Temperature dependence of the electron–phonon scattering time of charge carriers in *p*-Si/SiGe heterojunctions

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$\mathrm{Si/Si_{0.64}Ge_{0.36}}$ heterojunctions with *p*-type conductivity exhibit an electron overheating effect. An analysis of the damping of the amplitudes of the Shubnikov–de Haas oscillations upon a change in temperature and applied electric field yields the temperature dependence of the electron–phonon relaxation time: $t_{e\mathrm{ph}}=10^{-8}T^{-2}$ s. © *2000 American Institute of Physics.* [S1063-777X(00)00512-0]

Two-dimensional electron systems (inversion layers, delta layers, heterojunctions, and quantum wells) have unique properties in that, on the one hand, the charge carriers in them have a two-dimensional character of their motion and, on the other hand, this two-dimensional state is realized in the bulk of the semiconductor, i.e., the phonons interacting with the carriers are three-dimensional. Information about the electron–phonon interaction in inversion layers, delta layers, and heterojunctions at low temperatures (~ 1 K) cannot be obtained from the behavior of the quantum corrections to the conductivity due to the weak localization and electron interaction effects (see, e.g., Refs. 1–7), since at such temperatures the dominant inelastic relaxation process is electron–electron scattering.

The electron–phonon interaction time $\tau_{e\mathrm{ph}}$ can be determined by studying the electron overheating effect.[8] In the electron overheating effect the electron temperature T_e exceeds the phonon temperature T_{ph} under the influence of a high electric field (current) or of other, ''heating'' factors. The transfer of excess energy from the electron to the phonon system, even under conditions of strong elastic scattering, is governed by the time $\tau_{e\mathrm{ph}}$. Therefore the problem of determining $\tau_{e\mathrm{ph}}$ experimentally reduces to finding the value of the overheating of the electron gas $\Delta T_e=(T_e-T_{\mathrm{ph}})$ under conditions of high current flow. To realize the electron overheating effect it is necessary to ensure the free escape of phonons from the conducting layer into the surrounding crystal (i.e., to provide good acoustical coupling of the conducting layer and the crystal). This requirement is manifestly satisfied for inversion layers, delta layers, and heterostructures.

The electron overheating effect has been analyzed previously with the use of Shubnikov–de Haas (SdH) oscillations for inversion layers on silicon[9,10] and for $\mathrm{Si/Si_{0.7}Ge_{0.3}}$ heterostructures with *n*-type conductivity.[11] In the cited papers the falloff of the amplitude of the oscillations with increasing applied electric field was used to find a relation between the electron temperature and the rate of loss of the excess energy by the electrons; in Ref. 11 the dependence of the energy loss time on the overheating temperature was found, and it was concluded that the main channel of electron energy loss is the emission of acoustical phonons.

In this study we have realized the electron overheating effect in $\mathrm{Si/Si_{0.64}Ge_{0.36}}$ heterojunctions containing implanted boron, which led to a *p*-type conductivity. The samples were obtained by molecular beam epitaxy (MBE).[1] The conducting region had a width of 0.55 mm, and the distance between the potential contacts was 2.25 mm. The mobile charge carrier concentration n was 2×10^{11} cm^{-2}, and the Hall mobility of the carriers was ~ 12000 cm^{2}V^{-1}s^1.

At low temperatures (0.35–2 K) the heterojunctions exhibited pronounced SdH oscillations (Fig. 1). Naturally, as the temperature was raised, and also as the current flowing through the heterojunctions increased, the amplitude of the SdH oscillations fell off. The electron temperature T_e under conditions of high current flow can be determined from a comparison of the change in the amplitude of the SdH oscillations under the influence of current and under the influence of temperature. We carried out such an analysis for three extrema in the magnetic field interval 8–14 kOe. Figure 2 shows the change in the amplitude of the SdH oscillations with quantum numbers $\nu=5,6,7$ as the temperature and current are varied. From a comparison of the curves one can find T_e at each specified value of the current.

