J Low Temp Phys
DOI 10.1007/s10909-012-0627-2

On the Magnetoresistance Maximum Observed in the Intermediate Magnetic Field Region for the Two-Dimensional Hole Gas in a Strained $Si_{0.05}Ge_{0.95}$ Quantum Well

I.B. Berkutov · V.V. Andrievskii · Y.F. Komnik · T. Hackbarth · D.R. Leadley · O.A. Mironov

Received: 31 January 2012 / Accepted: 11 April 2012

Abstract The magnetoresistance (MR) of a two-dimensional hole gas in a quantum well of compressively strained $Si_{0.05}Ge_{0.95}$ has been investigated as a function of temperature. The MR shows a maximum at intermediate magnetic fields between the regions of weak localization and the Shubnikov-de Haas oscillations, which is discussed in terms of a recent theoretical study of the electron-electron interaction effect by Sedrakyan and Raikh (SR). The magnetic field MR dependence is clearly observed to cross over from quadratic to linear at $T = 7.8$ K and $B \approx 0.3$ T. It is shown that the SR theory provides a good description of both the measured quadratic and positive linear MR, but over estimates the field position of the MR maximum and does not account for the shift in position with temperature that is observed. Earlier theories of electron-electron interaction (by Altshuler and Aronov, Gornyi and Mirlin) show a better agreement with the experimentally observed behavior of the MR maximum, but fit the low field MR less accurately.

Keywords Heterostructure · Electron-electron interaction · Friedel oscillations

I.B. Berkutov (✉) · V.V. Andrievskii · Y.F. Komnik
B. Verkin Institute for Low Temperature Physics and Engineering,
National Academy of Sciences of Ukraine, 47 Lenin Ave., Kharkov 61103, Ukraine
e-mail: berkutov@ilt.kharkov.ua

T. Hackbarth
Forschungszentrum Ulm, Daimler AG, Wilhelm-Runge-Str. 11, 89081 Ulm, Germany

D.R. Leadley · O.A. Mironov
Department of Physics, University of Warwick, Coventry CV4 7AL, UK

O.A. Mironov
International Laboratory of High Magnetic Fields and Low Temperatures, P.O. Box 4714,
50-985 Wroclaw 47, Poland

Published online: 17 May 2012

1 Introduction

The magnetoresistance (MR) of a two-dimensional (2D) electron, or hole, gas often exhibits effects of quantum origin at low temperatures. In weak magnetic fields ($\omega_c\tau \ll 1$, where $\omega_c = eB/m^*$ is the cyclotron frequency, τ is elastic scattering time, and m^* is effective mass) an effect of weak localization (WL) is observed [1, 2], which is due to quantum-interference. The WL effect gives a quantum correction $\Delta\sigma^{loc}(B,T)$ to the conductivity that depends on the temperature and magnetic field. The magnetic field region where the WL effect is observed is characterized by the condition $L_B/l \simeq 1$, where $L_B = (\hbar/eB)^{1/2}$ is the magnetic length, and l is the mean free path [3, 4]. In higher magnetic fields ($\omega_c\tau \gg 1$), magnetic quantization leads to Shubnikov-de Haas (SdH) oscillations that are exhibited in the conductivity of high quality samples. In addition, there is a contribution from electron-electron interactions (EEI) over a wide region of magnetic fields up to the quantum limit [5–7] that, like WL, is of a quantum-interference nature. The EEI gives a quantum correction to the conductivity of $\Delta\sigma^{int}(B,T)$ again determined by the temperature and magnetic field.

The well known Altshuler-Aronov theory describes EEI corrections [7] in the diffusion limit, described by the inequality $k_BT\tau/\hbar < 1$ when the effective EEI time $\hbar/k_BT$ is larger than momentum relaxation time and is sufficiently long for the two electrons involved to each scatter at several impurities. EEI corrections at an arbitrary correlation between k_BT and $\hbar/\tau$ were obtained by Zala, Narozhny and Aleiner who considered the temperature dependence of EEI corrections [8], the EEI correction to the Hall coefficient [9] and the correction to MR in a parallel field [10]. The theory was developed for electron scattering at point impurities having a short-range potential. In the case of $k_BT\tau/\hbar > 1$ electrons can only be scattered at a single impurity during the EEI time. This regime of scattering, which occurs at high electron energies, has been called the ballistic regime [8–10], although in the diffusion regime an electron also performs a ballistic motion between two impurity scattering events. A general theory of EEI corrections, including electron scattering at a short-range potential and the Coulomb interaction with scattering centers, was developed by Gornyi and Mirlin [11]. It is applicable in all of the diffusion, intermediate and ballistic regimes. This theory (see also Ref. [12]) applies to a perpendicular magnetic field, but it actually concentrates on temperature variations of the EEI corrections. The magnetic field is characterized using a factor $\omega_c^2\tau^2$ whose meaning is as follows. In a magnetic field the inversion of the conductivity tensor to the resistance tensor gives the following correction to the resistance [13]:

