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Exact and quasiclassical Green's functions of two-dimensional electron gas with Rashba–Dresselhaus spin–orbit interaction in parallel magnetic field

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ABSTRACT

New exact and asymptotical results for the one particle Green's function of 2D electrons with combined Rashba–Dresselhaus spin–orbit interaction in the presence of in-plane uniform magnetic field are presented. A special case that allows an exact analytical solution is also highlighted. To demonstrate the advantages of our approach we apply the obtained Green's function to calculation of electron density and magnetization.

1. Introduction

One particle Green's functions (GF) is widely used in quantum mechanics and solid state physics [1,2]. Although the simple analytical formulas for GFs of the free electron gas can be found in some textbooks (see, for example, [2]), in the presence of interactions and in external fields in most cases only complicated integral representations of GF are available. Normally using the GF in the form of a multiple integral is extremely uncomfortable both for analytical analysis and for numerical computations. Therefore the search of exact, simplified and asymptotic results for GFs attracts the constant attention of theorists and mathematicians (see, for example, [3–5]).

In last decade the spintronics development has triggered off investigations of two-dimensional (2D) electron systems with spin–orbit interaction (SOI) (for review see [6,7]). Particularly the 2D systems with combined Rashba and Dresselhaus (R–D) SOI possesses new perspective properties (for review see [8]). The 2D electron gas with R–D SOI can be formed nearby the heterostructure interface between two semiconductors one of which possesses the bulk inversion asymmetry giving rise to the Dresselhaus SOI [9]. The Rashba SOI [10] results from an asymmetry of a confinement potential in a vicinity of the interface. The electrons occupy only the first quantum level in the potential well and freely move in the interface plane. For example, R–D SOI takes place in 2D electron systems in the heterostructures made of GaAs/GaAlAs [11,12], InAs/AlGaSb [11], AlGaN/GaN [13], GaAs/InGaAs [14], InGaAs/InP [15]. In spite of the fact that SOI is the relativistic effect and its influence to electron propertied should be small, the real values of SOI constants allow to observe SOI-related phenomena [8]. So, in Ref. [13] one can find the typical parameters for GaAs/AlGaAs 2D electron gas: $\alpha, \beta \simeq 10$ eVÅ, $\alpha/\beta \simeq 1 \div 7.6$, and the electron density $n_s \simeq 10^{11}$ cm^{-2}.

The GF for 2D electrons with R–D SOI without magnetic field and its asymptotic behavior have been discussed in the papers [16,17]. Explicit GF for each Rashba and Dresselhaus spin–orbit Hamiltonians with uniform perpendicular magnetic field have been derived in Ref. [3]. In our paper we obtain some exact and asymptotic expressions at zero temperature for the time independent GF of 2D electron gas with combined Rashba–Dresselhaus SOI for arbitrary values of SOI constants and arbitrary uniform magnetic field strength and direction parallel to the plane of the conductor. The structure of the paper is as it follows. In Sec. 2 we discuss the Hamiltonian of the system, its eigenvalues and eigenfunctions. The GF in coordinate space is presented as the sum of two parts describing separated contributions of two spin–orbit split branches of electron energy spectrum. In Sec. 3 we reduce the general expression for the GF to single integral of well-known special functions. The obtained formula is valid at arbitrary values of parameters. In Sec. 4 we derive asymptotic formulas for GF for large value of coordinate variable. In Sec. 5 we find the exact GF for the spe-

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cial case of equal SOI constants and certain direction of magnetic field. In Sec. 6 as a demonstration we apply our results for calculation of electron density of states and density of magnetization. We conclude the paper with the final remarks and summing up main results in Sec. 7.

2. Basic formulas

Let us consider the Hamiltonian of a two-dimensional (2D) noninteracting electron gas with Rashba and Dresselhaus SOI in the magnetic field $B=\left(B_x,B_y,0\right)$. Using the Coulomb gauge $\mathbf{A}=\left(0,0,B_x y-B_y x\right)$, $\nabla\cdot\mathbf{A}=0$, we write 2D Hamiltonian of the system as the sum of four terms

$$\hat{H}=\hat{H}_0+\hat{H}_R+\hat{H}_D+\hat{H}_B. \qquad (1)$$

Here $\hat{H}_0=\frac{\hbar^2\left(\hat{K}_x^2+\hat{K}_y^2\right)}{2m}\sigma_0$ is the Hamiltonian of 2D free electron gas, $\hat{H}_R=\alpha(\sigma_x\hat{K}_y-\sigma_y\hat{K}_x)$ and $\hat{H}_D=\beta(\sigma_x\hat{K}_x-\sigma_y\hat{K}_y)$ are Hamiltonians of Rashba and Dresselhaus SOI, respectively, $\hat{H}_B=\frac{g^*}{2}\mu_B\left(B_x\sigma_x+B_y\sigma_y\right)$ is the Hamiltonian of interaction between electron spin and magnetic field, $\hat{K}_{x,y}=-i\nabla_{x,y}$ is the wave vector operator, m is effective electron mass, $\sigma_{x,y,z}$ are Pauli matrices, $\hat{\sigma}_0$ is unit matrix 2×2, α and β are Rashba (α) and Dresselhaus (β) constants of SOI, μ_B is the Bohr magneton, g^* is an effective g-factor of the 2D system. We rewrite the total Hamiltonian (1) in the following form