The value of $\tau_{e\mathrm{ph}}$ can be calculated from the data for the electron overheating effect with the use of the heat balance equation, which assumes that the electrical power $P=E^2\sigma$ released in a unit volume is equal to the amount of energy transferred by the electrons to the lattice per unit time:

$$E^2\sigma=\int_{T_{\mathrm{ph}}}^{T_e}\frac{C_e(T)\,dT}{\tau_{e\mathrm{ph}}(T)}. \qquad (1)$$

If it is assumed that $\tau_{e\mathrm{ph}}^{-1}=\alpha T^p$ and that the electronic heat capacity $C_e(T)=\gamma T$, then it follows from (1) that[12]

$$E^2\sigma=\frac{\alpha\gamma}{p+2}\left(T_e^{p+2}-T_{\mathrm{ph}}^{p+2}\right). \qquad (2)$$

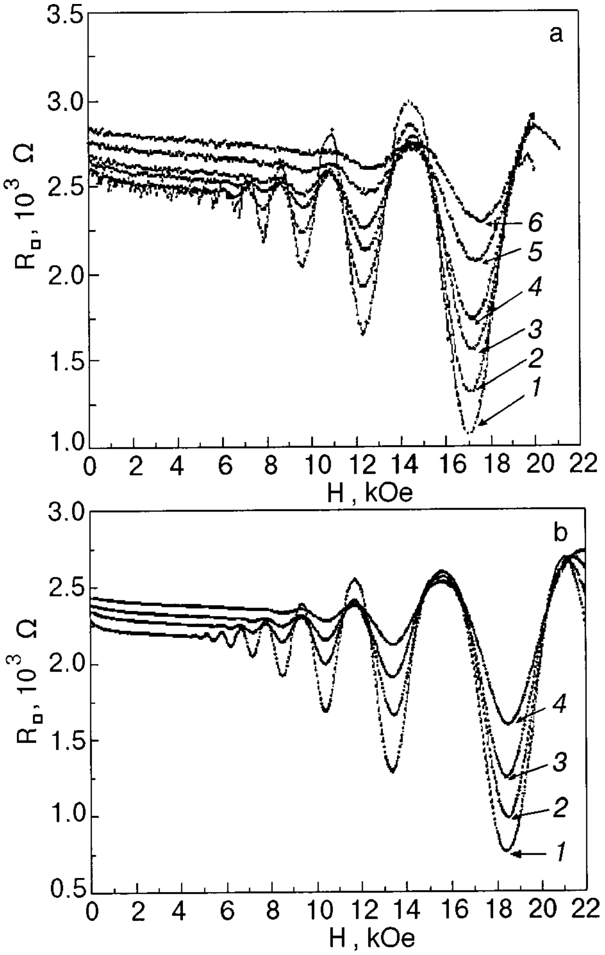

FIG. 1. Shubnikov–de Haas oscillations at various temperatures T [K]: 0.334 (*1*), 0.619 (*2*), 0.834 (*3*), 0.984 (*4*), 1.281 (*5*), and 1.514 (*6*) (a) and at various currents [nA]: 100 (*1*), 1000 (*2*), 1790 (*3*), and 3000 (*4*) (b).

Under the condition $T_e \gg T_{\rm ph}$ one should observe a dependence $T_e \propto E^{2/(p+2)}$. Figure 3 shows the experimental dependence of T_e on the applied electric field E (points) together with the relation $T_e \propto E^{1/2}$ (continuous curve), which approximates the experimental data quite well (except for the points at small E, for which the condition $T_e \gg T_{\rm ph}$ is not satisfied). Thus we arrive at the preliminary finding that the exponent p in the relation $\tau_{\rm eph}^{-1} = \alpha T^p$ is equal to 2.

We attempted to find the temperature dependence $\tau_{\rm eph}(T)$ directly from the experimental data presented above. Let us assume that in the steady state $\tau_{\rm eph}$ in (1) corresponds to a certain temperature $T_{\rm eph}$ that characterizes the electron–phonon interaction under conditions of electron overheating. Then from (1) we have

$$E^2\sigma = (T_e - T_{\rm ph})\,\frac{T_e + T_{\rm ph}}{2}\,\frac{\gamma}{\tau_{\rm eph}(T_{\rm eph})}. \tag{3}$$

For $T_{\rm eph} = (T_e + T_{\rm ph})/2$ Eq. (3) implies the following relation, which was first given in Ref. 13:

$$T_e - T_{\rm ph} = \frac{E^2\sigma}{\gamma T_{\rm eph}}\,\tau_{\rm eph}(T_{\rm eph}). \tag{4}$$

Since the value of γ is unknown for our objects of study, we must turn to the relation[14]

