Weak localization and charge-carrier interaction effects in a two-dimensional hole gas in a germanium quantum well in a SiGe/Ge/SiGe heterostructure

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The weak localization and interaction effects for charge carriers in a two-dimensional hole gas in a pure germanium quantum well in a SiGe/Ge/SiGe heterostructure with a hole density of 5.68×10^{11} cm^{-2} and mobility of 4.68×10^4 cm^2 V^{-1} s^{-1} are investigated. The resistance measurements were made at temperatures from 46 mK to 10 K and magnetic fields up to 15 T. The magnetic-field dependence of the resistivity exhibits Shubnikov-de Haas oscillations and quantum Hall effect steps. At very low magnetic fields ($B < 0.1$ T) a weak localization effect for holes is revealed, which makes for a negative magnetoresistance and growth of the resistance with decreasing temperature (at $T < 2$ K). The manifestation of the interaction effect is observed and analyzed over a wide range of temperatures and magnetic fields. With increasing temperature the manifestation of the interaction-induced quantum correction passes from the diffusive regime to an intermediate and then to the ballistic regime. In all regions the behavior of the interaction quantum correction is in good agreement with the modern theoretical predictions. *© 2006 American Institute of Physics*. [DOI: 10.1063/1.2216282]

I. INTRODUCTION

It is known that the electrical conductivity of a weakly disordered two-dimensional electron gas is given by the classical value of the conductivity plus two types of quantum corrections to the conductivity: a correction due to the weak localization effect for electrons,[1,2] and corrections due to electron-electron interaction.[3–5] In the low-temperature region these quantum corrections make for a peculiar variation of the electrical resistance of a two-dimensional electron system upon changes of the temperature and magnetic field. A theory of these effects developed successfully two and a half decades ago[1–6] considered a diffusive regime of electron relaxation and interaction. That theory made it possible not only to give an interpretation of the anomalous behavior of the low-temperature resistance of two-dimensional electron systems but also yielded information about the relaxation times of the electron phase and spin and about the interaction parameters. The main objects of study were thin films of metals[7,8] and later electrons in inversion layers, δ layers, and heterostructures in semiconductors.

For the last decade there has been interest in electronic phenomena in Si-MOSFET structures and heterojunctions with low carrier density and high mobility—in particular, the presence of metallic behavior of the conductivity and a metal-insulator transition upon change of the carrier density.[9] These phenomena require a deeper theoretical treatment of the spin-orbit effects in such structures[10] and also the construction of a theory of the interaction effects in the ballistic regime.[11,12]

The diffusive regime corresponds to the condition $k_B T\tau/\hbar < 1$, which physically presupposes[13] that the effective interaction time $\hbar/k_B T$ is longer than the momentum relaxation time τ and, hence, two interacting electrons undergo scattering on many impurities. The ballistic regime corresponds to the condition $k_B T\tau/\hbar > 1$; in this case one is considering the interaction of two electrons scattering on isolated impurities. In the ballistic regime the temperature behavior of the interaction correction changes.

In the present study we investigated a heterostructure based on SiGe with a quantum well of pure germanium with hole-type conductivity. By varying the temperature in that heterostructure we were able to study the transition from the diffusive to the intermediate and then to the ballistic regime of interaction of the charge carriers.

The heterostructure was fabricated by low-energy plasma deposition.[14] It had a quantum well width of 15 nm. The Ge layer was bordered on both sides by layers of $Si_{0.3}Ge_{0.7}$. The thickness of the spacer separating the well

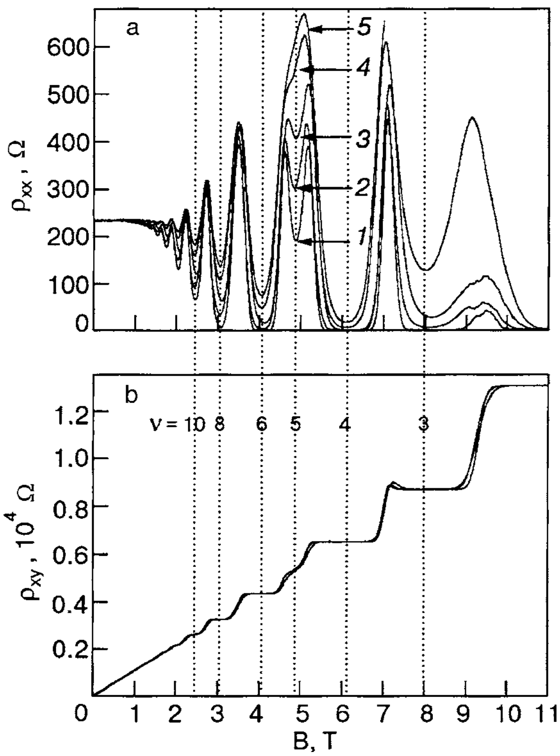

FIG. 1. Magnetic-field curves of the resistivity ρ_{xx} (a) and ρ_{xy} (b) at different temperatures: 52 mK (*1*), 0.5 K (*2*), 0.9 K (*3*), 2 K (*4*), 3 K (*5*).