$$\hat{H}=\hat{H}_0+\hat{\mathbf{R}}\sigma, \qquad (2)$$

where $\sigma=\left(\sigma_x,\sigma_y,\sigma_z\right)$ is the Pauli vector,

$$\begin{aligned}&\hat{R}_x=h_x+\alpha\hat{K}_y+\beta\hat{K}_x;\quad \hat{R}_y=h_y-\alpha\hat{K}_x-\beta\hat{K}_y,\\&\hat{R}_z=0;\quad h_{x,y}=\frac{g^*}{2}\mu_B B_{x,y}.\end{aligned} \qquad (3)$$

The eigenvalues and the eigenfunctions of the Hamiltonian (2) are (see, for example, [18])

$$\epsilon_{1,2}\left(\mathbf{k}\right)=\epsilon_0\pm R\left(k_x,k_y\right); \qquad (4)$$

$$\psi_{1,2}\left(\mathbf{r}\right)=\frac{1}{2\pi\sqrt{2}}e^{i\mathbf{kr}}\begin{pmatrix}1\\ e^{i\theta_{1,2}}\end{pmatrix}\equiv\frac{1}{2\pi}e^{i\mathbf{kr}}\phi\left(\theta_{1,2}\right). \qquad (5)$$

We introduce notations

$$\sin\theta_1=\frac{R_y}{R};\quad \cos\theta_1=\frac{R_x}{R},\quad \theta_2=\theta_1+\pi, \qquad (6)$$

$$\mathbf{k}=\left(k_x,k_y,0\right),\quad \epsilon_0=\frac{\hbar^2k^2}{2m},\quad k=\sqrt{k_x^2+k_y^2}, \qquad (7)$$

$$\begin{aligned}R&=\sqrt{R_x^2+R_y^2}\\&=\sqrt{\left(h_x+\alpha k_y+\beta k_x\right)^2+\left(h_y-\alpha k_x-\beta k_y\right)^2}.\end{aligned} \qquad (8)$$

The angles $\theta_{1,2}$ define the average spin direction for two branches of energy spectrum (4)

$$\mathbf{s}_{1,2}\left(\theta\right)=\phi^{\dagger}\left(\theta_{1,2}\right)\sigma\phi\left(\theta_{1,2}\right)=\left(\cos\theta_{1,2},\sin\theta_{1,2},0\right). \qquad (9)$$

The electron GF corresponding Hamiltonian (2) for complex ϵ in coordinate representation can be written as

$$\begin{aligned}&\hat{G}\left(\epsilon,\mathbf{r}\right)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{d\mathbf{k}e^{i\mathbf{kr}}}{\left(\epsilon-\epsilon_0\right)\sigma_0-\mathbf{R}\sigma}=\\&\frac{1}{2(2\pi)^2}\sum_{j=1,2}\int_{-\infty}^{\infty}d\mathbf{k}\frac{e^{i\mathbf{kr}}}{\epsilon-\epsilon_j}\left(\sigma_0+\sigma_x\cos\theta_j+\sigma_y\sin\theta_j\right),\end{aligned} \qquad (10)$$

where $\epsilon\in\mathcal{C}$. We used Eqs. (4), (6) and the identities $\epsilon_{1,2}-\epsilon_0=\pm R$,

$$\begin{aligned}&\left(\left(\epsilon-\epsilon_0\right)\sigma_0-\mathbf{R}\sigma\right)\left(\left(\epsilon-\epsilon_0\right)\sigma_0+\mathbf{R}\sigma\right)=\\&\left[\left(\epsilon-\epsilon_0\right)^2-R^2\right]\sigma_0=\left(\epsilon-\epsilon_1\right)\left(\epsilon-\epsilon_2\right)\sigma_0;\end{aligned} \qquad (11)$$

$$\begin{aligned}&\frac{\left(\left(\epsilon-\epsilon_0\right)\sigma_0+\mathbf{R}\sigma\right)}{\left(\left(\epsilon-\epsilon_0\right)\sigma_0-\mathbf{R}\sigma\right)\left(\left(\epsilon-\epsilon_0\right)\sigma_0+\mathbf{R}\sigma\right)}=\\&\frac{1}{2R}\left\{\frac{\left(\epsilon_1-\epsilon_0\right)\sigma_0+\mathbf{R}\sigma}{\epsilon-\epsilon_1}-\frac{\left(\epsilon_2-\epsilon_0\right)\sigma_0+\mathbf{R}\sigma}{\epsilon-\epsilon_2}\right\}.\end{aligned} \qquad (12)$$

In the Eq. (10) GF splits up into two independent parts describing separate contributions of every branch of energy spectrum (4).