$$\Delta\rho^{int}(B,T) \approx -\frac{1}{\sigma_0^2}\left(1-\omega_c^2\tau^2\right)\Delta\sigma^{int}(B,T), \tag{1}$$

where σ_0 is the conductivity in zero magnetic field. In the case of strong magnetic fields ($\omega_c^2\tau^2 \gg 1$) the factor $1-\omega_c^2\tau^2$ can be relapsed with $\omega_c^2\tau^2$ and (1) becomes

$$\frac{\rho^{int}(B,T)-\rho_0}{\rho_0} = \frac{1}{\sigma_0}\omega_c^2\tau^2\Delta\sigma^{int}(B,T). \tag{2}$$

Since the correction $\Delta\sigma^{int}(T)$ is negative, (2) describes a negative field-quadratic magnetoresistance. In the case of weak magnetic fields, a simplified expression $\rho^{int}(B)/\rho_0 = -\sigma^{int}(B)/\sigma_0$ can be used ($\omega_c\tau < 1$) for the inversion of conductivity tensor to the resistance tensor.

The current theories of EEI have been supplemented by Sedrakyan and Raikh (SR) [14] who investigated quantum EEI corrections for magnetic fields in the region between those where WL effects and SdH oscillations are observed. These authors consider electron scattering at an impurity with a short-range potential in a sample having a long mean free path ($k_F l \gg 1$, k_F is the Fermi wave number). As in the previous study [8–10], the EEI correction to the conductivity is examined due to interference arising from back scattering off the impurity and Friedel oscillations of the electron density induced by the electric field of the impurity. Note that the interference of the electron waves reflected from the impurity and the Friedel oscillations is caused by the coherence because the radius of the Friedel oscillations are a multiple of the electron wavelength. The advantage of this study [14] is that the theoretical computation arrives at an explicit expression for the EEI correction in a perpendicular magnetic field which compares favorably with experimental results. These processes, each involving two scattering acts are much more sensitive to magnetic field than the one described by (2), even for $\omega_c\tau < 1$. In this case the interaction contribution is

$$\frac{\Delta\sigma^{SR}(B,T)}{\sigma_0} = \frac{4\lambda^2}{(k_F l)^{3/2}} F_1\left(\frac{\omega_c}{\Omega_l}\right), \tag{3a}$$

where

$$\Omega_l \tau = (k_F l)^{-1/2},$$

here Ω_l is the cyclotron frequency which originates from the new physical process: double backscattering from the impurity-induced Friedel oscillations, λ is the interaction constant, k_F is Fermi momentum. The function F_1 can be approximated by the expressions

$$F_1(x) = \begin{cases} -x^2/8, & x \ll 1, \\ -2x/3, & x \gg 1. \end{cases} \tag{3b}$$

Since the region of the Friedel oscillations is restricted by the size $r_T = v_F\hbar/(2\pi k_B T)$ (v_F is the Fermi velocity), the authors of Ref. [14] describe the temperature-dependent EEI correction to the conductivity in the region between weak localization and the SdH oscillations as:

$$\frac{\Delta\sigma^{SR}(B,T)}{\sigma_0} = 4\lambda^2\left(\frac{\pi k_B T}{\varepsilon_F}\right)^{3/2} F_2\left(\frac{\omega_c}{2\pi^{3/2}\Omega_T}\right), \tag{4a}$$

where

$$\Omega_T = \frac{(k_F T)^{3/2}}{\varepsilon_F^{1/2}},$$

ε_F is Fermi energy and the function F_2 can be approximated by the expressions

$$F_2(x) = \begin{cases} -0.7x^2, & x \ll 1, \\ -2x/3, & x \gg 1. \end{cases} \tag{4b}$$

It is seen that the correction decreases with rising temperature. As the magnetic field increases the correction to conductivity, which exhibited a quadratic growth in very low fields, starts to grow linearly with increasing field. The most important result of the SR theory [14] is this prediction of positive magnetoresistance for 2D electron systems that is linearly dependent on the magnetic field.

The experimental observation of MR maxima in various 2D electron systems (inversion layers in silicon, GaAs/AlGaAs heterostructures and quantum wells of GaAs and InGaAs) [15] were compared with the SR predictions [14]. The theory and experiment exhibit some qualitative agreement: (i) the magnetoresistance is positive in weak magnetic fields; (ii) the magnetoresistance has a maximum in stronger magnetic fields; (iii) the order of MR magnitude agrees with the prediction. However some experimental findings disagree with theory [14]: (i) the magnetic fields of the MR maxima exceed the predicted ones; (ii) the MR maximum moves to higher magnetic fields as temperature increases, in contrast to predictions; (iii) the magnitude of the effect increases with temperature rather than decreasing as the theory [14] predicts.