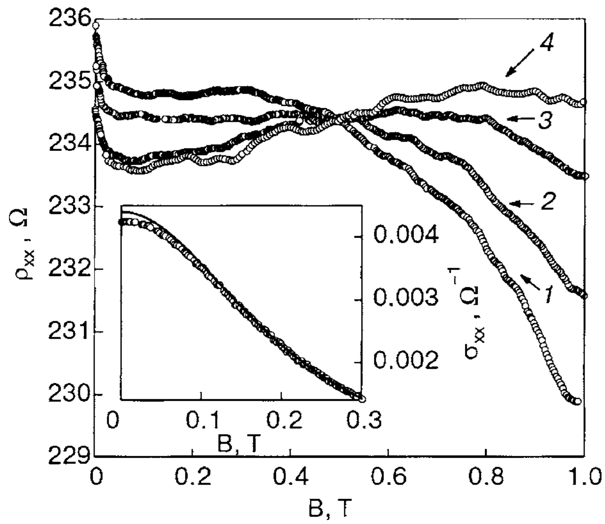

FIG. 2. Magnetic-field variation of the resistivity $\rho_{xx}(B)$ at temperatures of 46 mK (*1*), 0.5 K (*2*), 1.1 K (*3*), and 2 K (*4*). The inset shows an example of separation of the Drude contribution to the conductivity at a temperature of 46 mK; the solid curve is a calculation according to Eq. (3).

from the boron-doped $Si_{0.3}Ge_{0.7}$ layer was 10 nm. The density of carriers (holes) in the structure studied was 5.68×10^{11} cm^{-2}, and the mobility reached 4.68×10^4 cm^2 V^{-1} s^{-1}. The hole effective mass obtained by analysis of the Shubnikov-de Haas oscillations is $m^* = 0.112 m_0$ (m_0 is the free electron mass). The transport scattering time τ reaches the value 3×10^{-12} s corresponding to $k_B T \tau / \hbar = 1$ at $T = 2.55$ K.

Measurements of the diagonal R_{xx} and off-diagonal R_{xy} components of the resistance as a function of magnetic field were made at values up to 15 T in the temperature interval 46 mK–10 K. Figure 1 shows the form of the $\rho_{xx}(B)$ and $\rho_{xy}(B)$ curves for different temperatures. (The symbol ρ denotes the resistance per square of the two-dimensional electron system.) The curves shown demonstrate pronounced Shubnikov-de Haas oscillations (at $B \geqslant 1$ T) and also the quantum Hall effect (at $B \geqslant 2$ T). In the low-field region the curves of the magnetoresistance for different temperatures below ~2.5K intersect at a single point at $B \sim 0.5$ T and a value $\rho_{xx}^0 \cong 234.5$ Ω (Fig. 2). That value of the magnetic field essentially separates the low-field region from the region where magnetic quantization is manifested. The condition $\omega_c \tau = 1$ (ω_c is the cyclotron frequency) is satisfied at $B = 0.212$ T, i.e., somewhat lower than 0.5 T. In the low-field region ($B < 0.1$ T) a manifestation of the weak localization effect for holes is revealed; at fields below 0.3 T the interaction effects are clearly present.

II. DIFFUSIVE REGIME OF MANIFESTATION OF QUANTUM CORRECTIONS

In the region of very low magnetic fields $B < 0.1$ T the initial part of the curve of the resistance of our heterostructure versus magnetic field demonstrates negative magnetoresistance (see Fig. 2), which is evidence of a weak localization effect on $\rho_{xx}(B)$.