3. Exact results for Green's function

For $\alpha\neq\beta$ we introduce new variables of integration $\tilde{k}$, f as follows

$$k_x=k_{x0}+\tilde{k}\cos f,\quad k_y=k_{y0}+\tilde{k}\sin f, \qquad (13)$$

$$k_{x0}=\frac{\alpha h_y+\beta h_x}{\alpha^2-\beta^2};\quad k_{y0}=-\frac{\alpha h_x+\beta h_y}{\alpha^2-\beta^2}, \qquad (14)$$

where $\mathbf{k_0}=\left(k_{x0},k_{y0}\right)$ is the point of branch touch (see, for example Ref. [19]). In coordinates (13) spin angles (6) depend only on the wave vector direction, the angle f, and SOI constants

$$\begin{aligned}&\sin\theta_{1,2}\left(f\right)=\mp\frac{\alpha\cos f+\beta\sin f}{\sqrt{\alpha^2+\beta^2+2\alpha\beta\sin 2f}}\\&\cos\theta_{1,2}\left(f\right)=\pm\frac{\alpha\sin f+\beta\cos f}{\sqrt{\alpha^2+\beta^2+2\alpha\beta\sin 2f}}.\end{aligned} \qquad (15)$$

It can be shown that spin direction is symmetric with respect to the center $\theta_{1,2}(f+\pi)=\theta_{1,2}(f)+\pi$, indicating a convenience of Eq. (13). The values k_{x0}, k_{y0} have been found out of the system of equations $R_x\left(k_{x0},k_{y0}\right)=0$, $R_y\left(k_{x0},k_{y0}\right)=0$. In a shifted polar coordinates $\tilde{k}$, f (13) the energies $\epsilon_{1,2}$ take the form

$$\epsilon_{1,2}\left(\tilde{k},\tilde{f}\right)=\frac{\hbar^2\tilde{k}^2}{2m}-\frac{\hbar^2\tilde{k}}{m}\lambda_{1,2}\left(f\right)+E_0. \qquad (16)$$

Here

$$\begin{aligned}&\lambda^{(1,2)}\left(f\right)=-k_{x0}\cos f-k_{y0}\sin f\\&\mp\frac{m}{\hbar^2}\sqrt{\alpha^2+\beta^2+2\alpha\beta\sin(2f)};\end{aligned} \qquad (17)$$

$$E_0=\hbar^2\frac{h^2(\alpha^2+\beta^2)+4\alpha\beta h_x h_y}{2m(\alpha^2-\beta^2)^2}. \qquad (18)$$

The Eq. (16) make it possible to write the poles $\epsilon_{1,2}\left(k_{\pm}^{(1,2)},f\right)=\epsilon$ of the Green function (10) in a simple form

$$k_{\pm}^{(1,2)}=\lambda^{(1,2)}\pm\sqrt{\xi^{(1,2)}}. \qquad (19)$$

$$\xi^{(1,2)}=\left(\lambda^{(1,2)}\right)^2+\frac{2m(\epsilon-E_0)}{\hbar^2}. \qquad (20)$$

By using the roots (19) in coordinates (13) one can write a simple expansion of the functions $\left(\epsilon-\epsilon_{1,2}\right)^{-1}$

$$\frac{1}{\epsilon-\epsilon_{1,2}}=-\frac{2m}{\hbar^2\tilde{k}}\sum_{\pm}\frac{k_{\pm}^{(1,2)}}{k_{\pm}^{(1,2)}-k_{\mp}^{(1,2)}}\frac{1}{\tilde{k}-k_{\pm}^{(1,2)}}. \qquad (21)$$

By means of identity (21) the GF (10) is written as the double sum

$$G(\epsilon,\mathbf{r}) = -\frac{m}{(2\pi\hbar)^2}\exp\left[i(k_{x0}\cos\varphi_r + k_{y0}\sin\varphi_r)r\right]\times \sum_{j=1,2}\oint d\tilde{f}\left(1+\sigma_x\cos\theta_j+\sigma_y\sin\theta_j\right) \sum_{\pm}\frac{\tilde{k}_{\pm}^{(j)}}{\tilde{k}_{\pm}^{(j)}-\tilde{k}_{\mp}^{(j)}}\int_0^{\infty}\frac{d\tilde{k}}{\tilde{k}-\tilde{k}_{\pm}^{(j)}}e^{i\tilde{k}r\cos(\tilde{f}-\varphi)} \quad \epsilon\in\mathcal{C}. \tag{22}$$

At $r\neq 0$ the integral over $\tilde{k}$ in Eq. (22) can be expressed by means of trigonometric integral functions (see, for example [20]),

$$F(k_0,r) = \int_0^{\infty}\frac{d\tilde{k}}{\tilde{k}-k_0}e^{i\tilde{k}r} = e^{ik_0 r}\left[-Ci\left(-k_0|r|\right)+iSi\left(k_0 r\right)+\frac{i\pi}{2}sgn\, r\right], \quad r\in\mathcal{R}, \tag{23}$$

where $Si(z)$ and $Ci(z)$ are sine and cosine integrals,

$$Ci(z) = -\int_z^{\infty}\frac{\cos z}{z}; \quad Si(z) = -\int_z^{\infty}\frac{\sin z}{z}+\frac{\pi}{2}; \quad \lim_{\delta\to+0}Ci(x\pm i\delta) = Ci(|x|)\pm i\pi\Theta(-x), \tag{24}$$

where $z\in\mathcal{C}$; $|\arg z|<\pi$; $x\in\mathcal{R}$. The Eq. (23) is valid for any $k_0\in\mathcal{C}$ excepting semiaxis $\mathrm{Im}\,k_0=0$, $0<\mathrm{Re}\,k_0<\infty$, for which usually one introduces the retarded and advanced Green functions $G^{R(A)}(E,\mathbf{r})$ as the limit

$$G^{R(A)}(E,\mathbf{r}) = \lim_{\delta\to+0}G(E\pm i\delta,\mathbf{r}), \quad E\in\mathcal{R}. \tag{25}$$