As the magnetic field grows to the region $\omega_c\tau > 1$, the factor $\omega_c^2\tau^2$ should be taken into account because according to (2), it is responsible for negative and quadratic magnetoresistance. The competition of those two functions results in a maximum in MR.

2 Object of Investigation

In our experiments the magnetoresistance of a 2D hole gas in a modulation-doped strained SiGe quantum well has been studied as a function of temperature with particular emphasis on the MR maximum in the region of magnetic fields between the WL effect and the SdH oscillations. Here we analyze the experimental results obtained and compare them with the SR theory [14]. The sample was a quantum well of $Si_{0.05}Ge_{0.95}$ with hole conduction and a distinct MR maximum (Fig. 1). It was prepared by molecular beam epitaxy, starting with a 4650 nm thick SiGe relaxed buffer layer grown onto a (001) Si substrate in which the Ge concentration increased linearly from 5 % to 63 %. The subsequent layers were arranged as follows: a 500 nm thick relaxed $Si_{0.37}Ge_{0.63}$ constant composition layer, a 11 nm thick compressively strained $Si_{0.05}Ge_{0.95}$ layer (the active quantum well), a 10 nm thick $Si_{0.37}Ge_{0.63}$ spacer layer, a 10 nm thick supply layer of the same composition doped with boron (volume concentration 2×10^{18} cm^{-3}), a 10 nm thick $Si_{0.37}Ge_{0.63}$ layer and finally a 3 nm thick Si cap layer. The sample configuration was a double cross. In the experiments the magnetic field was applied perpendicular to the quantum channel. Since the quantum well in the heterostructure does not contain impurity atoms, the main scattering mechanism in this structure is remote Coulomb interaction of the holes with the boron acceptor atoms, which are located in a layer separated from the quantum well by an impurity-free spacer layer 10 nm thick. At $T = 0.355$ K the charge carriers had the following

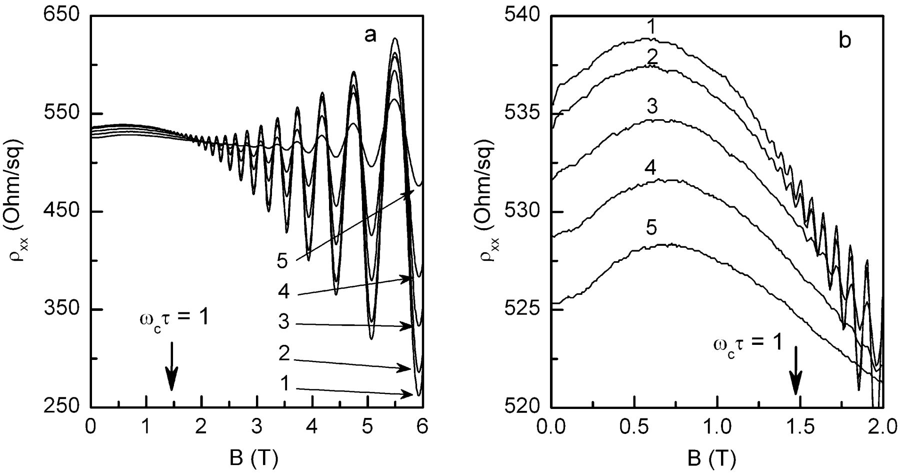


Fig. 1 Magnetic field dependence of resistance ρ_{xx} at temperatures: (*1*) 0.355 K, (*2*) 1.1 K, (*3*) 2.73 K, (*4*) 4.55 K, and (*5*) 7.8 K. *Vertical arrow* shows the condition $\omega_c\tau = 1$. (**b**) The low field WL and MR maximum at intermediate fields are more clearly seen in the expanded field view

parameters: effective hole mass $m^* = 0.156m_0$ (estimated from the temperature variation of SdH oscillations amplitude), hole concentration $p_H = 1.76 \times 10^{12}$ cm^{-2} (obtained from Hall effect) and $p_{SdH} = 1.62 \times 10^{12}$ cm^{-2} (obtained from the SdH oscillation period), elastic scattering time $\tau = 6.05 \times 10^{-13}$ s, quantum scattering time $\tau_q = 2.77 \times 10^{-13}$ s, mean free path $l = 149.6$ nm, mobility $\mu = 6.8 \times 10^3$ cm^2 V^{-1} s^{-1}. The object investigated had the parameter $k_F l \simeq 50$ and the transition to the ballistic regime with rising temperature, determined by $k_B T/\hbar = 2/(\pi\tau)$, occurs at $T \simeq 7.7$ K [8, 9]. Therefore, the low temperature results discussed below refer to the diffusion and intermediate regimes. The following effects were observed in different regimes of magnetic field: a weak localization effect (in the weakest magnetic fields), a MR maximum (at intermediate magnetic fields) due to the change from positive to negative MR as the magnetic field increases, and Shubnikov-de Haas oscillations (in strong magnetic fields). All these magnetoresistance features are discussed in more detail below.

