Eur. Phys. J. B (2017)
DOI: 10.1140/epjb/e2017-80194-4

THE EUROPEAN PHYSICAL JOURNAL B

Regular Article

Quantum high-frequency conductivity oscillations in graphene multilayers and nodal semimetals in a tilted magnetic field

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Received 1 April 2017
Published online (Inserted Later) – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2017

Abstract. A new type of angular oscillations of the high-frequency conductivity for conductors with a band-contact line has been predicted. The effect is caused by groups of charge carriers near the self-intersection points of the Fermi surface, where the electron energy spectrum is near-linear and can be described by anisotropic Dirac cone model. The amplitude of the resonance peaks satisfies the simple sum rule. The ease in changing the degree of anisotropy of the Dirac cone due to the angle of inclination of the magnetic field makes the considered type of oscillations attractive for experimental observation of relativistic effects.

1 Introduction

Recently, there has been growing interest in the study of nodal semimetals having band-contact lines. First principles calculations indicates the existence of ring-shaped nodal lines in Ca, Sr, Yt [1]. Also, the topological transition 3 $\frac{1}{2}$ kind is known for the conductors with band-contact line and thus possible in graphite conductors family, Be, Mg, Zn, Cd, Al and other materials [2]. Usual graphite have nodal lines [3].

In this paper we call attention to the effects of anisotropic Dirac cones without an inversion center (tilted Dirac cones) in nodal semimetals. The Hamiltonian, corresponding to the linear energy spectrum of Dirac-type charge carriers has the form [4–6]:

$$\epsilon(p_x, p_y) = v_0(\alpha\sigma_x p_x + \sigma_y p_y + \eta p_y) \qquad (1)$$

where the absence of an inversion center $\eta \neq 0$ is either a consequence of the internal symmetry of the conductor or may be achieved artificially, e.g. in strained graphene or in a problem of Dirac electron drift in crossed electric and magnetic fields [4]. The so-called "tilt" η can describe the relativistic effects [7]. Furthermore, the "collapse" ($|\eta| > 1$) of the Hamiltonian (1) is naturally explained in terms of relativistic rotations ("Lorentz boosts") [8].

The implementation of the Hamiltonian (1) for graphene requires a relatively strong electric fields $\sim 10^6$ V/m and relatively large strain values in the sample $\sim$10% [7]. In case of natural anisotropy, particularly in the compound $\alpha-(BEDT-TTF)_2I_3$ [4,5,9–11], changing of parameters of the electron energy spectrum is difficult, since the latter is due to the intrinsic properties of the conductor. Thus the experimental observation of effects that require a parameter η to be changed is associated with certain difficulties in these conductors. All the above mentioned conductors have a pronounced two-dimensional nature.

At the same time physical phenomena characteristic of the Hamiltonian (1), will take place in nodal semimetals near the self-intersection points of Fermi surfaces. It can be noticed that in a tilted magnetic field, the electron energy spectrum in a Larmor orbit's plane will be given by the model (1), where the value of η which determines the degree of anisotropy of the electron energy spectrum, can be easily changed by simply changing the tilt angle of a quantizing magnetic field. The attractiveness of the graphite and its derivatives is determined by the fact that for simple chemical compounds the high purity of the conductor required for the observation of high harmonics of the quantum cyclotron resonance [12] can be more easily achieved.

The goal of the present work is new oscillation phenomena caused by tilted Dirac cone effects in conductors having band-contact (nodal) lines. In Section 2 we choose the model of the electron energy spectrum. The model has the qualitative accordance with the Fermi surface of a number of nodal semimetals. The conditions limiting the applicability of the model are given. In Section 3 a new type of angular oscillations of the high-frequency conductivity for conductors with nodal lines has been predicted. The physical mechanism of these oscillations is explained. Section 4 shows that the amplitude of the resonance peaks satisfies the simple sum rule or the "magic square rule", which follows directly from properties of Pauli matrices.

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In Section 5 the paper is summarized and concluded. The possibility to observe the predicted oscillation effects is discussed. We provide a brief overview of articles related to the results of the present work.