Using Eqs. (21), (23) one can rewrite the Green function (10) as

$$G(\epsilon,\mathbf{r}) = -\frac{m}{(2\pi\hbar)^2}\exp\left[i(k_{x0}\cos\varphi_r + k_{y0}\sin\varphi_r)r\right]\times \sum_{j=1,2}\oint df\left(1+\sigma_x\cos\theta_j+\sigma_y\sin\theta_j\right)\times \sum_{\pm}\frac{k_{\pm}^{(j)}}{k_{\pm}^{(j)}-k_{\mp}^{(j)}}F\left(k_{\pm}^{(j)}, r\cos(f-\varphi_r)\right), \quad \epsilon\in\mathcal{C}, \tag{26}$$

where angle φ_r defines a direction of coordinate $\mathbf{r}=r(\cos\varphi_r, \sin\varphi_r, 0)$. The formula (26) turns to the result of the Ref. [16] for the zero magnetic field. The obtained GF (26) is suitable for numerical calculation under arbitrary values of all parameters. It also gives analytical formulas in the quasiclassical case $r\to\infty$.

For equal SOI constants, $\alpha=\beta$, one cannot use coordinates (13) and we introduce usual polar coordinates

$$k_x = k\cos\varphi_k; \quad k_y = k\sin\varphi_k. \tag{27}$$

After transformations similar to those performed above we find

$$\hat{G}(\epsilon,\mathbf{r}) = \frac{m^2}{\pi^2\hbar^4}\sum_{n=1}^{4}\oint d\varphi_k\left[\left(\epsilon-\frac{\hbar^2k_n^2}{2m}\right)\sigma_0 + L_x(k_n)\sigma_x + L_y(k_n)\sigma_y\right]\frac{k_n F\left(k_n, r\cos(\varphi_k-\varphi_r)\right)}{\partial Q/\partial k|_{k=k_n}}, \tag{28}$$

where k_n are the roots of quartic polynomial equation without cubic term

$$Q(k_n,\varphi_k) = 0, \tag{29}$$

$$Q(k,\varphi_k) = k^4 - k^2\frac{4m}{\hbar^2}\left[\epsilon+\frac{2m\alpha^2}{\hbar^2}(1+\sin 2\varphi_k)\right] - k\frac{8m^2\alpha h}{\hbar^4}\left[\sin(\varphi_k-\varphi_h)+\cos(\varphi_k+\varphi_h)\right] + \frac{4m^2}{\hbar^4}\left(\epsilon^2-h^2\right), \tag{30}$$

$$L_x = \left(h\cos\varphi_h + \sqrt{2}\alpha k\sin\left(\varphi_k+\frac{\pi}{4}\right)\right), \quad L_y = \left(h\sin\varphi_h - \sqrt{2}\alpha k\sin\left(\varphi_k+\frac{\pi}{4}\right)\right), \tag{31}$$

the function $F(k,r)$ is given by Eq. (23), φ_h defines the magnetic field direction, $\mathbf{h}=h(\cos\varphi_h,\sin\varphi_h,0)$. Though the Eq. (29) has exact analytical solutions (see, for example, [21]) they are very complicated and not suitable for analytical calculation. Nevertheless the Eq. (28) may be practically convenient in numerical analysis. As for the particular case of magnetic field direction along the symmetry axis $k_x=-k_y$ we consider it in Sec. 5. and the GF has been expressed by means of Bessel functions.

4. Quasiclassical Green's function

Quasiclassical approximation can be applied in physical investigations, if characteristic length scales of the problem are much larger than Fermi wavelength λ_F which has of the order of inverse wave vector k^{-1} at Fermi level. Since the GF oscillates as a function of the coordinate r on a scale $r\sim k^{-1}$ in framework of quasiclassical approximation in most cases the asymptotic formulas for large kr the GF could be used. Below we find the asymptotic expressions for GF (26) at $r\to\infty$. For real $\epsilon=E$ the equality