3 Weak Magnetic Fields: Weak Localization Effect

The quantum correction to the conductivity of 2D electron system that comes from the weak localization effect varies in a perpendicular magnetic field follows from the theory[6]:

$$\Delta\sigma^{loc}(B) = \frac{e^2}{2\pi^2\hbar}\left[\frac{3}{2}f_2\left(\frac{4eDB}{\hbar}\tau_\varphi^*\right) - \frac{1}{2}f_2\left(\frac{4eDB}{\hbar}\tau_\varphi\right)\right], \tag{5a}$$

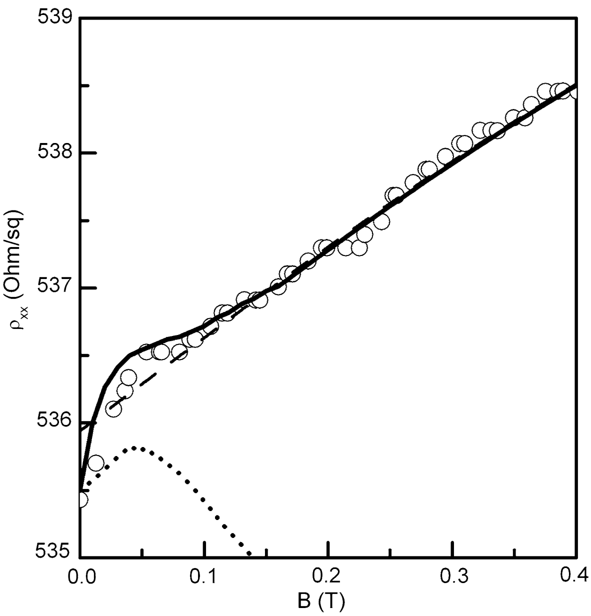


Fig. 2 Magnetic field-induced variation of resistance ρ_{xx} in low magnetic fields at $T = 0.355$ K. The *doted line* is the contribution of the WL (calculated as in [16], including phase and spin-orbit relaxation times) correction, the *dashed line* describes the SR-EEI contribution [14] (calculated by (7) at $\lambda = 0.6$). The *solid line* is a combined contribution of WL and SR-EEI effects

where $f_2(x) = \ln(x) + \Psi(1/x + 1/2)$, Ψ is the logarithmic derivative of the Γ-function,

$$f_2 = \begin{cases} x^2/24, & x \ll 1, \\ \ln(x) + \Psi(1/2), & x \gg 1. \end{cases} \tag{5b}$$

In (5a) D is the diffusion coefficient of carriers, $(\tau_\varphi^*)^{-1} = \tau_\varphi^{-1} + 4/3\, \tau_{so}^{-1}$, τ_φ is the phase relaxation time, and τ_{so} is spin-orbit relaxation time. The terms of different signs in (5a) correspond to the triplet and singlet states of the interfering charges. To calculate the corrections due to the weak localization of holes we employed the theoretical model of Ref. [16], which is based on the theory of Ref. [6] and can be applied to undeformed and deformed bulk p-type semiconductors and quantum-well structures based on them.

In the investigated object, the WL-related quantum correction appeared in very weak magnetic fields ($B < 0.1$ T) as a feature in $\rho_{xx}(B)$ (Fig. 2) close to the condition $L_B \simeq l$ which is obeyed at a magnetic field of 0.03 T. The dashed line in Fig. 2 describes the EEI correction obtained in accordance with theory [14] (see below), which can be added to the WL contribution calculated according to Ref. [16] and shown as a dotted line, to reveal good agreement with the experimental $\rho_{xx}(B)$. The typical form of the expected WL correction corresponds to the case of similar values of τ_φ and τ_{so}. As the field grows, the relationship between the singlet and triplet contributions in (5a) changes and positive MR transforms into negative MR. This peculiarity of the MR, determined by the spin-orbital effects, was quite distinct when the EEI contribution was small [17–19] or less evident [20] than in the present sample. The analysis of the WL-related correction at different temperatures gives the dependence $\tau_\varphi(T)$ which can be approximated by the expression $\tau_\varphi(T) = 1.17 \times 10^{-12} T^{-p}$, where $p = 0.87$. This dependence is expected to describe the hole-hole scattering for which the theory predicts $p = 1$ in the case of a 2D gas of charge carriers [21].