2 Model

The model of graphene multilayers with a crystal lattice of AA type stacking is convenient for the observation of the resonant effects near the Dirac cone, since the character of the energy spectrum of the charge carriers can be considered to be linear in a broad range of energies [13,14]. AA stacked graphite is unstable and cannot exist under natural conditions. Although the nanoparticles with the number of layers of about ten grown on the border of the diamond may be available for direct observation [15]. Nevertheless, the energy spectrum of AA type graphite is widely used in theoretical works as the simplest and the most convenient theoretical model due to its characteristics, such as layering and Dirac energy spectrum of charge carriers near the Fermi surface (see the work) [13] (see also Ref. [16]). The Hamiltonian of low-energy charge carriers carriers corresponding to the model has the form:

$$H(\mathbf{p}) = v_{\parallel}(\sigma_x p_x + \sigma_y p_y) - 2t\cos(\frac{a_z p_z}{\hbar}), \qquad (2)$$

where a_z is the interlayer distance and t is the overlap integral of the wave functions in adjacent layers, that we consider to be positive. This model was proposed for the conductors with a graphitelike energy spectrum (2) in reference [13], where a linear magnetoresistance of a layered conductor with a small overlap integral t was investigated.

One can also easily see the qualitative accordance of the model (2) with a fragment of the Fermi surface of a number of nodal semimetals near the point of self-intersection of Fermi surfaces (see Fig. 1 of Ref. [2]). In particular the topological transition of 3 $\frac{1}{2}$ kind [2] occurs when $\epsilon_F = \pm 2t$ for the model (2).

This model of the electron energy spectrum can be also suitable for a number of graphite intercalates with AA type stacking of graphene layers [17]. For example, recent ARPES studies have reported about the direct observation of a linear energy spectrum of charge carriers in KC_8 compounds. Along with the observed data concerning the traditional quantum oscillation effects, the ARPES results reveal the applicability of the Dirac cone model for the energy spectrum of the charge carriers in conductors of this type [18]. The dependence of the energy of the charge carriers on the momentum components in the plane of the layers with a good degree of accuracy can be considered to be linear in the energy area of the order of fractions of ev, which is significantly higher than in graphite (several mevs) [18,19]. The Fermi velocity in the layers plane $v_F \approx (0.82\text{–}0.97) \times 10^6$ m/s (see for example [12]), i.e. is close to the value of the Fermi velocity of conduction electrons in graphene. Unfortunately, a strong shift of the Fermi level is often observed in intercalated graphite. Therefore the Dirac singularity can be deep below the Fermi level, that takes place for intercalation by alkali metals in particular. Nevertheless, the wide variety of intercalated graphite compounds gives the possibility to observe the effects of an anisotropic Dirac cone for the other members of this family of compounds. The model (2) was later used in reference [16] to study the quantum cyclotron resonance in the case of not so high frequencies $\hbar\omega < \epsilon_1$, where ϵ_1 is the energy difference between the zeroth and first Landau levels, when the influence of the electron-hole transitions can be neglected.

In a tilted magnetic field $\mathbf{B} = (0, B_0 \sin\theta, B_0 \cos\theta)$, near the self-intersection point of the Fermi surface $\mathbf{p} = (0, 0, p_{z0})$, the dependence of the charge carriers energy on the components of the momentum in Larmor orbit's plane can be described by the expression (1) with the parameter values

$$\eta = -\frac{v_\perp}{v_\parallel}\tan\theta, \qquad v_0 = v_\parallel \cos\theta,$$
$$\alpha = \frac{1}{\cos\theta}, \qquad v_\perp = \frac{2ta_z}{\hbar}\sin\frac{a_z p_{z0}}{\hbar}, \qquad (3)$$

($v_\perp$ is the Fermi velocity of conduction electrons along the normal to the layers). We assume that the inequality $|\epsilon_F| < 2t$ holds, in which the Fermi surface has self-intersection points. We concentrate on the frequency region $\hbar\omega > \epsilon_1$, so that the representation of the cyclotron resonance is determined by electron-hole transitions. The quantum cyclotron resonance and the classical contribution to the high frequency conductivity in the frequency region $\hbar\omega \ll \epsilon_1$, where the influence of electron-hole transition is negligible, have already been considered in reference [16] for the case of the magnetic field normal to the layers. The deviation from the linear dependence (1) can be neglected for Landau levels with $\epsilon_n \sim \hbar\omega$, (A.1) for angles θ of the magnetic field $\mathbf{B}$ satisfying the following inequality, which is considered to hold from now onwards:

$$\epsilon_1 < \hbar\omega \ll \min\{\epsilon_1/\eta,\ (2t \pm \epsilon_F)\}. \qquad (4)$$

We only consider the case of a sufficiently large relaxation time τ and relatively low temperatures T:

$$\frac{2t \pm \epsilon_F}{\hbar\omega} \ll \omega\tau \ll \frac{\hbar v_0}{a_z T \tan\ \theta}. \qquad (5)$$

The right side of the inequality allows us to neglect the deviation from the linear dependence (1) in the region of temperature smearing of the Fermi surface (2), near the latter's points of self-intersection. Also later we will consider only the diagonal matrix elements of the conductivity tensor σ_{ii} in the plane $(\tilde{x}, \tilde{y})$ orthogonal to the vector $\mathbf{B}$,

$$\tilde{x} = x,\ \tilde{y} = y\cos\theta - z\sin\theta,\ \tilde{z} = z\cos\theta + y\sin\theta. \qquad (6)$$

Here in after the sign "tilde" will be used to denote the components in the rotated coordinate system (6). While calculating the conductivity tensor components, we will use quantum kinetic equation in the relaxation time approximation.