In the diffusive regime the quantum correction due to the weak localization for a two-dimensional system in a perpendicular magnetic field has the form[4]

$$\Delta\sigma_{xx}^{WL} = \frac{e^2}{2\pi^2\hbar}\left[\frac{3}{2} f_2\left(\frac{4eDB}{\hbar}\tau_\varphi^*\right) - \frac{1}{2} f_2\left(\frac{4eDB}{\hbar}\tau_\varphi\right)\right]. \qquad (1)$$

where $f_2(x) = \ln(x) + \Psi(1/x + 1/2)$, Ψ is the logarithmic derivative of the Γ function, τ_φ is the phase relaxation time of the electrons, $(\tau_\varphi^*)^{-1} = \tau_\varphi^{-1} + 4/3\tau_{so}^{-1}$, τ_{so} is the spin-orbit interaction time for elastic scattering of electrons, and D is the electron diffusion coefficient. This last can be estimated from the Einstein formula $D = 1/2 \cdot (v_F^2 \tau)$, where v_F is the Fermi velocity, which can be found from the formula $v_F = (h/m^*) \times (2\pi n)^{1/2}$ for a two-dimensional system (n is the carrier density). To go from the measured values of the resistance to the quantum corrections to the conductivity one can use the relation $-\Delta\sigma_{xx} = [\rho_{xx}(B) - \rho_{xx}^0(0)]/\rho_{xx}(B)\rho_{xx}^0$.

The terms in expression (1) reflect the spin states of the electrons on the conjugate trajectories forming the interference contribution to the conductivity. The first term corresponds to the triplet spin state (total spin $j=1$) and the second, to the singlet state ($j=0$). The triplet state is characterized by three possible values of the projection of the total moment ($M = 0, \pm 1$), which, as a result of the spin-orbit scattering, varies in a random manner. The spin-orbit scattering suppresses the coherence of the electron states in the triplet channel. The triplet term forms a negative magnetoresistance. The singlet term appears in expression (1) with a minus sign, and in the case $\tau_\varphi \gg \tau_{so}$ (strong spin-orbit interaction), when the singlet term is dominant in Eq. (1), the localization correction gives an anomalous positive magnetoresistance.

The negative magnetoresistance of the heterostructure studied gives way in a small field interval to growth of the resistance (see Fig. 2), which is due to the influence of the magnetic-field-induced change of the classical (Drude) resistance,

$$\sigma_{xx}(B) = \sigma_{xx}^{D}(B) + \Delta\sigma_{xx}^{WL}(B) + \Delta\sigma_{xx}^{EEI}, \tag{2}$$

where

$$\sigma_{xx}^{D}(B) = \frac{\sigma_0}{1 + (\omega_c \tau)^2}. \tag{3}$$

The superscript WL corresponds to the weak localization of electrons, while EEI stands for the electron-electron interaction; $\omega_c = eB/m^*$ is the cyclotron frequency. In expression (3) instead of $\omega_c\tau$ one can use the product μB (μ is the mobility) and match the corresponding curve with the experimental dependence of $\sigma_{xx}(B)$ in the logarithmic saturation region of the function (1). An example of the separating out of the classical contribution to the magnetic-field-induced change of the conductivity of the heterostructure is given in the inset of Fig. 2.

With the goal of determining the values of the characteristic relaxation times τ_φ and τ_{so} the isolated localization correction to the conductivity can be analyzed in accordance with Eq. (1). However, as was mentioned in Ref. 15, in Si, Ge, and III–V semiconductors and in heterostructures based on them the valence band forms on account of a strong spin-orbit interaction, and the total moment turns out to be coupled to the quasimomentum of the particle. As a result, the spin and momentum relaxation times turn out to be of the same order. Furthermore, for heterostructures characterized by the existence of an internal potential gradient, the spin-orbit processes occur differently in the directions perpendicular to and parallel to the heterojunction. For this reason, in calculating the corrections due to the weak localization of holes in the system under study, we employed the theoretical model constructed in Ref. 15 to treat undeformed and deformed bulk p-type semiconductors and quantum-well structures based on them. According to that theoretical model, the magnetic-field dependence of the localization correction to the conductivity is described by the following expression:

$$\Delta\sigma_{xx}^{WL}(B) = \frac{D_{ij}^0}{D_a^0} G_0 \left[f_2\left(\frac{4eDB}{\hbar} \frac{\tau_\varphi \tau_\parallel}{\tau_\varphi + \tau_\parallel} \right) + \\ - \frac{1}{2} f_2\left(\frac{4eDB}{\hbar} \frac{\tau_\varphi \tau_-}{\tau_\varphi + \tau_\perp} \right) - \frac{1}{2} f_2\left(\frac{4eDB}{\hbar} \tau_\varphi \right) \right], \tag{4}$$

where $G_0 = e^2/(2\pi^2\hbar)$; $\tau_\parallel$ and $\tau_\perp$ are, respectively, the longitudinal and transverse spin relaxation times; the role of the preferred axis here is played by the normal to the plane of the quantum well; the ratio D_{ij}^0/D_a^0 characterizes the relative values of the components of the diffusion coefficient; in the calculations it was taken equal to 1. We carried out a numerical description of the experimental data using relation (4), taking τ_φ, $\tau_\parallel$, and $\tau_\perp$ as fitting parameters. Examples of such a description are presented in Fig. 3.