4 Intermediate Magnetic Fields: Magnetoresistance Maximum

It is interesting to compare the experimental results on the MR maximum at intermediate fields with the predictions of the SR theory [14]. Note that experimentally a MR maximum is observed in magnetic fields which are 16–28 % lower (at the $T = 7.8$–0.355 K range) than the predicted temperature independent $B_{max} = 0.84$ T from the condition $\omega_c\tau = 1/\sqrt{3}$ that from the theory [14]. In the investigated case the linear approximation for large x (4b) is applicably. Then, from (1) and (4a) we have:

$$\frac{\Delta\rho^{SR}(B)}{\rho_0} = \frac{4}{3}\lambda^2\left(1 - \omega_c^2\tau^2\right)\left(\frac{\omega_c}{\varepsilon_F}\right). \tag{6}$$

At low temperatures ($T < 2$ K) the region of quadratic approximation of (4b) is negligibly small but it grows with increasing temperature. In order to take it into account in the region of the transition from quadratic to linear approximation of (4b), the calculation was made using an approximate function F_2 obtained by matching those approximations. For calculating magnetoresistance we use a complete form of (4a) and take into account the full factor $1 - \omega_c^2\tau^2$. The calculated equation is

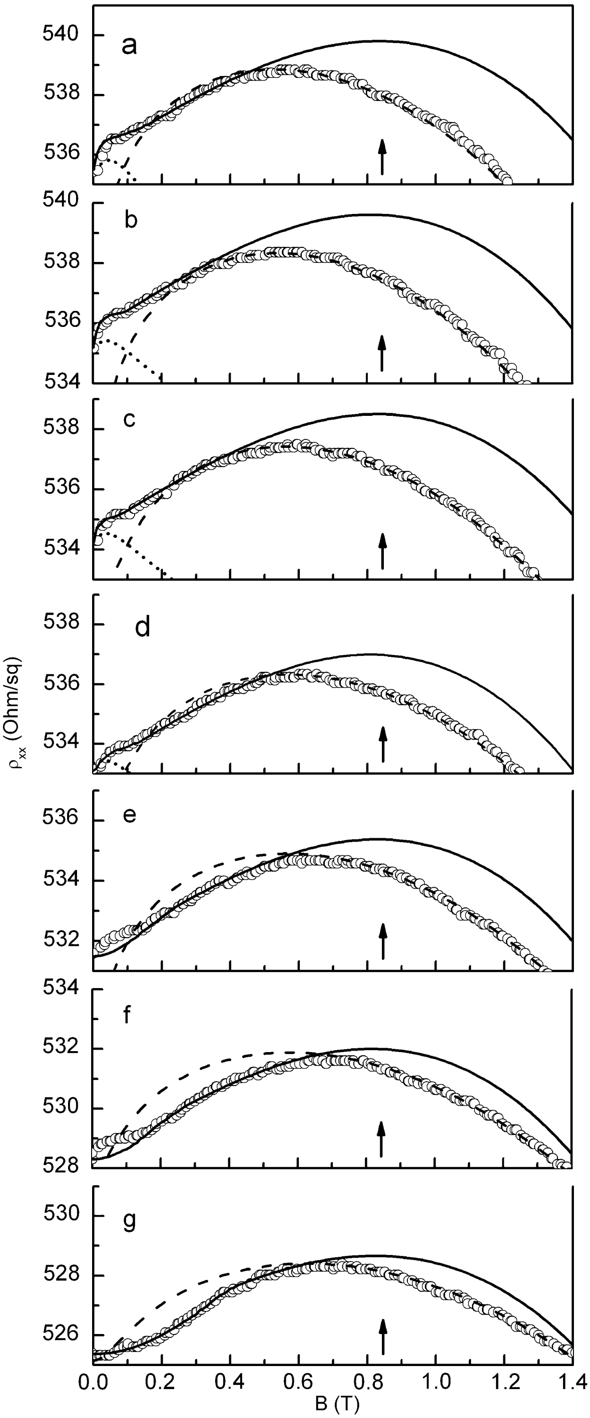
$$\frac{\Delta\rho^{SR}(B)}{\rho_0} = 4\lambda^2\left(\frac{\pi k_B T}{\varepsilon_F}\right)^{3/2}\left(1 - \omega_c^2\tau^2\right)F_2(\omega_c, T). \tag{7}$$

The curves in Fig. 3 (solid lines) were plotted using only one fitting parameter λ. Its most suitable estimate was $\lambda = 0.6$. Note that the curves calculated with this single λ-value coincide with the initial parts of the experimental dependences including the crossover from quadratic to linear magnetic field dependence at all the temperatures used; however, they all have a maximum that appears at much higher magnetic fields than observed in the experiment. This MR maximum can be seen to shift slightly with increasing temperature towards higher magnetic fields. To visualize the efficiency of the SR theory [14] in describing the initial regions of MR variation, the calculated curves also include the resistance in zero magnetic fields where the WL contribution is essential.

