{\tau} = \frac{4\hbar\pi^2 C_{\text{imp}} n_{\text{imp}} f_{\text{imp}}^2}{ma}$$

is the relaxation time due to the impurity[19] and is numerically equal to the nonoscillatory part of expression (15); f_{imp} is the total scattering amplitude (13), $C_{\text{imp}} = 1$ if $f_{\text{imp}} \ll a$ and $C_{\text{imp}} = (a/f_{\text{imp}})^2$ if $f_{\text{imp}} \gg a$.

As was noted in Ref. 20, the quantum oscillations of the impedance and the quantities characterizing the propagation of an electromagnetic wave in a conductor are determined mainly by the quantum nature of the collision integral associated with scattering on impurities. In the collisionless limit the impedance oscillations are due to oscillations of the magnetization. In Ref. 20 only the oscillations at the fundamental harmonics were considered. Low-frequency oscillations at the combination frequencies do not appear in the magnetization oscillations. Thus, to a sufficient degree of accuracy one can assume that the total conductivity tensor has the form $\sigma^{\pm} = \sigma_{\text{cl}}^{\pm} + \sigma_q^{\pm}$, where $\sigma_{\text{cl}}^{\pm} = n_e e^2/[m(-i\omega \pm i\Omega + 1/\tau)]$ is the classical, nonoscillatory part of the conductivity tensor.

Since we are considering only the case of the normal skin effect, the expressions given for the conductivity completely describe the process of electromagnetic wave propagation. In the approximation of local coupling of the current density with the electric field, the latter is damped exponentially in the sample:

$$E^{\pm}(z,t) = E_0^{\pm}\exp(ik_z^{\pm}z - i\omega t),$$

where

$$k_z^{\pm} = \left(\frac{4\pi i\omega\sigma_{\pm}}{c^2}\right)^{1/2}.$$

The impedance and the penetration depth of the field into the conductor are related to $k_z^{\pm}$ by the relations

$$Z_{\pm} = \frac{4\pi}{c^2}\frac{\omega}{k_z^{\pm}}, \quad \delta_{\pm} = (\text{Im}\,k_z^{\pm})^{-1}.$$

When the smallness of the amplitude of the quantum oscillations is taken into account, the expression for the impedance can be rewritten as

$$
\begin{aligned}
Z_{\pm} = Z_{\pm}^{cl} \Bigg\{ 1 &- \frac{i}{(\omega \mp \Omega)\tau} \Bigg[\left(1 \pm \frac{3i}{\Omega\tau} \right) \\
&\times \sum_{k=1}^{\infty} \exp\left(2\pi i k \frac{\omega}{\Omega} \right) J_0^2 \left(\frac{2\pi k A}{\hbar\Omega} \right) C_D^2 \\
&+ \left(1 \pm \frac{3}{2}\frac{i}{\Omega\tau} \right) \sum_{k=1}^{\infty} \frac{(-1)^k i\Omega}{\pi k\omega} \cos\left(\frac{2\pi k \varepsilon_f}{\hbar\Omega} \right) \\
&\times \left[\exp\left(\frac{2\pi i k\omega}{\Omega} \right) - 1 \right] J_0\left(\frac{2\pi k A}{\hbar\Omega} \right) C_D C_t \Bigg] \Bigg\},
\end{aligned} \quad (30)
$$

where $Z_{\pm}^{cl}$ is the part of the impedance which is nonoscillatory in the inverse magnetic field.

The unusual temperature dependence of the low-frequency oscillations has a simple physical explanation. The low-frequency oscillations of the impedance are formed by the interference of the oscillations from two extremal cross sections, giving rise to a factor

$$
\cos\left(\frac{S_1 - S_2}{eB\hbar/c} \right) + \cos\left(\frac{4\pi A}{\hbar\Omega} \right),
$$

where $S_{1,2} = 2\pi m(\varepsilon_f \pm A)$, which results in a dependence of the low-frequency contribution on the magnetic field. If the overlap integral of the wave functions of electrons belonging to neighboring layers is nearly independent of energy, i.e., $A(\varepsilon) \simeq$ const, then taking the temperature smearing of the Fermi function into account will not lead to a decrease in the amplitude of the oscillations. Even if $A(\varepsilon)$ does depend on the energy of the charge carriers, then $dA(\varepsilon)/d\varepsilon \simeq \eta$, and with increasing temperature the oscillation amplitude, which is proportional to $\exp[-(2\pi^2 k_B T)(|dA/d\varepsilon|)/(\hbar\Omega)]$, falls off much more slowly than do the amplitudes of the remaining harmonics. This gives reason to hope that low-frequency quantum oscillations of the impedance might be observed in synthetic complexes based on tetrathiafulvalene even at liquid hydrogen temperature, where their fundamental harmonics are utterly small. In spite of the fact that the low-frequency oscillations of the impedance at the combination frequencies appear in higher orders in the magnetic small parameter $\hbar\Omega/(\eta\varepsilon_f)$, these oscillations have been successfully observed at liquid helium temperatures for the case of the static magnetoresistance,[22] and Prof. V. G. Peschansky has informed us that these observations agree with the theoretical calculation.[23]

[*]E-mail: kirichenko@ilt.kharkov.ua

[1] O. V. Kirichenko, Yu. A. Kolesnichenko, and V. G. Peschansky, Phys. Rev. **18**, 4 (1998).