$$\epsilon_{1,2}\left(k_{\pm}^{(1,2)}, f\right) = E \tag{32}$$

gives two branches of the electron energy spectrum. The positive roots of Eq. (32) describe the isoenergetic contours $k=k_{\pm}^{(j)}(E,f)$ corresponding physical electron states in k-space for given energy E. If $E>E_0$, the roots $k_{+}^{(1,2)}>0$ for any values of f, while roots $k_{-}^{(1,2)}<0$. For $E<E_0$ reals roots of equation (32) exist, if inequality $\frac{2m(E_0-E)}{\hbar^2}\leq\left(\lambda^{(j)}\right)^2$ is hold. Both roots $k_{\pm}^{(j)}$ take positive values for the angles f in which $\lambda_j>0$. Below we will not consider values of energies E for which the GF exponentially decreases with coordinate r assuming $k_{\pm}^{(j)}, E\in\mathcal{R}$. At first we substitute asymptotic expansions for $Si(z)$ and $Ci(z)$ at the large z (see, for example [20]) to the function $F(k_0,r)$ (23). At large $r\to\infty$ and real k_0 the main term of expansion reads as

$$F(k_0\pm i0,r)\approx\frac{i\pi}{2}e^{ik_0r}\left[(1+sgn(k_0))(sgn(r)\pm 1)\right] + O\left(\frac{1}{|k_0r|}\right); \quad |k_0r|\gg 1. \tag{33}$$

As the second step one derive the asymptotic formula for (23) by the stationary phase method [22]. Stationary phase points $f=f_{st}^{(j)}$ must be found from equation

$$\frac{d}{df}\left(k_{\pm}^{(j)}\cos(f-\varphi_r)\right)\Big|_{f=f_{st}^{(j)}} = \dot{k}_{\pm}^{(j)}\cos(f-\varphi_r) - k_{\pm}^{(j)}\sin(f-\varphi_r)\Big|_{f=f_{st}^{(j)}} = 0, \tag{34}$$

which leads to the condition $\mathbf{r}\parallel\mathbf{n}_v$, where $\mathbf{n}_v$ is the unit vector along the electron velocity $\mathbf{v}^{(j)}=\nabla_{\mathbf{k}}\epsilon_j/\hbar$ (see also [23,24])

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$$\mathbf{rn}_\nu|_{f=f_{st}^{(j)}}=r;\quad \mathbf{n}_\nu(f)=\frac{\mathbf{v}^{(j)}}{\left|\mathbf{v}^{(j)}\right|}=$$
$$\mp\left(-\frac{\dot{k}_\pm^{(j)}\sin f+k_\pm^{(j)}\cos f}{\sqrt{k_\pm^{(j)2}+\dot{k}_\pm^{(j)2}}},\frac{\dot{k}_\pm^{(j)}\cos f-k_\pm^{(j)}\sin f}{\sqrt{k_\pm^{(j)2}+\dot{k}_\pm^{(j)2}}}\right).\tag{35}$$

Here and in all formulas below the point above functions denotes the derivative on angle f.

As the result of standard calculations we find the asymptotic of the GF (26)

$$G(\epsilon,\mathbf{r})\simeq-\frac{i}{2\sqrt{2\pi}}\exp\left[i(k_{x0}\cos\varphi_r+k_{y0}\sin\varphi_r)r\right]\times$$
$$\sum_{j=1,2}\sum_s\frac{\left(1+\sigma_x\cos\theta_j+\sigma_y\sin\theta_j\right)}{\hbar v^{(j)}\sqrt{|K_j|r}}\times\tag{36}$$
$$\exp\left[iS_jr\mp\frac{i\pi}{4}sgnK_j\right]\Big|_{f=f_{st}^{(j)}}+O\left(\frac{1}{r}\right);\quad r\to\infty,$$

$$S_j(f)=k_\pm^{(j)}(f)\cos(f-\varphi_r),\tag{37}$$

$$\ddot{S}_j\left(f_{st}^{(j)}\right)=\mp K_j\left(f_{st}^{(j)}\right)\left(\dot{k}_\pm^{(j)}\left(f_{st}^{(j)}\right)^2+k_\pm^{(j)}(f_{st}^{(j)})^2\right).\tag{38}$$

We assume $S_j(f)\in\mathcal{R}$, $r>0$, $S_j\left(f_{st}^{(j)}\right)\neq 0$, $\ddot{S}_j\left(f_{st}^{(j)}\right)\neq 0$. All functions in Eq. (36) are calculated in stationary phase points $f=f_{st}^{(j)}$ for which $\mathbf{rv}>0$. Summation over s takes into account the existence of few solutions of Eq. (34) (few stationary phase points $f_{st}^{(2)}(s)$ for given direction of vector $\mathbf{r}$ (see Ref. [17])). It is possible in the cases when isoenergetic contour $k=k_+^{(2)}(E,f)$ is nonconvex. In Eq. (36), $K_{1,2}(f)\neq 0$ is the curvature of the isoenergetic curve $\epsilon_{1,2}(f)=E$,

$$K_j(f)=\frac{k_\pm^{(j)}(f)^2+2\dot{k}_\pm^{(j)}(f)^2-k_\pm^{(j)}(f)\ddot{k}_\pm^{(j)}(f)}{\left(\dot{k}_\pm^{(j)}(f)+k_\pm^{(j)}(f)^2\right)^{3/2}};\tag{39}$$

$$v_j=\frac{1}{\hbar}\sqrt{\left(\frac{\partial\epsilon_j}{\partial k_\pm^{(j)}}\right)^2+\frac{1}{k_\pm^{(j)2}}\left(\frac{\partial\epsilon_j}{\partial f}\right)^2}\tag{40}$$

is an absolute value of electron velocity. The Eq. (36) coincides with the results of Ref. [17] in the case of $B=0$. We do not adduce the GF for single inflection points, which can exist on the isoenergetic contour $\epsilon_2(f)=E$ for certain values of SOI constants. It can be simply derived in the same way.