3 Conductivity tensor

Within the relaxation time approximation, the conductivity tensor $\sigma_{ij}(\omega)$ can be written in the form:

$$\sigma_{ij}(\omega) = \frac{2e^3B}{(2\pi\hbar)^2c}\sum_{n,m=-\infty}^{\infty}\int d\tilde{p}_z\left(-\frac{f_n^0(\tilde{p}_z)-f_m^0(\tilde{p}_z)}{E_n(\tilde{p}_z)-E_m(\tilde{p}_z)}\right) \times \frac{v_{mn}^j(\tilde{p}_z)v_{nm}^i(\tilde{p}_z)}{-i\omega+\frac{i}{\hbar}(E_n(\tilde{p}_z)-E_m(\tilde{p}_z))+\frac{1}{\tau}}, \quad (7)$$

where $f_n^0(\tilde{p}_z) = f^0(E_n(\tilde{p}_z))$ is the Fermi-Dirac function, τ is the relaxation time, $v_{nm}^{i,j}$ are the matrix elements of the velocity operator and $\tilde{p}_z$ is the projection of the momentum vector onto the magnetic field vector $\mathbf{B}$. The factor of 2 in the numerator is obtained from summation over the conventional spin. If the inequalities (4), (5) are satisfied, the energy levels for model (2) can be represented as

$$E_n(\tilde{p}_z) = \epsilon_1(\tilde{p}_z)sign(n)\sqrt{|n|} - 2t\cos\left(\frac{a_z\tilde{p}_z}{\hbar\,\cos\,\theta}\right) + \delta E_n(\tilde{p}_z). \quad (8)$$

Caused by the deviation from the model (1) the amendment $|\delta E_n(\tilde{p}_z)| \ll \frac{1}{\tau}$, does not significantly affect the position of the resonance peaks and can be omitted. The energy of the first Landau level

$$\epsilon_1(\tilde{p}_z) = v_\parallel\sqrt{2\cos\theta\frac{eB\hbar}{c}\lambda^3(\tilde{p}_z)}, \quad (9)$$

where

$$\lambda(\tilde{p}_z) = \sqrt{1-\left(\frac{2ta_z}{\hbar v_\parallel}\sin\left(\frac{a_z\tilde{p}_z}{\hbar\cos\theta}\right)\tan\theta\right)^2}, \quad (10)$$

can be easily obtained from the expression (A.1) with the energy spectrum parameters in (3) evaluated at the point $p_x = p_y = 0$, $p_z = \tilde{p}_z/\cos\theta$, and is given here for succession.

In the frequency region delimited by the inequalities (4) and (5), the conductivity oscillations will be determined by the charge carriers near the self-intersection points of the Fermi surface, for which the dependence of the cyclotron frequency $\hbar\Omega_n(\tilde{p}_z) = E_{n+1}(\tilde{p}_z) - E_n(\tilde{p}_z)$ on $\tilde{p}_z$ can be neglected. Consequently, a real part of the conductivity tensor for this group of electrons can be written in the form:

$$\mathrm{Re}\,\sigma_{ii}(\omega) \approx N\sum_{n,m}\frac{\sigma_{nm}^{ii}}{\frac{\tau^2}{\hbar^2}(E_n(\tilde{p}_{z0})-E_m(\tilde{p}_{z0})-\hbar\omega)^2+1}, \quad (11)$$

where N is the number of self-intersection points of the Fermi surface, $N = 2$ for the model (2). The magnitudes $E_{n,m}$ are evaluated by the expression (8) given at the self-intersection point of the Fermi surface $\tilde{p}_{z0} = p_{z0}\cos\theta$, $p_{z0} = \frac{\hbar}{a_z}\arccos\left(-\frac{\epsilon_F}{2t}\right)$. The approximate value of the conductivity tensor differs from the exact value, which takes into account all groups of electrons, by a correction amendment $\Delta\sigma \ll \frac{t}{\hbar^2\omega^2\tau}\frac{e^2}{a_z}$, which is negligible in comparison with the characteristic values of the conductivity tensor (11) due to the left side of the inequality (5).

