Shubnikov-de Haas oscillations of the conductivity of a two-dimensional gas in quantum wells based on germanium and silicon. Determination of the effective mass and *g* factor

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(Submitted September 5, 2008; revised September 26, 2008)
Fiz. Nizk. Temp. **35**, 188–193 (February 2009)

The Shubnikov-de Haas oscillations of the conductivity of a two-dimensional gas of holes in quantum wells consisting of pure germanium and silicon with low germanium content (13%) are analyzed to determine the effective masses and the g factor in these regions. The magnetic-field dependences of the resistivity ρ_{xx} obtained at temperatures from 33 mK to 4 K in magnetic fields up to 11 T are used for the analysis.
[DOI: 10.1063/1.3075945]

It is necessary to know the characteristics of the mobile charge carriers in order to develop electronic devices based on systems with a two-dimensional electron gas. Some characteristics (carrier concentration and mobility) are easily determined from the conductivity and Hall's constant. However, characteristics such as the effective mass of the carriers, which determines the kinetic properties of the electronic system and makes it possible to calculate the Fermi energy, the Fermi velocity of the carriers, the density of states, and others, as well as the effective g factor, which characterizes the spin (Zeeman) splitting of the carrier states in a magnetic field must be known in order to have a complete picture of the properties of a two-dimensional electron gas. The Shubnikov-de Haas oscillations of the conductivity (SdH) in a magnetic field are used to determine these characteristics.[1,2]

In the present work the SdH oscillations of the conductivity of a two-dimensional gas of holes in quantum wells of pure germanium (in the heterostucture $Si_{0.3}Ge_{0.7}/Ge/Si_{0.3}Ge_{0.7}$) and silicon with low germanium content (in the heterostructure $Si/Si_{0.87}Ge_{0.13}/Si$) are studied. These heterostructures were obtained by different methods: sample A with a quantum well consisting of pure Ge was obtained by low-energy plasma deposition; sample B with a quantum well $Si_{0.87}Ge_{0.13}$ was obtained by molecular-beam epitaxy. The quantum well of sample A consists of a thin layer (15 nm thick) of pure germanium sandwiched between to $Si_{0.3}Ge_{0.7}$ layers. The layer with acceptor boron atoms is separated from the quantum well by a 10 nm thick spacer. Similarly, the quantum well of sample B consists of a 10-nnm thick $Si_{0.87}Ge_{0.13}$ layer, sandwiched between two layers of pure silicon, one of which is a 10-cm thick spacer and separates the quantum well from the layer doped with boron atoms.

The experimental dependences of the change of the diagonal ρ_{xx} and off-diagonal ρ_{xy} components of the resistivity of the structures investigated in a magnetic field at low temperatures demonstrate Shubnikov-de Haas oscillations and a quantum Hall effect (Fig. 1). Some characteristic parameters of the samples are presented in Table I.

The temperature and magnetic-field dependences of the amplitude of the SdH oscillations[1)] were analyzed, by the method described in Refs. 3 and 4, in order to find the effective mass m^* and the quantum scattering time τ_q.

The variation of the conductivity of a two-dimensional gas of charge carriers in the magnetic quantization region is examined theoretically in Refs. 5 and 6. According to the theory of Ref. 6, the variation of the resistivity is described by the relation

$$\rho_{xx}=\frac{1}{\sigma_0}\left[1+4\sum_{s=1}^{\infty}\left(\frac{\Psi s}{\mathrm{sh}\Psi s}\right)\times\exp\left(-\frac{\pi s}{\omega_c\tau_q}\right)\cos\left(\frac{2\pi s\varepsilon_F}{\hbar\omega_c}-\Phi\right)\right], \quad (1)$$