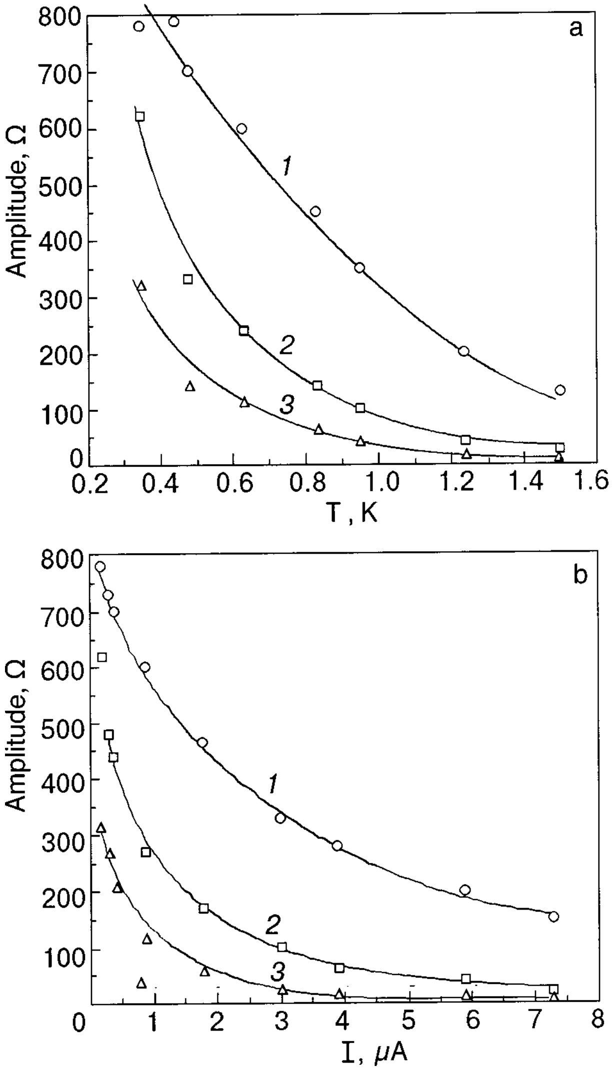

FIG. 2. Change in the amplitude of the SdH oscillations with quantum numbers $\nu = 5$ (*1*), 6 (*2*), 7 (*3*) upon changes in temperature (a) and current (b).

$$(kT_e)^2 = (kT_{\rm ph})^2 + \frac{6}{\pi^2}(eE)^2 D\,\tau_{\rm eph}, \tag{5}$$

where D is the electron diffusion coefficient, and E is the electric field that leads to heating of the electrons. This relation is obtained from Eq. (4) with the electronic heat capacity and conductivity expressed in terms of the density of states ν_{ds}: $C_e = (\pi^2/3)k^2\nu_{ds}T$ and $\sigma = e^2\nu_{ds}D$. For two-dimensional electrons $\nu_{ds} = m^*/(\pi\hbar^2)$, $D = (1/2)v_F^2\tau$, and the Fermi velocity $v_F = (\hbar/m^*)(2\pi n)^{1/2}$. The elastic scattering time can be determined from the formula $R_\square^{-1} = ne^2\tau/m^*$. For the effective mass we take the value $m^* = 0.242m_0$ (m_0 is the free electron mass), obtained from an analysis of the SdH oscillations. The electric field in a conducting channel of length L and width a can be found from the values of the current I and the resistance per square $R_\square$: $E = IR/L = IR_\square/a$ (since $R = R_\square L/a$). For $T_{\rm ph}$ one should take the temperature of the crystal (in our case $T_{\rm ph} = 0.37$ K).

From calculations based on Eq. (5) we obtained the temperature dependence of $\tau_{\rm eph}$ (Fig. 4). We assumed, as in Refs. 15 and 16, that $T_{\rm eph} = 1/2(T_{\rm ph} + T_e)$. The temperature

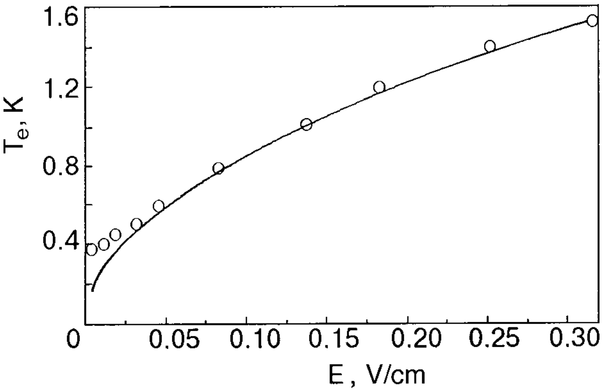

FIG. 3. Electron temperature T_e versus the applied electric field E (points) and the approximating function $T_e \propto E^{1/2}$ (continuous curve).

dependence $\tau_{eph}(T)$ presented in Fig. 4 can be approximated at temperatures above ~0.4 K by a power law: $\tau_{eph} = 10^{-8} T^{-p}$, where $p \approx 2$.