The value found for the phase relaxation time varies smoothly from 1.1×10^{-11} s at a temperature of 52 mK to 3.5×10^{-12} s at 1.1 K. This temperature variation of τ_φ can be approximated by a dependence $\tau_\varphi \sim T^{-0.45}$, analogous to that found[16] for heterostructures with quantum

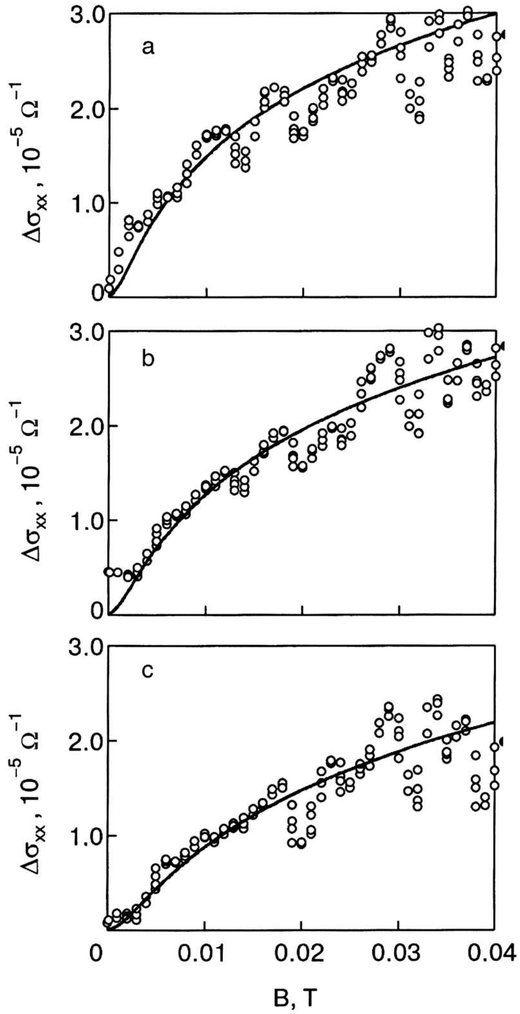

FIG. 3. Magnetic-field variation of the localization correction to the conductivity $\Delta\sigma_{xx}^{WL}$ at temperatures of 52 mK (a), 0.2 K (b), and 1.1 K (c).

wells with the consituents $Si_{0.2}Ge_{0.8}$ and $Si_{0.05}Ge_{0.95}$. The reason for a temperature dependence of the form $\tau_\varphi \propto T^{-1/2}$ instead of the dependence $\tau_\varphi \propto T^{-1}$ expected[17] for an electron-electron interaction in a two-dimensional system is not clear. The longitudinal and transverse spin relaxation times are independent of temperature and equal to $\tau_\parallel = 1.6 \times 10^{-11}$ s and $\tau_\perp = 3 \times 10^{-11}$ s. The values of these times are somewhat greater than the value of τ_φ over the whole temperature interval investigated, making for a negative magnetoresistance due to the weak localization effect.

The temperature dependence of the resistance ρ_{xx} in the temperature region where the diffusive and intermediate regimes of interaction take place depends substantially on the value of the magnetic field (Fig. 4). At $B=0$ T the resistivity falls off with increasing temperature. In a magnetic field the form of the dependence $\rho_{xx}(T)$ is transformed, and at $B \geqslant 0.5$ T the temperature coefficient of resistance changes sign. Significantly, all of the curves cross in a single region at $T \sim 2.0-2.5$ K with a resistance value $\rho_{xx}^0 \sim 234\ \Omega$. The characteristic temperature $T=2.55$ K corresponds to a transition from the diffusive to the ballistic regime.