[2] M. V. Kartsovnik, V. N. Laukhin, and V. N. Nizhankovskiĭ, A. A. Ignat'ev, JETP Lett. **47**, 363 (1988).

[3] M. V. Kartsovnik and P. A. Kononovich, V. N. Laukhin, I. F. Shchegolev, JETP Lett. **48**, 541 (1988).

[4] I. D. Parker, D. D. Pigram, R. H. Friend, M. Kurmo, and P. Day, Synth. Met. **27**, A387 (1988).

[5] K. Oshima, T. Mori, H. Inokuchi, H. Urayama, H. Yamochi, and C. Sato, Phys. Rev. B **38**, 938 (1988).

[6] W. Kang, G. Montanbaux, J. R. Cooper, D. Jerome, P. Batail, and C. Lenoir, Phys. Rev. Lett. **62**, 2559 (1989).

[7] I. F. Schegolev, P. A. Kononovich, M. V. Kartsovnik, V. N. Laukhin, S. I. Pesotskii, B. Hilti, and C. W. Mayer, Synth. Met. **39**, 537 (1990).

[8] R. Yagi, Y. Iue, T. Osada, and S. Kagoshima, Phys. Soc. Jpn. **59**, 3069 (1990).

[9] N. D. Kurshch, L. I. Buravov, M. V. Kartsovnik, V. N. Laukhin, S. I. Pesotskii, R. P. Rozenberg, E. B. Jagubskii, and A. V. Zvarikina, Synth. Met. **46**, 271 (1992).

[10] A. E. Kovalev, M. V. Kartsovnik, R. P. Shibaeva, R. P. Rozenberg, and I. F. Shcegolev, Solid State Commun. **89**, 575 (1994).

[11] M. V. Kartsovnik, A. E. Kovalev, V. N. Laukhin, I. F. Shchegolev, H. Ito, T. Ishiguro, N. D. Kushch, H. Mori, and G. Saito, Synth. Met. **70**, 811 (1995).

[12] J. Woznitza, *Fermi Surfaces of Low-Dimensional Organic Metals and Superconductors*, Springer-Verlag, Berlin (1996); M. V. Kartsovnik and V. N. Laukhin, J. Phys. I **6**, 1753 (1996); J. Singelton, Rep. Prog. Phys. **63**, 1111 (2000).

[13] J. Singleton, F. L. Pratt, M. Dorotto, T. J. B. M. Janssen, M. Kurmoo, J. A. A. J. Perenboom, W. Hayes, and P. Day, Phys. Rev. Lett. **68**, 2500 (1992).

[14] S. V. Demishev, N. E. Sluchanko, A. V. Semeno, and N. A. Samarin, JETP Lett. **61**, 313 (1995).

[15] S. V. Demishev, A. V. Semeno, N. E. Sluchanko, N. A. Samarin, I. B. Voskoboinikov, V. V. Glushkov, J. Singleton, S. J. Blundell, S. O. Hill, W. Hayes, M. V. Kartsovnik, A. E. Kovalev, M. Kurmoo, P. Day, and N. D. Kushch, Phys. Rev. B **53**, 12794 (1996).

[16] A. Polisski, J. Singleton, and N. Kushch, Czech. J. Phys. **46**, 2623 (1996).

[17] S. V. Demishev, A. V. Semeno, N. E. Sluchanko, N. A. Samarin, I. B. Voskoboĭnikov, M. V. Kartsovnik, and A. E. Kovalev, N. D. Kushch, Zh. Éksp. Teor. Fiz. **111**, 979 (1997) [JETP **84**, 540 (1997)].

[18] A. M. Kosevich, V. V. Andreev, Zh. Éksp. Teor. Fiz. **38**, 882 (1960) [Sov. Phys. JETP **11**, 637 (1960)].

[19] V. V. Andreev, A. M. Kosevich, Zh. Éksp. Teor. Fiz. **43**, 1061 (1962).

[20] V. G. Skobov, Zh. Éksp. Teor. Fiz. **39**, 689 (1960) [Sov. Phys. JETP **12**, 483 (1961)].

[21] V. G. Peschanskiĭ, I. V. Kozlov, K. Jiasemides, Fiz. Nizk. Temp. **26**, 225 (2000) [Low Temp. Phys. **26**, 169 (2000)].

[22] T. G. Togonidze, M. V. Kortsovnik, J. A. A. J. Perenboom, N. D. Kurshch, and H. Kobayashi, Physica B **294–295**, 435 (2001).

[23] P. D. Grigoriev, M. V. Kartsovnik, W. Biberacher, and P. Wyder, cond-mat/0108352 (2001).

Translated by Steve Torstveit