# On the 60th anniversary of the Lifshitz-Kosevich theory

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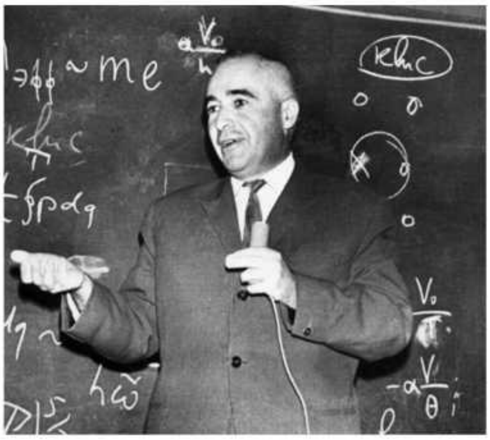
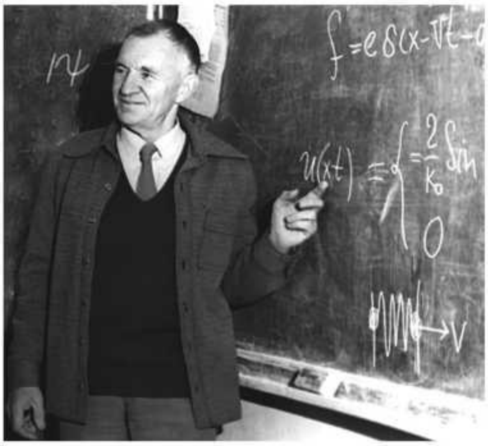


Ilya Mikhailovich Lifshitz, 1960s.



Arnold Markovich Kosevich, 1980s.

This year marks sixty years since Doklady Akademii Nauk SSSR (DAN SSSR) published a study by Ilya Mikhailovich Lifshitz and Arnold Markovich Kosevich "On the theory of the de Haas–van Alphen effect for particles with an arbitrary dispersion law."[1] This work played a key role in the creation of a new direction in solid state physics, which has been termed "fermiology"—the science of the Fermi surface (FS) structure and characteristics, such as the cross-sectional area, diameters, effective and cyclotron mass, velocities at different points on the Fermi surface, etc.

An oscillatory magnetic field dependence of the magnetic susceptibility of metals $\chi(H)$, associated with the magnetic quantization of the energy of the orbital motion of charge carriers, was theoretically predicted by Landau in his work "Diamagnetism of metals," published in 1930.[2] In the same year, independently, there appeared a communication by de Haas and van Alphen "Note on the dependence of the susceptibility of diamagnetic metal on the field," reporting the observation of the oscillating field dependence of $\chi(H)$ in single crystals of bismuth.[3] This effect was named after the authors of the experimental discovery. By the early fifties of the last century, the de Haas–van Alphen (dHvA) effect had been observed in many metals (see the monograph by Shoenberg[4] and references therein), and in this regard the theoretical study of quantum oscillation effects became very important.

The possibility of determining the FS extremal sections through the oscillation period of the dHvA oscillation was first noticed by Onsager in 1952 in his "Interpretation of de Haas van Alphen effect."[5] On the basis of the Bohr-Sommerfeld quantization rule, Onsager wrote the relation between the number of the maximum $n$ in the oscillatory dependence $\chi(H)$, which corresponds to a magnetic field $H_n$, and the extreme cross-sections of the FS by the planes $\mathbf{pH} = \text{const}$ ($\mathbf{p}$ is the electron momentum)[5]

$$n + \Theta = (hc/e)A/H_n. \tag{1}$$

The rigorous solution for the problem of the dependence of the magnetic susceptibility of a metal on the strong (quantizing) magnetic field under the most general assumptions concerning the dispersion law of the conduction electrons has been obtained by Lifshitz and Kosevich[1] (a more detailed account of these results was published a year later in the "Journal of Experimental and Theoretical Physics"[6]). The general equation for the oscillations of the magnetic susceptibility $\chi(H)$ is now known in the scientific literature as the Lifshitz–Kosevich equation.[4] In the same volume of DAN SSSR in 1954, a work by Lifshitz and Pogorelov[7] was published, which has showed that by knowing all the extremal sections of an arbitrary convex FS, we can uniquely determine its shape. The authors of the theory[1] not only have rigorously obtained the relation between the periodicity of dHvA oscillations versus inverse field $1/H$ and the extremal FS cross-sections, but also have identified the range of applicability of Eq. (1). Moreover, they also showed that the temperature dependence of the oscillation amplitude allows us to determine the cyclotron mass of an electron. Equation (1), which allows, in a simple way, to find the area of an extremal section of the FS through the period of the dHvA oscillations is still widely used by researchers. Various authors call it differently—the Onsager equation or the Lifshitz–Onsager equation.

The work by Lifshitz and Kosevich[1] has stimulated a stream of theoretical studies that addressed different properties of metals for an arbitrary law of the dispersion of charge carriers, assuming it *a priori* known. As a result, a whole set of different methods has been developed for determining the electronic energy spectrum of metals by experimental studies of various thermodynamic and kinetic characteristics of metals and comparing the experimental results with the corresponding theory. The achievements of fermiology reached by the time of its "maturity" (in the 1970s), by which time the FS of most metallic elements had been reconstructed, are detailed in the books Refs. 8–10 and references therein.