Let us discuss the possible causes of a temperature dependence $\tau_{eph}^{-1} \propto T^2$. For three-dimensional conductors in the "clean" limit one should have $\tau_{eph}^{-1} \propto T^3$ (Refs. 17 and 18), while for strong disordering, in the "dirty" limit, the theory predicts a weakening of the electron–phonon interaction and the appearance of a temperature dependence of the form $\tau_{eph}^{-1} \propto l T^4$, where l is the mean free path of the electrons[17,19,20] (the transition from the "clean" to the "dirty" limit corresponds to a transition from the inequality $q_{ph} l > 1$ to the inequality $q_{ph} l < 1$, so that in the latter case the mean free path of the electrons is shorter than the wavelength of a thermal phonon, $\lambda_{ph} = 2\pi/q_{ph} = 2\pi\hbar s/kT$, where q_{ph} is the phonon wave vector and s is the phonon velocity).

Our experimental dependence $\tau_{eph}^{-1} \propto T^2$ is very often observed for thin films in an analysis of the behavior of the quantum corrections to the conductivity due to weak localization and electron interaction effects (see, e.g., Refs. 16 and 21–25) or in the analysis of experiments on electron overheating (see, e.g., Refs. 15 and 26–29). The appearance of a temperature dependence of this kind for films can be attributed to modification of the phonon spectrum in thin films. Let us clarify this statement. The time τ_{eph} is determined by the Éliashberg function $\alpha^2(\omega)F(\omega)$ in the frequency region corresponding to the energy of thermal phonons:[30]

$$\tau_{eph}^{-1} = 4\pi \int d\omega \frac{\alpha^2(\omega) F(\omega)}{\mathrm{sh}(\hbar\omega/kT)}. \tag{6}$$

For a quantized phonon spectrum the density of states is a linear function of ω, i.e., $F(\omega) \propto \omega$. In that case (with allowance for the weak dependence of α on ω) it turns out that $\tau_{eph}^{-1} \propto T^2$. An analogous result was observed in an analysis of the possibility that shear waves (waves of the Love type) with an unusual dispersion relation $\omega \propto q^{1/2}$ can exist in a film–substrate system.[31] We might also point out Refs. 12 and 32, in which a dependence $\tau_{eph}^{-1} \propto l T^3$ was obtained in the "dirty" limit for the case of a two-dimensional phonon spectrum. Accordingly, in the "clean" limit one would expect a dependence of the form $\tau_{eph}^{-1} \propto T^2$. However, these variant explanations of the behavior $\tau_{eph}^{-1} \propto T^2$ cannot be directly applied to the heterostructures studied here, since in them the two-dimensionality is inherent to the electron system, while the phonons remain three-dimensional.

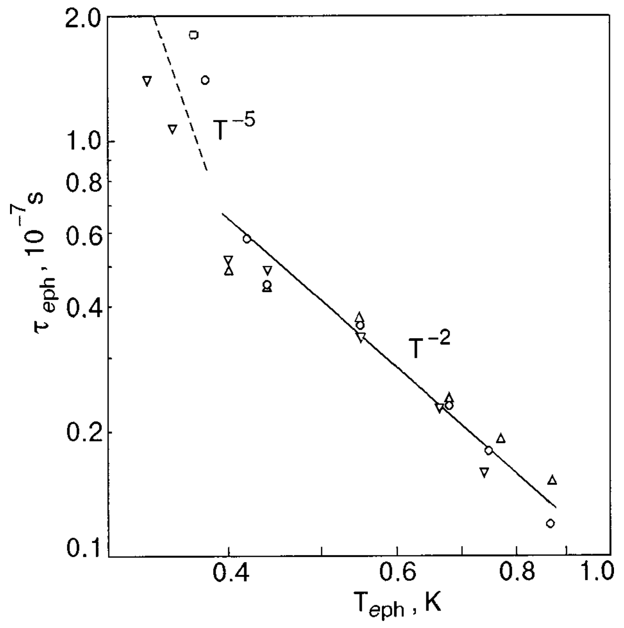

FIG. 4. Temperature dependence of the electron–phonon relaxation time τ_{eph} found from the decrease in amplitude of the SdH oscillations with quantum numbers $\nu = 5$ (△), 6 (▽), and 7 (○) under the influence of a current.