Each contribution σ_{nm}^{ii} is determined only by the transitions between the Landau levels with numbers n,m and correspond to the maximum of the conductivity $Re\,\sigma_{ii}(\omega)$ at the resonance frequency $\hbar\omega = \epsilon_n - \epsilon_m$ if the mutual overlap of the resonance peaks is omitted. In the region delimited by (4) only resonance frequencies corresponding to electron-hole transitions of the charge carriers are found where their energy spectrum are approximately linear. For the harmonics of the quantum cyclotron resonance with not too high order numbers

$$|n|, |m| \ll 1/(k^2\eta^2), \qquad k = |n| - |m|, \quad (12)$$

that means the linear approximation for the electron energy spectrum in the calculation of the matrix elements of the velocity operator, the contributions σ_{nm}^{ii} can be written in the form:

$$\sigma_{nm}^{ii} = \frac{2e^3B\tau\cos\theta}{(2\pi\hbar)^2c|v_\perp|}|v_{nm}^i|^2, \quad (13)$$

here $v_\perp$, (3) and v_{nm}^i (A.6) are determined by the linear energy spectrum (1) with the parameters (3) and (A.1) evaluated at $\mathbf{p} = (0, 0, p_{z0})$. The phase independence of the quantum oscillations of the conductivity tensor (11) and (13) on the Fermi energy under the conditions of quantum cyclotron resonance and the absence of temperature damping of the oscillations at not so high temperatures, when the electron-phonon scattering can be neglected, are associated with the fact that in case of the linear energy spectrum (1), the cyclotron frequency of the charge carriers of Dirac type depends strongly on the number of Landau levels, but is practically the same for charge carriers which have different momentum component along the magnetic field direction (see the part 4.C of the Ref. [20]). In case of a tilted magnetic field we can can neglect the difference between the linear relation (1) and the exact energy spectrum within the limits of the temperature smearing of the Fermi level when inequality (5) is satisfied. In a quantized magnetic field orthogonal to the layers, the possibility of observing a high-temperature effect for the conductors of the graphite family was predicted in reference [16], where preliminary evaluations were provided.

Figure 1 shows the behavior of the diagonal components of the conductivity tensor as a function of the magnetic field magnitude for the fixed magnetic field tilt θ and frequency ω. The relation is numerically built taking into account all the groups of electrons, $-\frac{\pi\hbar}{a_z} < p_z \le \frac{\pi\hbar}{a_z}$ using the expressions (2), (3), (7), (A.1) and (A.6). The similar dependence built using the approximate expression (11) with the same values of the parameters is not visually different from that one shown in the figure. The pair of numbers (n, m) at each peak in Figure 1 and its reflection $(-m, -n)$ correspond to the Landau level numbers and the most important contributions to σ_{nm}^{ii} (13) forming the shown resonant peak. In a tilted magnetic

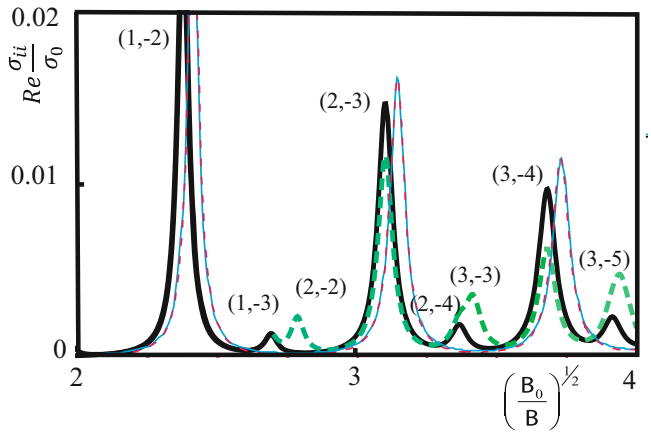

Fig. 1. The dependence of the conductivity tensor $\mathrm{Re}\sigma_{xx}$ (continuous line) and $\mathrm{Re}\sigma_{\tilde{y}\tilde{y}}$ (dashed line) on the magnetic field B for a constant electromagnetic field frequency, when $\tan\theta = 0$ (thin line) and $\tan\theta = 2/3$ (thick line). The constants used on the axes labels are $\sigma_0 = 2e^2/(\hbar a_z)$, $B_0 = \frac{c\hbar\omega^2}{2ev_\parallel^2}$. The parameter values used are $t/(\hbar\omega) = 8$, $\epsilon_F/(\hbar\omega) = 1$, $v_\perp/v_\parallel = 0.3$, $\omega\tau = 100$. The pairs of numbers (n, m) determine the numbers of a pair of Landau levels forming the given resonance peak.