The dependence $\rho(T)$ in the diffusive region in the absence of magnetic field is governed by the contribution of the weak localization and interaction effects. The temperature

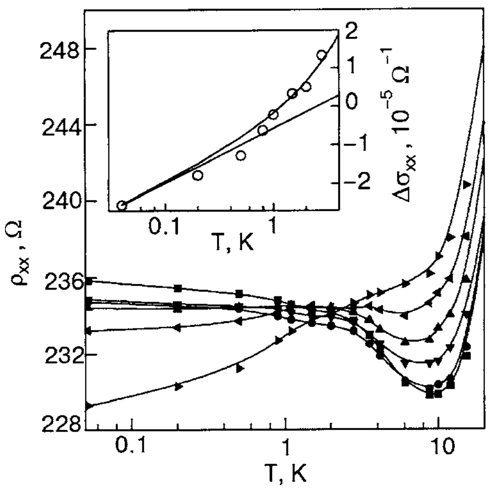

FIG. 4. Temperature dependence of the resistivity ρ_{xx} in magnetic fields [T]: 0 (■), 0.1 (●), 0.3 (▼), 0.5 (▲), 0.7 T (◀), 1 (▶). The inset shows the temperature dependence of the quantum correction to the conductivity, $\Delta\sigma_{xx}$, at $B=0$ T. The dashed curve is a calculation of the localization correction $\Delta\sigma_{xx}^{WL}$ according to Eq. (5), and the solid curve represents the sum $\Delta\sigma_{xx}^{WL}+\Delta\sigma_{xx}^{EEI}$.

dependence of the localization correction to the conductivity for a 2D electron system in the case of weak spin-orbit scattering ($\tau_\varphi<\tau_{so}$) has the following form:[2,4]

$$\Delta\sigma_{xx}^{WL}(T)=\frac{e^2}{2\pi^2\hbar}\rho\ln\left(\frac{k_BT\tau}{\hbar}\right), \tag{5}$$

where p is the exponent in the temperature dependence of the inelastic scattering time, $\tau_\varphi\propto T^{-p}$ (in the case of strong spin-orbit scattering an additional factor of $-1/2$ appears in expression (5)). Since $k_BT\tau/\hbar<1$, the correction $\Delta\sigma_{xx}^{WL}$ is negative and increases in absolute value with decreasing temperature.

The temperature dependence of the interaction correction for a 2D electron system in the diffusive regime ($k_BT\tau/\hbar<1$) has an analogous form:[5]

$$\Delta\sigma_{xx}^{EEI}(T)=\frac{e^2}{2\pi^2\hbar}\lambda\ln\left(\frac{k_BT\tau}{\hbar}\right), \tag{6}$$

where λ is the interaction constant.

For illustration of the contributions $\Delta\sigma_{xx}^{WO}$ and $\Delta\sigma_{xx}^{EEI}$ to the dependence $\rho(T)$ below $T=2.5$K, in the inset in Fig. 4 we show the experimental values of the correction $\Delta\sigma_{xx}$ (dots), the temperature dependence of the localization correction $\Delta\sigma_{xx}^{WO}$ calculated according to Eq. (5) for the value $p=1/2$ (dashed line), and the total correction $\Delta\sigma_{xx}^{WL}+\Delta\sigma_{xx}^{EEI}$ (solid curve). For calculation of the interaction correction we used a dependence $\lambda=\{1+3[1-(\ln(1+F_0^\sigma))/F_0^\sigma]\}$ (Ref. 11), where F_0^σ is the Fermi-liquid interaction constant in the triplet channel (see below). We took the value $F_0^\sigma=-0.228$, which ensures agreement of the calculation and the experimental points up to a temperature ~2 K. It is seen for the inset in Fig. 4 that in the temperature region corresponding to the diffusive regime the weak localization effect plays a noticeable role in addition to the interaction effect, but its contribution to the behavior of the resistivity upon variation of the temperature and magnetic field vanishes completely at $T\cong 2.5$ K (Fig. 4) and $B\simeq 0.1$ T (Fig. 2).

We note that in the calculation of all the conductivity corrections shown in the inset in Fig. 4, as a point of reference we have taken the value of the conductivity corresponding to a resistivity $\rho_{xx}^0=234.5$ Ω at 2 K. In reality the interaction correction exists over a wide temperature range and, remaining negative, decreases in absolute value with increasing temperature; this makes for a decrease of the resistivity to a minimum at ~10 K. The reference point for determining the interaction correction should most likely be a value of the conductivity corresponding to a lower resistivity than the minimum at ~10 K, since with increasing magnetic field the resistivity at the lowest experimental temperature is lower than the resistivity at ~10 K (see Fig. 4). The choice of the reference level for the correction does not affect its functional dependence on the variable parameter (temperature, magnetic field) but requires only a shift of the calculated values of the correction by a certain amount for comparison with the experimental data. It is also clearly seen in Fig. 2 that in the region where the interaction-induced correction is manifested (at $B>0.5$ T) the resistivity decreases with increasing magnetic field on account of the decrease of the absolute value of the negative correction to the conductivity. The magnetic field destroys the quantum interference in the carrier interaction effect. This is responsible for the negative magnetoresistance in the indicated field region.