In our view, the temperature dependence $\tau_{eph}^{-1} \propto T^2$ obtained in this study is due to the two-dimensional nature of the electron system. In heterojunctions, as a result of the discontinuity of the bands at the heterointerface and the appearance of internal electric fields, a potential well (which to a first approximation is triangular in shape) is formed in which the motion of the electrons (or holes) in the direction transverse to the well (along the z axis) is quantized, while in the plane of the interface (xy) the motion remains free. The electrons occupy size-quantization levels (subbands) and are described by the dispersion relation

$$\varepsilon = \frac{p_x^2 + p_y^2}{2m} + \varepsilon_i. \tag{7}$$

At low temperatures the absorption or emission of phonons is accompanied by a change in the electron momentum components p_x, p_y. At high temperatures, intersubband transitions can occur. According to Ref. 33, for $kT > kT_2 = \sqrt{8ms^2W}$ (where $W = \pi^2\hbar^2/(2md^2)$ is, in order of magnitude, the ground-state energy of the size quantization, s is the speed of sound, and d is the characteristic width of the well) the electron–phonon scattering processes are quasielastic and are characterized by a temperature dependence $\tau_{eph}^{-1} \propto T$. An estimate of T_2 for our objects (for $s = 9 \times 10^5$ cm/s,[34] $d \sim 100$ Å) gives a value ~40 K. At low temperatures ($T < T_2$) a situation can arise in which the wave momentum of a thermal phonon, $q_T = kT/(\hbar s)$, is sufficient to change the electron wave vector by the maximum value $2k_F$, since at lower temperatures one has $q_T < 2k_F$, and only small-angle scattering of the electrons is possible. The temperature T_1 separating these regions corresponds to the condition $q_T = 2k_F$. In the region of partial inelasticity ($T > T_1$) an electron upon interacting completely changes its momentum and absorbs (emits) a phonon with a wave vector predominantly perpendicular to the interface. Conservation of total momentum and energy for the two-dimensional electron system implies a dependence $\tau_{\mathrm{eph}}^{-1}\propto T^2$ (Ref. 33). In the small-angle scattering region ($T<T_1$) the wave vector of the phonon participating in the interaction is arbitrary in direction and limited in magnitude by the temperature. In this case the scattering is similar to small-angle scattering in a three-dimensional metal and is described by a temperature dependence $\tau_{\mathrm{eph}}^{-1}\propto T^5$ (Refs. 33 and 35).

The temperature dependence $\tau_{eph}^{-1}\propto T^2$ found in the present study corresponds to the region of partial inelasticity $T>T_1$. In Fig. 4 one can discern a tendency for $\tau_{\mathrm{eph}}(T)$ to become steeper for $T<0.4$ K. For purposes of illustration, the dashed line in Fig. 4 shows the dependence $\tau_{\mathrm{eph}}^{-1}\propto T^5$. Thus the experimental results are described rather successfully in terms of the concept developed for electron–phonon relaxation of two-dimensional charge carriers.[33] At the same time, numerical estimates of the characteristic temperature T_1 for the transition from a dependence of the form $\tau_{eph}^{-1}\propto T^2$ to $\tau_{eph}^{-1}\propto T^5$ did not give a unique value. For example, the value $k_F=0.04\times 10^6$ cm^{-1} found from the condition $q_T=2k_F$ for $T_1\sim 0.4$ K turned out to be substantially smaller than the value of the wave vector determined from the well-known relation for a two-dimensional electron system: $k_F=(2\pi n)^{1/2}$. In the latter case the value $k_F=1.12\times 10^6$ cm^{-1} was obtained. This discrepancy is possibly due to the fact that the total concentration of charge carriers is distributed between the size-quantization subbands, and at low temperatures the subband with the low occupation becomes important in electron–phonon relaxation processes. Our values of τ_{eph} (10^{-7}–10^{-8} s at T_{eph} in the interval 0.37–1 K) are entirely reasonable.

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1) The samples were prepared at the Advanced Semiconductors Group, University of Warwick, Coventry, UK.

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