5. Exact results for special case of equal SOI constants

Let us consider the special case: $\alpha=\beta$ and the magnetic field is directed along the $y=-x$ axis, $\mathbf{B}=\frac{B}{\sqrt{2}}(-1,1,0)$. Under these conditions the Eq. (10) can be presented in the form

$$\hat{G}(\epsilon,\mathbf{r})=\frac{1}{2(2\pi)^2}\sum_\pm\left(\sigma_0\pm\frac{\sigma_y-\sigma_x}{\sqrt{2}}\right)\int_{-\infty}^{\infty}d\mathbf{k}\frac{e^{i\mathbf{kr}}}{\epsilon-\epsilon_\pm},\tag{41}$$

where we introduce new functions of energy dimension

$$\epsilon_\pm=\frac{\hbar^2}{2m}\left[\left(k_x\pm\frac{\sqrt{2}m\alpha}{\hbar^2}\right)^2+\left(k_y\pm\frac{\sqrt{2}m\alpha}{\hbar^2}\right)^2\right]-\frac{2m\alpha^2}{\hbar^2}\mp h.\tag{42}$$

The spin angles (6), that correspond with parts of the sum with $\epsilon_\pm$, keep the constant directions $\theta_+=\frac{3\pi}{4}$ and $\theta_-=-\frac{\pi}{4}$. The Eq. (41) can be rewritten as

$$\hat{G}(\epsilon,\mathbf{r})=\frac{1}{2}\sigma_0\left(G_+(\epsilon,\mathbf{r})+G_-(\epsilon,\mathbf{r})\right)$$
$$+\frac{\sigma_y-\sigma_x}{2\sqrt{2}}\left(G_+(\epsilon,\mathbf{r})-G_-(\epsilon,\mathbf{r})\right),\quad \epsilon\in\mathcal{C},\tag{43}$$

where for the function $G_\pm(\epsilon)$ one obtains

$$G_\pm(\epsilon)=\int_{-\infty}^{\infty}\frac{dk_xdk_y}{(2\pi)^2}$$
$$\frac{e^{i\mathbf{kr}}}{\left(\epsilon+\frac{2m\alpha^2}{\hbar^2}\pm h\right)-\frac{\hbar^2}{2m}\left[\left(k_x\pm\frac{\sqrt{2}m\alpha}{\hbar^2}\right)^2+\left(k_y\pm\frac{\sqrt{2}m\alpha}{\hbar^2}\right)^2\right]}\tag{44}$$
$$=\exp\left(\pm i\frac{\sqrt{2}m\alpha}{\hbar^2}(x+y)\right)G_{2D}\left(\epsilon+\frac{2m\alpha^2}{\hbar^2}\mp h\right).$$

We point out that $G_{2D}(\epsilon,r)$ is well-known GF of free 2D electrons. Particularly the retarded GF reads as

$$G_{2D}^R(\epsilon,r)=-\frac{m}{2\hbar^2}\begin{cases}iH_0^{(1)}\left(\sqrt{2m\epsilon}|r|/\hbar\right); & \epsilon>0\\ \frac{2}{\pi}K_0\left(\sqrt{2m|\epsilon|}|r|/\hbar\right); & \epsilon<0\end{cases}\tag{45}$$

where $H_0^{(n)}(x)$ is the Hankel function and $K_0(x)$ is the McDonald function.

6. Densities of electron states and magnetization

As an example of our results applications we calculate the electron density of states $\rho(E)$ and the density of vector magnetization $\mathbf{m}(E)$ at $\alpha\neq\beta$, which are important characteristics of 2D conducting system (compare with results of Ref. [18]).

Density of states can be found from the relation

$$\rho(E)=-\frac{1}{\pi}\operatorname{Im}\operatorname{Tr}\left[\hat{G}^R(E,\mathbf{r})\right]\Big|_{\mathbf{r}=0}.\tag{46}$$

Substituting the retarded GF (25) at $r=0$ from Eqs. (22), (25) one can derive electron density of states,

$$\rho(E)=\frac{m}{\pi\hbar^2};\quad E\geq E_0,\tag{47}$$

$$\rho(E)=\sum_{j=1,2}\rho_j(E)=$$
$$\frac{m}{2\pi^2\hbar^2}\sum_{j=1,2}\oint df\,\Theta\left(\lambda^{(j)}\right)\Theta\left(\xi^{(j)}\right)\sum_\pm\frac{\pm k_\pm^{(j)}}{k_\pm^{(j)}-k_\mp^{(j)}}=\tag{48}$$
$$=\frac{m}{2\pi^2\hbar^2}\sum_{j=1,2}\oint df\frac{\lambda^{(j)}}{\sqrt{\xi^{(j)}}}\Theta\left(\lambda^{(j)}\right)\Theta\left(\xi^{(j)}\right);\quad E<E_0,$$

where $\lambda^{(j)}$ and $\xi^{(j)}$ are defined by Eqs. (17) and (20). Note the importance of the relation between electron energy E and the energy E_0 (18) of branch touch point. The Eq. (47) shows that the density of states is the same as for free 2D electron gas for energies $E\geq E_0$. In opposite case $E<E_0$ the density of states $\rho(E)$ depends on the magnetic field and constants of SOI. The features of $\rho(E)$ related to minima (steps) and saddle points (peaks) on the energy contours (32). These points $\left(\tilde{k}_\nu,f_\nu\right)$ should be found out of the system of equations