where $\Psi=2\pi^2k_BT/\hbar\omega_c$ determines the temperature and

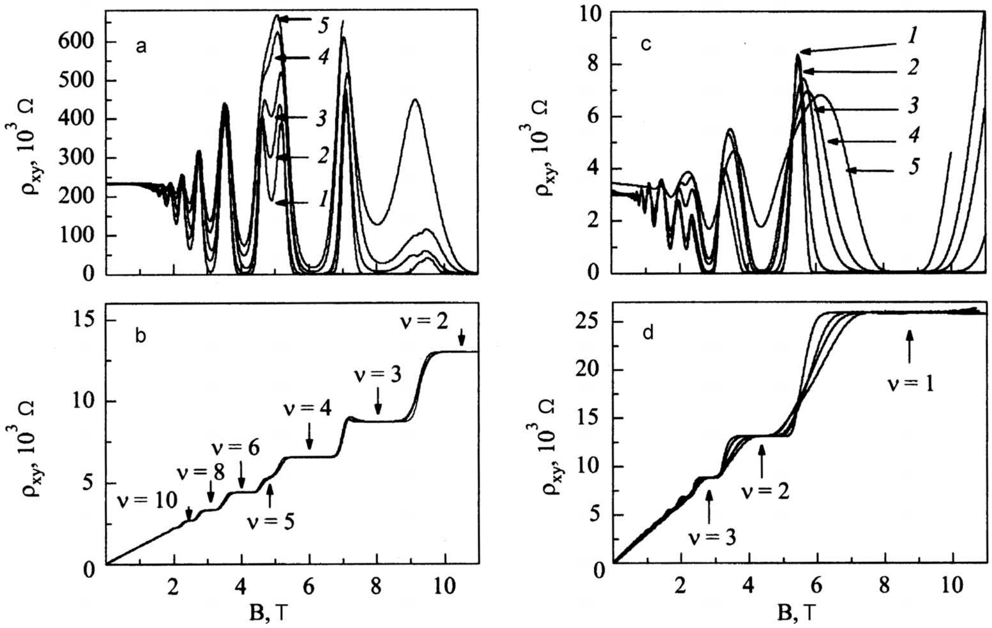

FIG. 1. Magnetic-field dependences of the diagonal ρ_{xx} and off-diagonal ρ_{xy} components of the resistivity of sample A (a, b) at T=52 mK (1), 0.5 (2), 0.9 K (3), 2 K (4), 3 K (5) and sample B (c, d) at T=33 mK (1), 0.3 K (2), 0.7 K (3), 0.89 K (4), 1.63 K (5).

magnetic-field dependences of the amplitude of the oscillations, $\omega_c = eB/m^*$ is the cyclotron frequency, τ_q is the quantum (single-particle) relaxation times of the charge carriers, which determines the collision broadening of the Landau levels, and Φ is the phase. The Fermi energy in a two-dimensional case is $\varepsilon_F = \pi\hbar^2 n/m^*$, where n is the charge-carrier concentration.[2)]

The functions $\ln[(\Delta R/R_0)(\sinh\Psi/\Psi)]$ versus $1/\mu B$ (or $1/\omega_c\tau$) were analyzed to determined the effective mass. The argument of the exponential function in Eq. (1) becomes $-\pi\alpha/(\omega_c\tau)$, where $\alpha = \tau/\tau_g$ and τ is the transport relaxation time. In this construction the points corresponding to extrema with different quantum numbers ν must lie on the same straight line. The effective mass m^* is an adjustable parameter which ensures that the points referring to different temperatures and magnetic fields on the same curve match one another. We note that according to the relation (1), for extremely strong magnetic fields with $1/(\omega_c\tau)\to 0$, these dependences should go to the value $\ln 4 = 1.386$, since then $\Psi/\sinh\Psi \to 1$. The matching of the points on the same curves (Figs. 2a and 2c) made it possible to determine the effective mass of the holes $m^* = 0.112 m_0$ and $m^* = 0.17 m_0$ (m_0 is the mass of a free electron) for samples A and B, respectively.