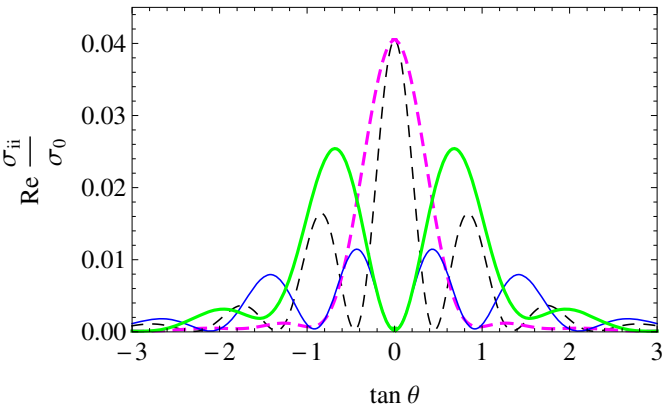

Fig. 2. The angular dependence of the maximum value of $\mathrm{Re}\sigma_{\tilde{y}\tilde{y}}$ (thick line) and $\mathrm{Re}\sigma_{xx}$ (thin line) near the resonance $(3, -4)$ (dashed line) and $(2, -4)$ (continuous line), normalized by the constant $\sigma_0 = 2e^2/(\hbar a_z)$, for a fixed value of the electromagnetic wave frequency ω. The parameter values used are $t/(\hbar\omega) = 7$, $\epsilon_F/(\hbar\omega) = 1.8$, $v_\perp/v_\parallel = 0.3$, $\omega\tau = 300$.

field besides the main peaks $|n| - |m| = \pm 1$, which determine the representation of the cyclotron resonance for $\theta = 0$, higher harmonics are added. In these harmonics, the amplitude of the sufficiently high peaks of the pair (n, m) shows oscillations as a function of the angle θ. Figure 2 shows the angular dependence of a resonance peak amplitude for a fixed frequency ω.

The physical mechanism of these oscillations can be explained as follows. The energy of the conduction electron $\epsilon(\tilde{p}_x, \tilde{p}_y)$ in Larmor orbit's plane $\tilde{p}_z = \text{const}$. can be described using the anisotropic Dirac cone model (1). It is well known, that the corresponding wavefunctions in a quantized magnetic field, which differ only by their Landau level number n, can be expressed through the Hermitian functions (A.2) with shifted center X_n (A.4) which magnitude depends only on the Landau level number. Hence, when $|n|, |m| \gg 1$ the expressions for the component of the velocity operator $v_{n,m}^{\tilde{x},\tilde{y}}$ contain the product of oscillating functions having a phase shift caused by the difference $X_n - X_m \neq 0$, which depends on the magnitude and direction of the magnetic field **B**. Their interference leads to the oscillatory dependence $v_{n,m}^{\tilde{x},\tilde{y}}$ (15), and, therefore, to the oscillations of the conductivity tensor component.

The representation of the oscillations of the conductivity tensor (13) would be clearer if we use asymptotic expressions for the velocity operator. One can admit that for the velocity operator components $v_{n,m}^{\tilde{x},\tilde{y}}$ [6], which are related to the electron-hole transitions $\mathrm{sign}(n) \neq \mathrm{sign}(m)$ and limited by the condition

$$|n| - |m| \ll 1/\eta, \sqrt{|n|}, \tag{14}$$

the known asymptotic expression $L_j^\alpha(x) \approx J_\alpha(2\sqrt{jx})$ can be applied yielding the components' simple asymptotic expression:

$$v_{nm}^{\tilde{y}} \approx v_0\lambda^2 J'_{|k|}(4\eta l), \quad v_{nm}^{\tilde{x}} \approx iv_0\lambda\alpha\frac{k}{4\eta l}J_{|k|}(4\eta l), \tag{15}$$