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$$\frac{\partial \epsilon_{1,2}}{\partial \tilde{k}} = \frac{\hbar^2}{m}\left(\tilde{k} - \lambda^{(1,2)}(f)\right) = 0; \tag{49}$$

$$\frac{\partial \epsilon_{1,2}}{\partial f} = -\frac{\hbar^2}{m}\tilde{k}\dot{\lambda}^{(1,2)}(f) = 0, \tag{50}$$

from which

$$\tilde{k}_\nu = \lambda^{(1,2)}(f_\nu), \quad \dot{\lambda}^{(1,2)}(f_\nu) = 0. \tag{51}$$

It is clear that the Eq. (49) can be satisfied, if $\xi^{(j)} = 0$ (see Eq. (20)). Therefore

$$\epsilon_{1,2}\left(\tilde{k}_\nu, f_\nu\right) = E_0 - \frac{\hbar^2}{2m}\left(\lambda^{(1,2)}(f_\nu)\right)^2. \tag{52}$$

The energy minima $\epsilon_{1,2}\left(\tilde{k}_\nu, f_\nu\right) = E^{\min}_{1,2}$ correspond to negative second derivative $\ddot{\lambda}^{(1,2)}(f_\nu) < 0$ and saddle points $\epsilon_{1,2}\left(\tilde{k}_\nu, f_\nu\right) = E^{sad}_{1,2}$ meet the case $\ddot{\lambda}^{(1,2)}(f_\nu) > 0$.

The electron density should be found by integration over all available energies below Fermi level E_F

$$n_e = \sum_{j=1,2} \int_{E_j^{\min}}^{E_F} dE \rho_j(E) = \frac{m}{\pi\hbar^2}\left[E_F + \frac{m}{2\hbar^2}\left(\alpha^2 + \beta^2\right)\right] \tag{53}$$

for $E_F \geq E_0$.

We find density of magnetization using its relations with retarded GF

$$m_{x,y}(E) = -\frac{1}{\pi} \operatorname{Im} \operatorname{Tr}\left[\sigma_{x,y}\hat{G}^R(E, \mathbf{r})\right]\Big|_{\mathbf{r}=0}. \tag{54}$$

Substituting the GF (22) into Eq. (54) after calculations similar to carried out above one obtain for $E \geq E_0$

$$m_{x,y}(E) = 0, \tag{55}$$

that follows from symmetry relations

$$\begin{aligned} &\lambda^{(1,2)}(f - \pi) = -\lambda^{(2,1)}(f); \\ &k_{\pm}^{(1,2)}(f - \pi) = -k_{\mp}^{(2,1)}(f). \end{aligned} \tag{56}$$

At $E < E_0$ the density of magnetization becomes

$$\begin{aligned} m_{x,y}(E) = {} & \frac{m}{2\pi^2\hbar^2} \sum_{j=1,2} \oint df \begin{Bmatrix} \cos\theta_i \\ \sin\theta_i \end{Bmatrix} \times \\ & \Theta\left(\lambda^{(j)}\right)\Theta\left(\xi^{(j)}\right) \sum_{\pm} \frac{\pm k_{\pm}^{(j)}}{k_{\pm}^{(j)} - k_{\mp}^{(j)}} \\ = {} & \frac{m}{2\pi^2\hbar^2} \sum_{j=1,2} \oint df \begin{Bmatrix} \cos\theta_i \\ \sin\theta_i \end{Bmatrix} \frac{\lambda^{(j)}}{\sqrt{\xi^{(j)}}} \Theta\left(\lambda^{(j)}\right)\Theta\left(\xi^{(j)}\right). \end{aligned} \tag{57}$$

The figure illustrates dependencies of densities of states (49) and magnetization (57) on the magnetic field B. In Fig. 1 the magnetic field is directed along the symmetry axis $k_x = -k_y$ of the energy spectrum at $B = 0$. In this case the vector of magnetization is directed along the magnetic field $\mathbf{m} \| \mathbf{B}$. The dependencies have the logarithmic singularity at $B = B_s$ which results from the saddle points of the energy surface $\epsilon = \epsilon_2(k_x, k_y)$, (van Hove singularity [25]). Another critical value of the magnetic field $B = B_c$ corresponds to an equality $E_0(B_c) = E_F$ (18) at which the smaller isoenergetic contour, $\epsilon_1 = E_F$ (4), disappears. In the field B_c first derivatives $\frac{\partial \rho}{\partial B}$, $\frac{\partial m_{x,y}}{\partial B}$ have a jump and at smaller magnetic fields $B < B_c$ one has $\rho = \rho_0$, $\mathbf{m} = 0$.