For sample A, formed with $m^* = 0.112 m_0$, the common line is not a straight line, as would follow from the relation (1). This requires a special explanation. The nonlinearity of the function constructed can be explained on the basis of the ideas developed in Ref. 7. It is asserted in this work that the potential and concentration of the carriers in a quantum channel can exhibit spatial nonuniformity. This effect of this is that the extrema of the oscillations on the magnetic field scale do not match in different regions of the sample, as a result of which the amplitude of the oscillations decreases somewhat in magnitude as compared with its value in a uniform sample, which corresponds to an additional effective broadening of the Landau levels, called "inhomogeneous broadening." The appearance of large-scale fluctuations of the carrier concentration (in the plane of the two-dimensional gas) could be due to natural thickness nonuniformity of the quantum channel as a result of the appearance of vicinal growth steps during the formation of the layer forming the quantum well. The theoretical analysis in Ref. 7 of the formation of SdH oscillations in the case where large-scale (in the plane of the two-dimensional gas) Gaussian fluctuations of the potential and concentration of the electrons exist showed that the expression (1) for the amplitude of the oscillations acquires an additional exponential contribution, so that the exponential factor in the relation (1) becomes

$$\exp\left[-\frac{\pi}{\omega_c\tau_q}-\left(\frac{\pi^2\hbar\,\delta n}{m^*\omega_c}\right)^2\right],$$

where δn is proportional to the fluctuations of the charge-carrier concentration.

The first term in the exponent of the exponential function, describing the collision broadening of the Landau levels, is inversely proportional to the magnetic field, and the second term, which takes account of the inhomogeneous broadening of the Landau levels, is inversely proportional to the squared field. This makes it possible to describe the experimental dependence constructed in Fig. 2a by a quadratic polynomial $Y = -a_1 X - a_2 X^2 + \text{const}$, where $a_1 = \pi a$ and $a_2 = (\pi^2\hbar\tau\delta p/m^*)^2$. The theoretical model of Ref. 7 has made it

TABLE I. Characteristic parameters of the samples.

Sample	Quantum channel	ρ_{xx}, kΩ	p_{Hall} (10^{11} cm^{-2})	p_{ShH} (10^{11} cm^{-2})	μ_{Hall}, 10^4 cm^2/V·s	τ, 10^{-12} s
A	Ge	0.23	5.81	5.68	4.68	2.98
B	$Si_{0.87}Ge_{0.13}$	3.01	1.89	2.04	1.17	1.05