where $k = |m| - |n|$, $\quad l = min(|m|, |n|)$ and $J'_k(x)$ is the derivative of the Bessel function. The asymptotic form of the velocity operator components is insignificantly different from (A.6) for the physical picture of the oscillations phase shift and does not account for the overwhelming multiplier $\exp(-2\eta^2 l) \approx 1$ when $\eta \ll \frac{1}{\sqrt{|n|}}$, as the condition (4) holds true. A more accurate, though awkward, the asymptotic expansion for associated Laguerre polynomials $L_n^a(z)$, in particular, for the oscillatory behavior of the region $0 < z < 4n + 2(a + 1)$, can be found in reference [21]. The expressions (15) maintain the physical structure of the velocity operator oscillations. This is the way the asymptotic value of $v_{nm}^{\tilde{x},\tilde{y}}$ (15), as well as its exact expression (A.6), will be significantly different from zero only in the region of $|\eta| < \frac{|k|}{|n|+|m|}$, and exponentially little beyond it (this condition is easier to obtain using the WKB approximation in conjunction with the method of a stationary phase). Figure 3 shows the dependence of the resonance peaks amplitude (13) on m and n numbers. The oscillations of values σ_{ii}^{nm} are the result of the anisotropy of the energy spectrum of Dirac type that is indirectly confirmed by qualitative similarity of the given figure and Figures 2a and 2c of reference [4]. While constructing Figure 3 the exact expressions for the matrix elements of the velocity operator v_{nm}^i for the energy spectrum (1) were used, however replacing them with the approximate values (15) describes correctly the oscillation dependence of the peak amplitude in terms of inequality (14).

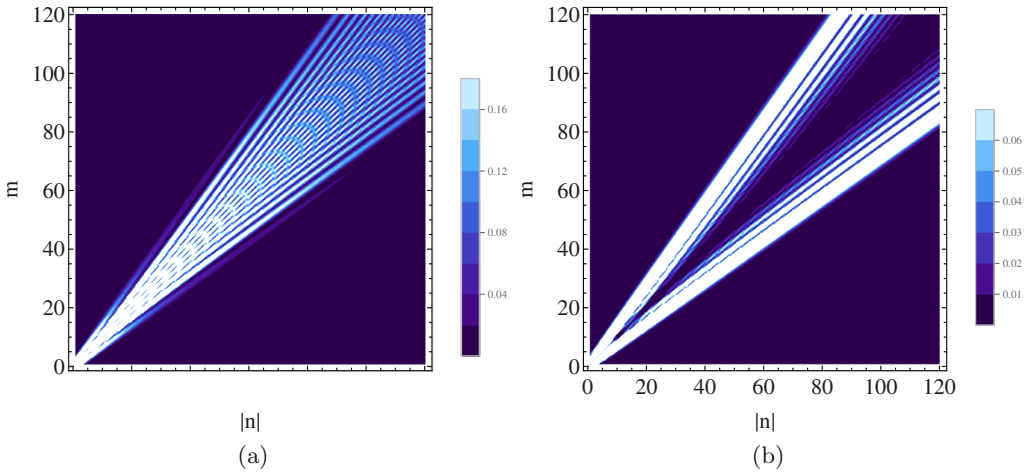

Fig. 3. The dependence of the contributions $\sigma^{\tilde{y}\tilde{y}}_{nm}$ (a) and σ^{xx}_{nm} (b), normalized by σ_Σ(17), that determine the conductivity tensor components (11) under the resonance conditions $\hbar\omega = \epsilon_n - \epsilon_m$ at the fixed magnitude and direction of the magnetic field, $\tan\theta = 0.3$, $v_\perp/v_\parallel = 0.3$, on the numbers (n, m), which determine the resonances caused by electron-hole transitions.

4 Sum rule

The tilt angle of the magnetic field θ, corresponding to the condition $\eta(\theta) \sim 1/l$, separates the two cases of the quantum cyclotron resonance. At the smaller angles θ, the quantum cyclotron resonance will be determined only by the fundamental harmonics of $k = |n| - |m| = \pm 1$. For the larger angles θ, in the frequency range (4) a lot of higher resonance harmonics will appear, while the amplitude of the fundamental harmonics caused by electron-hole transitions $n + m = \pm 1$ falls sharply.

It may be noticed, that there is a kind of rule of conservation of the resonance peaks of total amplitude, explaining the decrease of the amplitude of the fundamental resonances during the appearance of higher harmonics. Namely, for an arbitrary η and $n = const$ the relation:

$$\sum_{m=-\infty}^{\infty} |v^{\tilde{x}}_{nm}|^2 = v_0^2\alpha^2, \qquad \sum_{m=-\infty}^{\infty} |v^{\tilde{y}}_{nm}|^2 = v_0^2(1+\eta^2), \tag{16}$$

is valid, which follows directly from the properties of the Pauli matrices $\sigma^2_{x,y} = 1$. The expressions (16) remain valid when using the asymptotics (15) and pass to the known sum rule $J_0^2(x) + 2\sum_{n=1}^{\infty} J_n^2(x) = 1$. From the expressions (13) and (16) it follows that the maximums of the cyclotron resonance peaks due to the charge carriers near the Dirac singularity (4), (12) obtained for the same values of magnitude and direction of the magnetic field $\mathbf{B}$ (with different resonance frequencies ω) satisfy the relationship