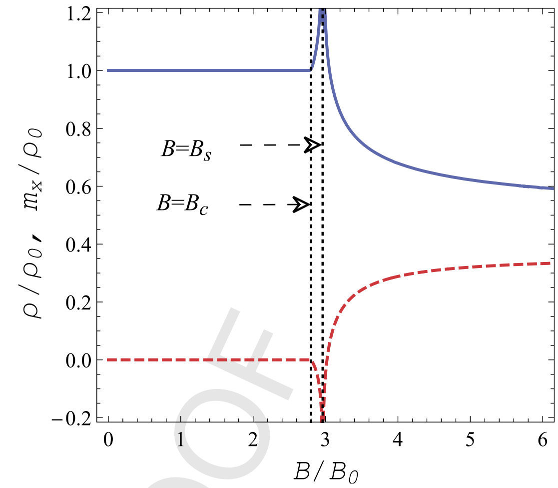

Fig. 1. Dependence of densities of states ρ (solid line) and magnetization m_x (dashed line) on the magnetic field B, normalized by constants $\rho_0 = \frac{m}{\pi\hbar^2}$ and $B_0 = \frac{2}{g*}\frac{\epsilon_F}{\mu_B}$. In the plot we used dimensionless values of parameters $\frac{m\alpha}{\hbar^2 k_F} = 1.2$, $\frac{\alpha}{\beta} = 6$ ($k_F = \sqrt{2m\epsilon_F}/\hbar$), $\varphi_h = -\frac{\pi}{4}$. The position of van Hove's singularities on the magnetic field scale are $B_s/B_0 = 2.96$, $B_c/B_0 = 2.8$.

If Fermi energy $E_F \geq E_0$ the magnetic moment corresponds to Pauli's paramagnetism of the free electron gas without SOI

$$M_{x,y} = \frac{g^*\mu_B}{2} \sum_{j=1,2} \int_{E_j^{\min}}^{E_F} dE m_{x,y;j}(E) = \frac{m(g^*\mu_B)^2}{4\pi\hbar^2} B_{x,y}, \tag{58}$$

and it does not depend on SOI constants. Here $m_{x,y;j}(E)$ are the two items in the sum over j in Eq. (57).

7. Summary

To sum up, the exact and asymptotical expressions for the Green's function (GF) of 2D noninteracting electron gas with combined Rashba–Dresselhaus spin–orbit interaction in parallel magnetic field at zero temperature are derived. We split the GF into two parts either of which depends only on characteristics of the one branch of spin–orbit split energy spectrum, Eq. (10). The GF in the form of double integral is reduced to the single integral of trigonometric integral functions, Eq. (26). This result should be helpful in numerical computations and in evaluating asymptotic expressions. We present the asymptotic of GF for large coordinate values which can be used in quantum mechanical quasiclassical calculations, Eq. (36). It is shown that asymptotic formula depends only on two local characteristics of energy spectrum (32): the curvature of isoenergetic curves and the electron velocity. For the equal SOI constants and magnetic field direction along one of the symmetry axis we express the GF by means of Bessel functions, Eq. (43). Although this exact result describes the special case, it may be used for qualitative analysis of different problems for other (but close) values of parameters. In the conclusion we demonstrate a usefulness of our results for calculation of physical quantities. We find the electron density of states, Eq. (48), and density of magnetization, Eq. (57). These results allow to obtain the clear interpretation of peculiarities of the dependencies of mentioned quantities on the energy and of their appearance conditions. We believe the results for the electron density (53) and the magnetization (58) for the combined Rashba–Dresselhaus spin–orbit interaction have been obtained for the first time.

In the present paper we have solved the 2D problem. In a framework of 2D model the Hamiltonian (1) does not depend on component of generalized momentum $P_z = p_z + eA_z/c$ and for the

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Coulomb gauge eigenvalues (3) and the eigenfunctions (4) are exact [18]. In this regard we claimed that obtained results are valid for an arbitrary value of the magnetic field. A real electron system in semiconductor heterostructures is quasi-two-dimensional one. It is characterized by a finite thickness in the perpendicular to the interface direction, which is of the order of the potential well width $d \sim 10$ nm [11,26]. In the in-plane magnetic field the variables a 3D Schrodinger equation cannot be separated and a coordinate dependence of the generalized momentum $\mathbf{P}$ should be taken into account. However, as it had been shown in Ref. [19], the Hamiltonian (1) can be used, if $(d/l_B)^2 \ll 1$ ($l_B = \sqrt{c\hbar/eB}$ is a magnetic length). The last inequality gives a restriction on the magnetic field value $B < 1$ T.

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Highlights

- The GF of 2D electron gas with Rashba-Dresselhaus SOI is reduced to a single integral
- Quasiclassical GF depends on local characteristics of energy spectrum
- Effect of magnetic field appears in GF by changes of geometry of isoenergetic contours
- The exact GF is found for equal SOI constants and magnetic field along symmetry axis

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