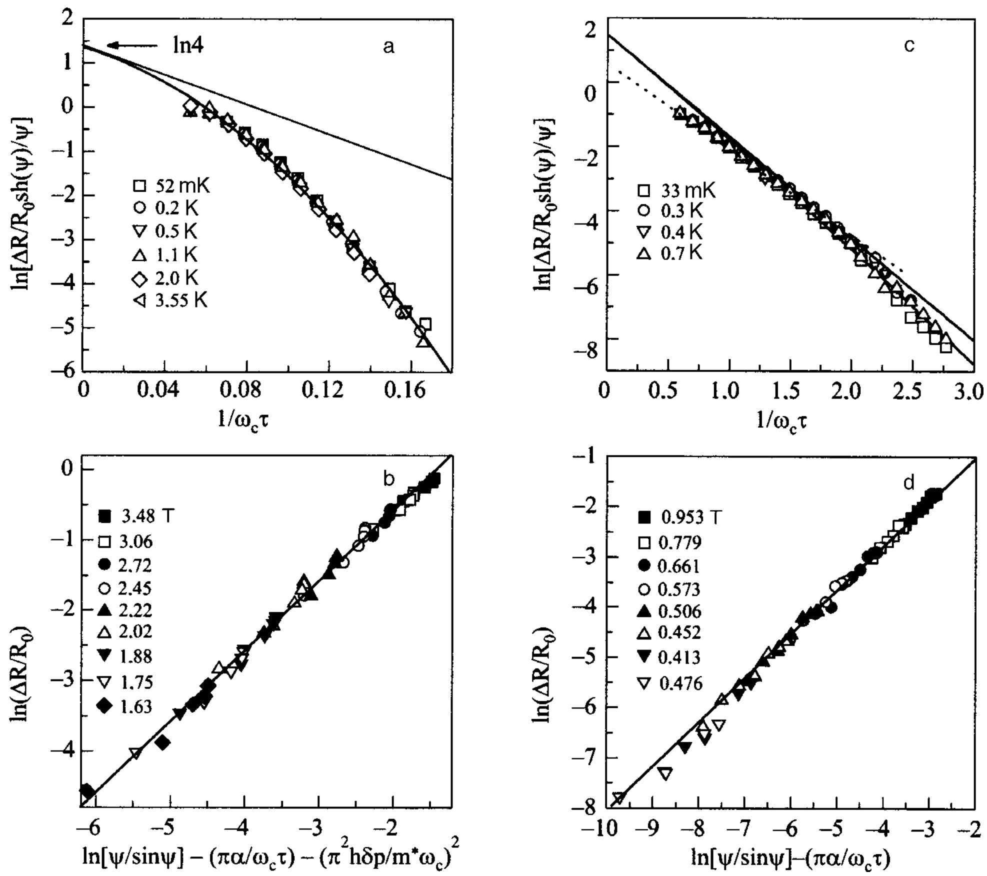

FIG. 2. Illustration of the procedure for determining the effective mass m^* and the parameter α for sample A at different temperatures (a) and in different magnetic fields (b) and sample B (c, d), respectively. The solid lines correspond to the theory of Ref. 6 and the dashed line is plotted taking account of the theory of Ref. 7. The slope of the solid lines in Figs. 2b and 2d equals 45°.

possible to describe quite accurately the experimental results for sample *A* (Fig. 2a) and obtain $\alpha=5.34$ and $\delta p=3.8\cdot 10^{10}$ cm^{-2}. Thus a quantum well obtained by low-energy plasma deposition is characterized by the thickness nonuniformity of the quantum channel. It should be noted that inhomogeneous broadening of the Landau levels occurs in sample B also but it is much weaker. For this sample $\alpha=1.02$ and $\delta p=4\times 10^9$ cm^{-2}.

The functions ln $\Delta R/R_0$ versus the temperature and magnetic field, in accordance with the relation (1) taking account of (for sample A) the contribution of inhomogeneous broadening of the Landau levels, are presented in Figs. 2b and 2d for the samples A and B. It is evident that all experimental points fit well on straight lines with slope angle tangent equal to 1. We note that attempts to describe the experimental results presented in Fig. 2b without taking account of the inhomogeneous broadening of the Landau levels have been unsuccessful.

For magnetic fields which are not strong, the points for sample B lie, in accordance with the relation (1), on a straight line passing through the value ln 4 on the abscissa in the limit $1/(\omega_c\tau)\to 0$. However, for strong magnetic fields $(1/(\omega_c\tau)<1.3)$ a small deviation of the points downward is observed; this is due to the decrease of the amplitude of the oscillations when spin (Zeeman) splitting occurs (see below).

The observation of spin splitting of the maxima (see Fig. 1) of the SdH oscillations makes it possible to determine the magnitude of the effective Landé g^* factor in the systems studied. A calculation of the magnitude of the impurity broadening of the Landau levels $\Gamma=\hbar/2\tau_g$ gives 6.8 and 3.6 K for samples A and B, respectively. The inequality $g^*\mu_B B\geqslant\Gamma$, where μ_B is the Bohr magneton, must be satisfied in order to observe spin splitting. Taking the equality in this expression, an approximate value can be obtained for g^*. For this, the value of the field B_2 corresponding to the minimum of the resistance on the experimental curves before the appearance of any indications of spin splitting must be used for B. This estimate gave $g^*=4.62$ and 14, respectively, for samples A and B (the field 0.41 T at which the experimental points are observed to deviate from the straight line constructed in accordance with the relation (1) is taken as B_2 for sample B).