$$\sum_{m=-\infty}^{\infty} \sigma^{\tilde{x}\tilde{x}}_{nm} = \alpha^2\sigma_\Sigma,$$
$$\sum_{m=-\infty}^{\infty} \sigma^{\tilde{y}\tilde{y}}_{nm} = (1+\eta^2)\sigma_\Sigma,$$
$$\sigma_\Sigma = \frac{2e^3 B\tau v_0^2\cos\theta}{(2\pi\hbar)^2 c|v_\perp|}, \tag{17}$$

in which summed contributions visually correspond to one of the horizontal in Figure 3. So we have the following "magic square rule" for a table built from the resonance peak amplitude values of the conductivity σ_{nm} (17): the sums of all the elements in the rows (n = const.) and columns (m = const.) do not depend on their numbers and they are equal.

5 Conclusions

The found oscillatory dependence of the conductivity tensor (11), (13), (15) has a quantum interference nature and is a consequence of the anisotropy of the electron energy spectrum in Larmor orbit's plane, which arises in a tilted magnetic field. The amplitude of the resonance peaks satisfies the simple sum rule or the "magic square rule", which follows directly from properties of Pauli matrices. The character of the oscillatory dependence is similar to those observed in reference [4] oscillations of the absorption coefficient of the electromagnetic field for a two-dimensional conductor of Dirac type with a natural anisotropy of the electron energy spectrum, or in crossed electric and magnetic fields, as a function of an electric field or the degree of deformation of the conductor. Unlike the two-dimensional case, in graphite family conductors the degree of the Dirac cone anisotropy η can be modified by simply changing the inclination angle of a quantized magnetic field, which substantially facilitates the conditions for the experimental observation of oscillatory phenomena that are related to the Dirac cone anisotropy.

Providing that the charge carrier velocity in the plane of the layers v_0 (2) is close to its value in graphene (see, for example, Tab. 2 in Ref. [22]), the resonance frequency corresponding to the transition between zeroth and the first Landau levels $\omega \sim 5 \times 10^{13} \times \sqrt{B[\mathrm{T}]}$ [Hz], when the magnetic field is directed by the normal to the layers, and decreases if magnetic field tilt angle θ is increasing according

to the expression (A.1). Thus, the region of the resonance frequencies in Figure 1 will be limited to the submillimeter and infrared diapason. Although the model (2) is suitable for multilayers of graphene with a AA type of stacking of the crystal lattice, but it can also be used for the description of the physical properties of other anisotropic conductors with a Dirac singularity in the electron energy spectrum, the characteristics of which may differ much from the similar values in graphene, including the region of resonance frequencies. However, the observed oscillatory effect is not restricted by the given model and can take place in different conductors with nodal line in the energy spectrum. In particular, the energy spectrum of graphite with AB type of stacking is also characterized by a non-zero Berry phase [3] and has a local structure (1) in the proximity of self-intersection points of the Fermi surface [23,24]. The purity of highly oriented pyrolytic graphite (HOPG) gives the possibility of experimental observation of higher harmonics of the quantum cyclotron resonance [12] and the angular oscillations of the kinetic coefficients in the frequency domain of the electromagnetic wave of millimeter and infrared range (see, for example Ref. [25] and references therein). The investigation of the angular oscillations of high-frequency kinetic coefficients which are caused by the charge carriers of Dirac type in graphite of AB type stacking is beyond the scope of this article and will be presented in a separate paper.

The absence of an inversion center ($\eta \neq 0$) of the model (1) in Larmor orbit's plane, being the cause of the oscillatory dependence on the matrix elements of the velocity operator (15), though does not lead to quantitative changes of the quantized energy spectrum (A.1). Therefore, the interference mechanism observed here may take place in the kinetic coefficients, which are related to electron transport phenomena (electrical conductivity, impedance) and at the same time can not cause the magneto-angular oscillations of the density of states and the related thermodynamic characteristics of a conductor. Naturally the effects specific to a Dirac anisotropic spectrum are not limited to high-frequency transport phenomena. Thus the phase transition of $3\frac{1}{2}$ kind in conductors with nodal lines in the energy spectrum of charge carriers, which is sensitive to the anisotropy of the Dirac electron energy spectrum, is described in reference [2].