**Доклады Академии Наук СССР**
**1954. Том XCVI, № 5**

*ФИЗИКА*

**И. М. ЛИФШИЦ и А. М. КОСЕВИЧ**

## К ТЕОРИИ ЭФФЕКТА ДЕ ГААЗ — ВАН АЛЬФЕНА ДЛЯ ЧАСТИЦ С ПРОИЗВОЛЬНЫМ ЗАКОНОМ ДИСПЕРСИИ

*(Представлено академиком Л. Д. Ландау 15 III 1954)*

1. В настоящее время периодическая зависимость магнитной восприимчивости от поля при низких температурах (эффект де Гааз — ван Альфена) наблюдена для большого числа металлов ([1]). Между тем, количественная теория этого явления разработана для случая электронного газа с квадратичным законом дисперсии, справедливым лишь у дна соответствующей энергетической зоны ([2]).

Представляется весьма существенным выяснение того, в какой мере особенности эффекта связаны с этим предположением. Некоторые качественные соображения по этому поводу были высказаны в последнее время в работе ([3]), однако примененный там путь рассуждений не позволяет провести полный анализ эффекта.

Известно, что при низких температурах электроны в металле, взаимодействующие друг с другом и с решеткой, в термодинамическом отношении могут быть заменены идеальным ферми-газом заряженных квази-частиц с некоторым законом дисперсии

$$\mathscr{E} = \mathscr{E}(p_x, p_y, p_z). \tag{1}$$

Поэтому все вычисления следует произвести для идеального газа квази-частиц с общим законом дисперсии (1). При этом первая задача, которая должна быть решена, заключается в отыскании уровней энергии такой частицы в магнитном поле.

Так как в дальнейшем нас будут интересовать в основном уровни энергии, отвечающие большим квантовым числам, то квантование движения в магнитном поле достаточно произвести в квази-классическом приближении.

Гамильтониан частицы с законом дисперсии (1) в магнитном поле **H**, направленном по оси $z$, формально получается заменой в (1) компонент импульса $p_i$, компонентами оператора кинетического импульса $\check{P}_i$, связанными между собой соотношениями коммутации:

$$[\check{P}_y, \check{P}_x] = \frac{e}{c} H, \quad [\check{P}_x, \check{P}_z] = [\check{P}_y, \check{P}_z] = 0. \tag{2}$$

Соотношение между $\check{P}_x$ и $\check{P}_y$ соответствует перестановочному соотношению между обобщенной координатой и обобщенным импульсом: $[\check{P}_y, \check{Q}_y] = 1$. Роль оператора обобщенной координаты $\check{Q}_y$ играет оператор $\frac{c}{eH}\check{P}_x$. Поэтому можно написать условие квази-классического квантования:

$$\oint P_y dQ_y = \frac{c}{eH}\oint P_y dP_x = (n + \gamma)\, h; \quad 0 < \gamma < 1. \tag{3}$$



First page of the seminal paper by Lifshitz and Kosevich.[1]

Today the Fermi surfaces of simple metals can be admired on the website "The Fermi Surface Database."[11]

Nevertheless, fermiology does not "age." Investigation of multicomponent compounds yielded a number of remarkable discoveries—metallic conductivity of complex organic compounds, high-temperature superconductivity of cuprates, coexistence of superconductivity and magnetism, two-band superconductivity of $MgB_2$, etc. One of the first questions that always comes up in the quest for explanations of the observed phenomena is the question about the FS characteristics. Nowadays, the toolkit of fermiology has greatly expanded. Tremendous progress in computer technology has made available first-principles calculations of the energy spectrum of degenerate conductors to the broad research community. Nowadays, Fermi surfaces are widely studied using angle-resolved photoemission spectroscopy (ARPES).[12] Direct visualization of the FS contours becomes possible by using scanning tunneling spectroscopy of the sample surface in the vicinity of single point defects located under the surface.[13,14]

Readers who wish to learn about the achievements in the study of the energy spectra of novel compounds with metallic conductivity during the last two decades can be referred, for example, to the reviews Refs. 15–22, although these publications certainly cannot offer a comprehensive account of all the information accumulated to date by fermiology.

This special issue of Low Temperature Physics entitled "Recent advances in fermiology" is dedicated to the 60th anniversary of the appearance of the Lifshitz–Kosevich theory. At that time, Ilya Mikhailovich Lifshitz headed the theoretical division at the Ukrainian Physico-Technical Institute (UPTI) and, simultaneously, held a chair at the Faculty of Physics and Mathematics of Kharkov State University, and Arnold Markovich Kosevich was his graduate student.

For a number of years, I. M. Lifshitz and A. M. Kosevich were members of the editorial board of LTP and made a significant contribution to the journal. One of the pioneers of experimental investigation of the de Haas–van Alphen effect in metals was the founder of LTP and its first Editor-in-Chief, Boris Ieremievich Verkin.[23] Studies by B. G. Lazarev, B. I. Verkin, and coworkers carried out at the UPTI in 1949–1953 have largely stimulated the fundamental theoretical investigations of I. M. Lifshitz and his students in the electronic theory of metals and fermiology.

*V. G. Peschanskii*
*Yu. A. Kolesnichenko*

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