The predicted linear MR [14] is bound above by the condition $\omega_c\tau = 1$ (for our sample the magnetic field corresponding to this condition is 1.46 T). However, the SR theory [14] does not allow for logarithmic saturation of the EEI correction to the magnetoconductivity predicted earlier [6, 7, 22] for EEI corrections in the diffusion regime. The saturation of the EEI corrections may be caused by the increasing curvature of hole trajectories as the field increases, which reduces the probability of interference for the wave functions of the interacting holes.

Model calculations were also made to investigate the role of the logarithmic saturation of the interaction correction on the formation of the MR maximum. Formulas for quantum EEI corrections in a perpendicular magnetic field in the diffusion and

Fig. 3 Magnetic field dependences of $\rho_{xx}(B)$ at temperatures: (*a*) 0.355 K, (*b*) 0.7 K, (*c*) 1.1 K, (*d*) 1.56 K, (*e*) 2.73 K, (*f*) 4.55 K, (*g*) 7.8 K (*open circles* as experimental data). The *solid lines* are SR-EEI calculation according to (7). The *dotted lines* represent the contribution of the WL correction [16]. The *dashed line* describes the EEI correction calculated according to combination of (9a) due to [7] and (1) due to [13]. *Vertical arrow* shows the condition $\omega_c \tau = 1/\sqrt{3}$



Cooper channels were chosen as the model functions. The correction to the magnetoconductivity due to interactions between charge carriers having close energies and a small momentum difference (the so called diffusion channel) was obtained by Lee and Ramakrishnan [22] as:

$$\Delta\sigma_D^{LR}(B,T) = -\frac{e^2}{2\pi^2\hbar}\lambda_D g_2(h), \tag{8a}$$

where $h = g\mu_B B/(k_B T)$, g is the Lande factor, μ_B is the Bohr magneton, and λ_D is the interaction constant in the diffusion channel. The function $g_2(h)$ has the following bounded approximations:

$$g_2(h) = \begin{cases} 0.084\, h^2, & h \ll 1, \\ \ln(h/1.3), & h \gg 1. \end{cases} \tag{8b}$$

The characteristic magnetic field where the quadratic dependence of $g_2(h)$ changes into the logarithmic one is specified by the expression $B_{D0} = \pi k_B T/(g\mu_B)$. The characteristic field in the diffusion channel is $B_{D0} \geq 1$ T where EEI corrections are small and so can be ignored.

The correction to magnetoconductivity determined by the interaction between charge carriers with close energies and a small sum of momentums (the so-called Cooper channel) was obtained by Altshuler and Aronov [7]:

$$\Delta\sigma_C^{AA}(B,T) = -\frac{e^2}{2\pi^2\hbar}\lambda_c \varphi_2(\alpha), \tag{9a}$$

where $\alpha = 2eDB/(\pi k_B T)$, λ_C is the interaction constant in Cooper channel:

$$\varphi_2(\alpha) = \int_0^\infty \frac{t\, dt}{\sinh^2(t)}\left[1 - \frac{\alpha t}{\sinh(\alpha t)}\right],$$
$$\varphi_2(\alpha) = \begin{cases} 0.3\alpha^2, & \alpha \ll 1, \\ \ln(\alpha), & \alpha \gg 1. \end{cases} \tag{9b}$$

The characteristic magnetic field $B_{C0} = \pi k_B T/(2eD)$ for our object at $T = 1$ K is 0.15 T. Hence, the logarithmic saturation $\Delta\sigma_C^{AA}(B,T)$ is within the region of the formation of the MR maximum.

The behavior of MR in our object was calculated at different temperatures using (1) and the correction $\Delta\sigma_c^{AA}$ from (9a) as the model function which saturates in strong magnetic fields. The result obtained is very instructive. The calculated MR curve (Fig. 3 dashed line) has a maximum which now practically coincides with the MR maximum in the experimental curve at all the temperatures used. To visualize the coincidence, the calculated curves were plotted after excluding the contribution of the WL correction from the resistance values taken in a zero magnetic field. It is important to note that, unlike the magnetic field dependence of corrections $\Delta\sigma^{SR}$, the correction $\Delta\sigma_c^{AA}$ does not describe the initial part of experimental MR curves, but it does provide a good description of the MR maxima.