The magneto-angular oscillations in bilayer graphene predicted in reference [26] have a similar physical nature as they are explained by interference of wave functions with the displacement of the centers of Larmor orbits in the graphene neighboring layers. However, the effect leads to the occurrence of the magneto-angular oscillations in the density of states of the electronic subsystem, which differs it from the mechanism of oscillations appearance Figure 2. Also in contrast to oscillations of the conductivity tensor (11), (13), (15), which period of oscillations is determined by the ratio of the Fermi velocities in directions perpendicular and parallel to the layers, the overlap integral between the layers does not affect the phase of the oscillations [26], although determines their amplitude. While working over the present article, we came across reference [27], where type-II Weyl semimetals in a tilted magnetic field were investigated and Landau quantization was proved to be possible even in the given conductors for magnetic field directions with the effective tilt $\eta < 1$. The existence of a new type of angular oscillations of kinetic coefficients for the conductors of the graphite family considered in the presented work was announced in the abstract [28].

The authors express the gratitude to FINCYT and CONCYTEC of Peru for financial support of this work.

Author contribution statement

The contributions of the three authors are equal.

Appendix A: Matrix elements of the velocity operator

The eigenvalues ϵ_n and wave functions $\varphi_\nu(\mathbf{r})$ of the Hamiltonian (1) in a quantized magnetic field with the gauge $A = (0, By, 0)$ have the form:

$$\epsilon_n = v_0 \,\mathrm{sign}(n)\sqrt{2\frac{eB\hbar}{c}\lambda^3\,\alpha\,|n|}, \tag{A.1}$$

$$\begin{aligned}\varphi_\nu(x,y) = {} & \frac{(\alpha\lambda)^{1/4}}{2(2\pi\hbar)\sqrt{a_H}}\sqrt{\frac{1+\delta_{0,n}}{1+\lambda}} \\ & \times \exp\left(\frac{i}{\hbar}P_y y\right)\left\{\begin{bmatrix} i\eta \\ 1+\lambda\end{bmatrix} h_{|n|}\left(\frac{\sqrt{\alpha\lambda}}{a_H}(x+X_n)\right)\right. \\ & \left. - \begin{bmatrix} i(1+\lambda) \\ \eta\end{bmatrix}\mathrm{sign}\,(n)\, h_{|n|-1}\left(\frac{\sqrt{\alpha\lambda}}{a_H}(x+X_n)\right)\right\},\end{aligned} \tag{A.2}$$

where $\nu = (n, P_y)$ is the complete quantum index set, $\mathbf{P}$ is the canonical momentum, the magnetic length

$$a_H = \sqrt{\frac{c\hbar}{eB}}, \qquad \lambda = \sqrt{1-\eta^2}, \tag{A.3}$$

the negative values of the Landau level numbers correspond to holes in the energy spectrum of charge carriers,

$$X_n = a_H\,\eta\,\mathrm{sign}\,(n)\sqrt{\frac{2|n|}{\alpha\lambda}} - \frac{cP_y}{eB} \tag{A.4}$$

is the centre of Larmor orbit of the conduction electrons,

$$h_n(\xi) = \frac{1}{\sqrt{2^n\sqrt{\pi}n!}}\exp(-\xi^2/2)H_n(\xi) \tag{A.5}$$

is the solution of the dimensionless harmonic oscillator problem, $H_n(\xi)$ is the nth Hermite polynomial and $\delta_{0,n}$ is the Kronecker symbol. It is considered that the contribution containing sign(n), (A.2) is equal to zero when $n = 0$.

The matrix elements of the velocity operator components have the form

$$v^y_{nm} = \lambda(\Phi_{nm} + \Phi_{mn}), \qquad v^x_{nm} = i\alpha(\Phi_{nm} - \Phi_{mn}) \quad \text{(A.6)}$$

where

$$\Phi_{nm} = v_0\lambda\sqrt{\frac{|n|}{2^{|m|-|n|+1}}}\sqrt{\frac{|m|!}{|n|!}}\Delta_{nm}^{|n|-|m|-1}e^{-\Delta^2_{nm}} \times L_{|m|}^{|n|-|m|-1}(2\Delta^2_{nm})\mathrm{sign}(n) \quad \text{(A.7)}$$

$$\Delta_{nm} = \frac{\eta}{\sqrt{2}}\left(\mathrm{sign}(n)\sqrt{|n|} - \mathrm{sign}(m)\sqrt{|m|}\right), \quad \text{(A.8)}$$

when $n \neq 0$ and $\Phi_{0m} = 0$.

The expressions similar to (A.1), (A.2), (A.6) are given in a series of works (for example, see [4,5]). In particular, the expression (A.6) corresponds to the formulae (A1)–(A2) of references [6] where the value of the parameter $\alpha = 1$, if the dependence on the latter is considered by simple coordinate transformation $y' = y,\ x' = x/\alpha$.

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