Two methods were used to determine g^* more accurately. The first one is based on comparing the value of the magnetic field for which SdH oscillations appear with the value of the magnetic field at which spin splitting becomes noticeable. An indication of the latter is not so much the appearance of obvious splitting of the maxima but rather, as shown above, a decrease of their height as compared with the expected value (i.e., a change of the character of the magnetic field dependence of the amplitude of the ShD oscillations). The ShD oscillations appear in a magnetic field B_1 for which the splitting $\hbar\omega_c$ between neighboring Landau levels is greater than Γ. Similarly, the spin splitting of the ShD peaks appear in a magnetic field B_2 for which $g^*\mu_B B>\Gamma$. Therefore $\hbar eB_1/m^*=g^*\mu_B B_2$. Since $\mu_B=e\hbar/2m_0$, we obtain $g^*=2(B_1m_0/B_2m^*)$. Calculations using this relation gave g^*

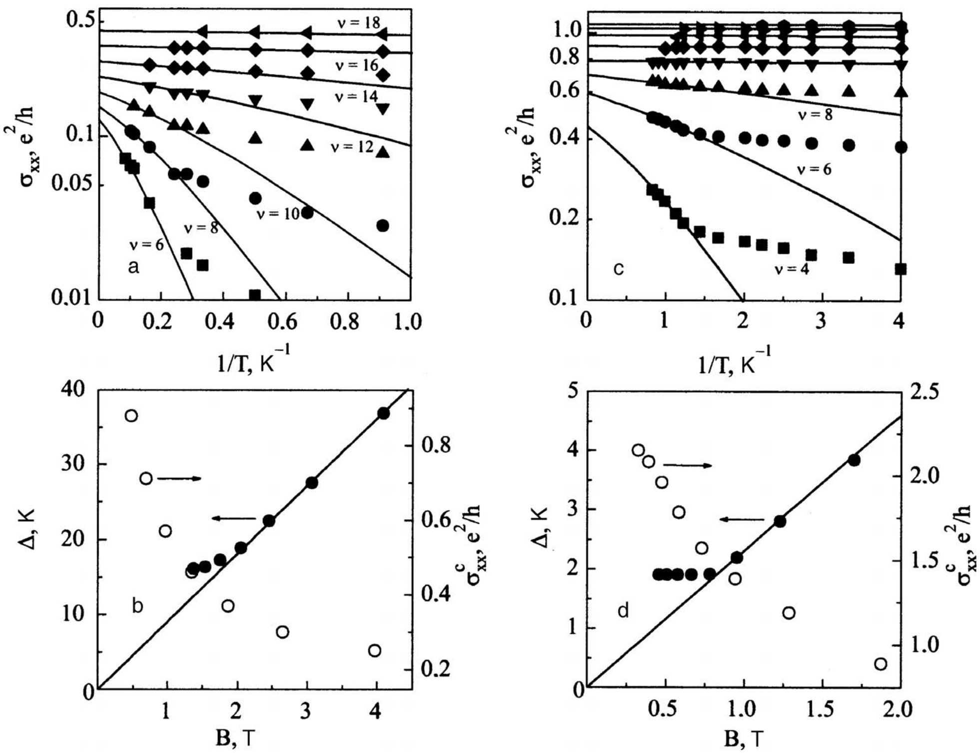

FIG. 3. Temperature dependence of the values of σ_{xx} that correspond to the minima of the resistivity in the traces of the SdH oscillations, for samples A (a) and B (c) and different values of ν. Solid lines—calculation according to Ref. 9. Magnetic-field dependences of the variation of the energy gap width Δ and the constant σ_{xx}^c for different values of ν for samples A (b) and B (d). The solid lines correspond to the relation $\Delta=\hbar\omega_c-g^*\mu_B B$.

$=4.31$ and 8.32 for the samples A and B, respectively.