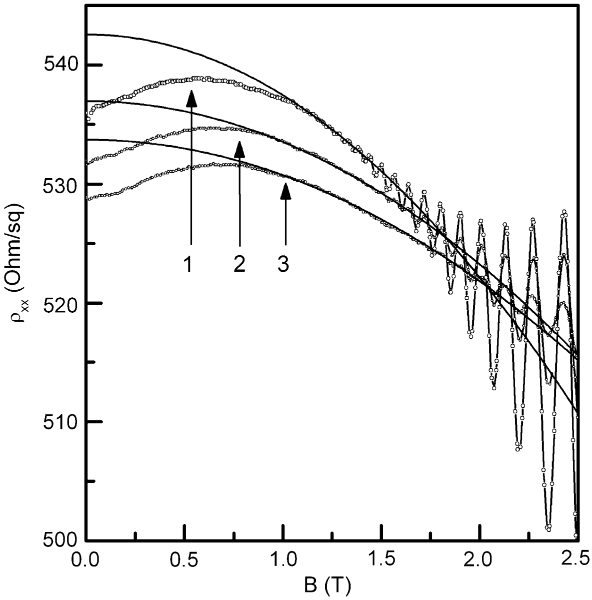


Fig. 4 Example of separating the magnetic field dependence of the EEI correction to $\rho_{xx}(B)$ at temperatures: (*1*) 0.355 K, (*2*) 2.73 K, and (*3*) 4.55 K. The *solid lines* are a calculation according to (2) due to [13] and (10) due to [11]

The interaction constant λ_c is used as the fitting parameter in this calculation and is found to decrease from 0.75 to 0.6 as the temperature increases from 0.355 to 7.8 K. This interaction constant is commonly expressed in terms of the Fermi-liquid constant F_0^σ, which accounts for the intensity of the spin-exchange interaction, although the functional relation between λ and F_0^σ depends on the particular situation. For example, λ_c is determined by the function $1 + 3[1 - \ln(1 + F_0^\sigma)/F_0^\sigma]$ in the diffusion regime and by $1 + 3F_0^\sigma/(1 + F_0^\sigma)$ in the ballistic regime [8]. Thus, the diffusion-to-ballistic change as temperature rises causes an appreciably decrease in the coupling constant, as we observe. However, the temperature variation of λ is commonly neglected and a particular constant λ-value is used for a limited temperature interval.

5 Strong Magnetic Fields

The interaction constant can also be obtained by analyzing MR in the strong magnetic field region, where the inequality $\omega_c\tau \gg 1$ is obeyed [23]. Figure 4 illustrates how (2) describes the monotonic part of the experimental dependence $\rho_{xx}(B)$, in the region $B > 1$ T at different temperatures. The monotonic part of $\rho_{xx}(B)$ is taken as the mid-point locus between the neighboring maxima and minima of SdH oscillations. The relative EEI-induced variation of resistance in a perpendicular magnetic field is described as [11]:

$$\frac{\Delta\rho^{GM}(B,T)}{\rho_0} = \frac{(\omega_c\tau)^2}{\pi k_B l}\left[G_F\left(\frac{k_B T\tau}{\hbar}\right) + G_H\left(\frac{k_B T\tau}{\hbar}; F_0^\sigma\right)\right], \tag{10}$$

where G_F and G_H are the functions describing the contribution of the exchange (Fock term) and direct (Hartree term) interactions, respectively, and their analytical

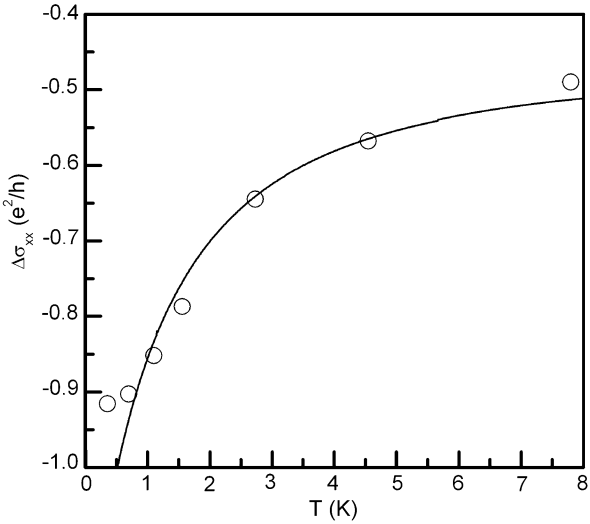


Fig. 5 Variations of the EEI correction to the conductivity with rising temperature. The *solid line* is a calculation according to the theory Ref. [11]

form is given in Ref. [11]. These contributions form singlet and triplet channels of interaction and together describe well the temperature variation of the EEI correction to the conductivity. It is shown theoretically [8, 11] that the general picture of interaction reduces to the triplet interaction channel in which the interaction is determined by the Fermi-liquid interaction constant F_0^σ present in the function G_H.

The agreement between the calculated temperature dependence of the correction to the conductivity (solid curve) and the EEI correction to conductivity $\Delta\sigma^{int}(T)$ (symbols) derived from the analysis of negative quadratic MR using (2) is illustrated in Fig. 5. It is seen that the magnitude of the EEI correction decreases rapidly with rising temperature and then becomes saturated.