The second method for determining g^* is based on the assumption that the splitting of the Landau levels engenders a contribution of activation processes, associated with transitions between Landau levels taking account of the spin splitting, to the temperature dependence of the conductivity.[8,9] To describe this contribution with integral values of the filling factor ν we used the relation $\sigma_{xx}(T) =\sigma_{xx}^c/[1+\exp(\Delta/2k_BT)]$, where $\sigma_{xx}^c=\sigma_{xx}(1/T=0)$.[9] Analysis of the experimental data using this relation makes it possible to determine the energy gap $\Delta=\hbar\omega_c-g^*\mu_B B$ for different values of ν (and, therefore, different values of the magnetic field). Figure 3a displays the experimental curves of σ_{xx} versus $1/T$ on a logarithmic scale for the experimental samples and the fit of the relation presented above to these curves for different values of ν. Such a fit for both samples was found to be successful for small values of ν only at relatively "high" temperatures for this sample (just as in Ref. 8). Figures 3b and 3d display the curves of Δ and σ_{xx}^c obtained versus the magnetic field. The values of Δ are present with the impurity broadening Γ of the Landau levels subtracted out, owing to which the dependences $\Delta(B)$ are straight lines emanating from zero. The value of g^* can be calculated from the slope of these straight lines. The results are $g^*=4.3$ and 8.3 for the samples A and B, respectively. The fact that the characteristic values of σ_{xx}^c are different for the samples studied attracts our attention: for sample A these values lie in the range $e^2\hbar\leqslant 1$, while for the sample B the values range from $1.4e^2/\hbar$ to $2e^2/\hbar$. Theoretically,[10,11] $\sigma_{xx}^c=1e^2/\hbar$ should be expected in the case where carriers are scattered by a short-range potential,[10] while for scattering by a long-range potential,[11] characteristically, $\sigma_{xx}^c=2e^2/\hbar$. The results obtained for the experimental samples (Figs. 3b and 3d) make it possible to conclude that the scattering by the nonuniformities of the boundaries of the quantum well, as found by determining the effective mass, predominates in sample A prepared by low-temperature plasma deposition, while scattering of holes by the potential of the impurity atoms in the doped layer lying far from the channel predominates in sample B prepared by molecular-beam epitaxy.

In conclusion we note that the values found for the effective mass of the mobile charge carriers and the effective g factor in the experimental samples turned out to be different because of the difference of the composition of the quantum wells. In addition, we note that they differ from the corresponding characteristics in bulk silicon and germanium crystals because the carriers comprise a two-dimensional hole gas. All other characteristics which were found—the concentration and mobility of the carriers and the transport and quantum relaxation times, information about the structure of the boundaries of the quantum wells, and so forth—reflect the specific structural features of the quantum wells studied and their fabrication technology.

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[1)]We take the amplitude of the oscillations to mean the deviation of the resistivity at the maximum or minimum from the monotonic variation of the average value of the resistivity.

[2)]For a real situation, it is sufficient to use the harmonic with $s=1$ in the relation (1).

[1]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 37 (1982).
[2]F. Schaffler, Semicond. Sci. Technol. **12**, 1515 (1997).

[3]Yu. F. Komnik, V. V. Andrievskiĭ, I. B. Berkutov, S. S. Kryachko, M. Mironov, and T. E. Voll, Fiz. Nizk. Temp. **26**, 829 (2000) [Low Temp. Phys. **26**, 609 (2000)].
[4]I. B. Berkutov, Yu. F. Komnik, V. V. Andrievskiĭ, O. A. Moronov, M. Nyronov, and D. R. Leadley, Fiz. Nizk. Temp. **32**, 109 (2006) [Low Temp. Phys. **32**, 82 (2006)].
[5]T. Ando, J. Phys. Soc. Jpn. **37**, 1233 (1974).
[6]A. Ishihara and L. Smrika, J. Phys. C **19**, 6777 (1986).
[7]S. D. Bystrov, A. M. Kreshchuk, Le Taun, S. V. Novikov, T. A. Polyanskaya, I. G. Savel'ev, and A. Ya. Shik, Fiz. Tekh. Poluprovodn. (St. Petersburg) **28**, 91 (1994) [Semiconductors **28**, 55 (1994)].
[8]R. B. Dunford, D. Popovich, F. H. Pollak, and T. F. Noble, J. Appl. Phys. **83**, 3144 (1998).
[9]V. N. Zverev, M. Muhammad, S. Rahman, and P. Debray, J. Appl. Phys. **96**, 6353 (2004).
[10]D. G. Polyakov and B. I. Shklovskii, Phys. Rev. Lett. **73**, 1150 (1994).
[11]D. G. Polyakov and B. I. Shklovskii, Phys. Rev. Lett. **74**, 150 (1995).

Translated by M. E. Alferieff