The analysis of the dependence $\Delta\sigma_{xx}(B,T)$ (Fig. 5) yields $F_0^\sigma = -0.225$, using the commonly accepted relation between λ and F_0^σ for the diffusion regime (see above). From this we obtain $\lambda = 0.6$, which coincides nicely with the value obtained from analysis of the magnetic field-resistance dependences obtained for our object on the basis of the SR theory [14].

6 Conclusions

The maximum of magnetoresistance of the investigated two-dimensional system has been a sort of "touchstone" for detecting corrections between predictions of different EEI theories and experimentally observed MR variations in different regions of magnetic fields. It is found that the equations of the Sedrakyan-Raikh theory [14] for EEI corrections are applicable in low magnetic fields (down to 0.3 $\omega_c\tau$) in which the predicted crossover from quadratic to linear positive MR is observed. In higher magnetic fields there is no consensus between experiment and theory Ref. [14] concerning the position of MR maximum. We suggest that the logarithmic saturation of the EEI correction to the magnetoconductivity in high magnetic fields can help to reach a better consensus.

Acknowledgements The authors thank Professor M.E. Raikh for helpful discussions. Measurements were made in International Laboratory of High Magnetic Fields and Low Temperatures, Wroclaw, Poland and University of Warwick, Coventry, UK.

References

1. E. Abrahams, P.W. Anderson, D.C. Licciardello, T.V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979)
2. B.L. Altshuler, D.E. Khmelnitzkii, A.I. Larkin, P.A. Lee, Phys. Rev. B **22**, 5142 (1980)
3. A. Kawabata, J. Phys. Soc. Jpn. **53**, 3540 (1984)
4. A. Cassamchenai, B. Shapiro, J. Phys. **4**, 1527 (1994)
5. B.L. Altshuler, A.G. Aronov, P.A. Lee, Phys. Rev. Lett. **44**, 1288 (1980)
6. B.L. Altshuler, A.G. Aronov, A.I. Larkin, D.E. Khmelnitsky, Sov. Phys. JETP **81**, 768 (1981)
7. B.L. Altshuler, A.G. Aronov, in *Electron–Electron Interactions Disordered Systems Modern Problems in Condensed Matter Science*, ed. by A.L. Efros, M. Pollak (North-Holland, Amsterdam, 1985)
8. G. Zala, B.N. Narozny, I.L. Aleiner, Phys. Rev. B **64**, 214204 (2001)
9. G. Zala, B.N. Narozny, I.L. Aleiner, Phys. Rev. B **64**, 201201 (2001)
10. G. Zala, B.N. Narozny, I.L. Aleiner, Phys. Rev. B **65**, 020201 (2002)
11. I.V. Gornyi, A.D. Mirlin, Phys. Rev. Lett. **90**, 076801 (2003)
12. I.V. Gornyi, A.D. Mirlin, Phys. Rev. B **69**, 045313 (2004)
13. A. Houghton, J.R. Senna, S.C. Ying, Phys. Rev. B **25**, 2196 (1982)
14. T.A. Sedrakyan, M.E. Raikh, Phys. Rev. Lett. **100**, 106806 (2008)
15. A.Yu. Kuntsevich, G.V. Minkov, A.A. Sherstobitov, V.M. Pudalov, Phys. Rev. B **79**, 205319 (2009)
16. N.S. Averkiev, L.E. Golub, G.E. Pikus, JETP Lett. **86**, 780 (1998)
17. T. Hassenkam, S. Pedersen, K. Baklanov, A. Kristensen, C.B. Sorensen, P.E. Lindelof, F.G. Pikus, G.E. Pikus, Phys. Rev. B **55**, 9298 (1997)
18. F.E. Meeijr, A.F. Morpugo, T.M. Klapeijk, T. Koda, J. Nitta, Phys. Rev. B **70**, 201307 (2004)
19. D.R. Leadley, V.V. Andrievskii, I.B. Berkutov, Yu.F. Komnik, T. Hackbarth, O.A. Mironov, J. Low Temp. Phys. **159**, 230 (2010)
20. V.V. Andrievskii, I.B. Berkutov, T. Hackbarth, Yu.F. Komnik, O.A. Mironov, M. Myronov, V.I. Litvinov, T.E. Whall, in *Molecular Nanowires and Other Quantum Objects*, ed. by A.S. Alexandrov, J. Demsar, I.K. Yanson. NATO Science Series, II: Mathematics, Physics and Chemistry, vol. 148 (Kluwer Academic, Norwell, 2004), p. 319
21. B.L. Altshuler, A.G. Aronov, D.E. Khmelnitsky, J. Phys. C **15**, 7367 (1982)
22. P.A. Lee, T.V. Ramakrishnan, Phys. Rev. B **26**, 4009 (1982)
23. I.B. Berkutov, V.V. Andrievskii, Yu.F. Komnik, O.A. Mironov, M. Myronov, D.R. Leadley, Low Temp. Phys. **32**, 683 (2006)