III. TRANSITION TO THE BALLISTIC REGIME OF MANIFESTATION OF THE INTERACTION CORRECTION

The interaction correction leads to magnetic-field dependence of the resistance because in the conversion of the conductivity tensor to the resistivity tensor the correction to the resistance acquires a factor of $-1(1-(\omega_c\tau)^2)$.[18–20] In this conversion we have taken into account the fact that in the diffusive regime the correction to the Hall conductivity can be neglected. Ultimately the resistance is described by the expression

$$\rho_{xx}(B,T)=\frac{1}{\sigma_0}-\frac{1}{\sigma_0^2}[1-(\omega_c\tau)^2]\Delta\sigma_{xx}^{EEI}(T). \tag{7}$$

At $\omega_c\tau\gg 1$ the relation (7) takes the form

$$\frac{\rho_{xx}(B,T)-\rho_0}{\rho_0}=\frac{1}{\sigma_0}\mu^2B^2\Delta\sigma_{xx}^{EEI}(T), \tag{8}$$

from which it is seen that the magnetoresistance is determined by the value and sign of the interaction correction to the conductivity. In the diffusive regime the correction to the conductivity is negative, since the argument of the logarithm is $k_BT\tau/\hbar<1$. Relation (7) makes for a negative magnetoresistance that is quadratic in the field. This was first demonstrated successfully by the authors of the experimental study reported in Ref. 21 (see also Refs. 13, 19, and 20).

In our heterostructure the negative quadratic magnetoresistance is observed in a wide interval of magnetic fields (see the inset in Fig. 5) with the onset of Shubnikov-de Haas oscillations this dependence describes the trend of the monotonic component (the geometric locus of midpoints between adjacent maxima and minima). We note that the contribution of the magnetic-induced change in the classical resistance is absent in the magnetoquantum region. The negative quadratic magnetoresistance is observed not only at $T<2.5$ K

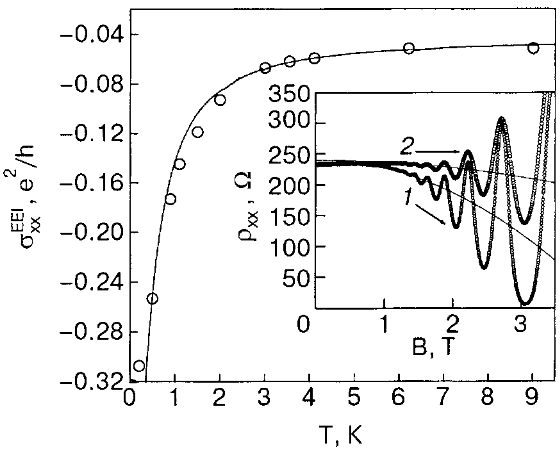

FIG. 5. Variation of the interaction correction with increasing temperature (up to 9 K). The solid curve is the calculation according to the theory of Ref. 12. The inset shows an example of separating out the interaction correction to the conductivity at temperatures of 52 mK (*1*) and 3 K (*2*). The solid curves were calculated according to Eq. (7).

but also at higher temperatures (up to 10 K). It would seem that when relation (6) is used under conditions $k_BT\tau/\hbar>1$, the correction should become positive. This contradiction was resolved recently by a new theory of the interaction effects in the ballistic regime.[11,12]

The scattering on point impurities with a short-range potential was considered in Ref. 11, and it was shown that in a comparatively wide interval of variation of the inequality $k_BT\tau/\hbar>1$ the interaction correction gives a negative magnetoresistance, but the character of its temperature dependence changes. The authors of Ref. 11 showed that in the ballistic regime the interaction correction appears as a result of the same physical causes as in the diffusive regime—the interference of electron waves in the scattering on the impurity and on the impurity-formed Friedel oscillations of the electron density.

The theory of the interaction, as is well known, includes the contributions of the exchange interaction (the Fock term) and the direct interaction (the Hartree term). The Fock contribution forms a singlet interaction channel; together with the singlet part of the Hartree contribution, it forms a "charge" singlet channel. The Hartree contribution forms a triplet interaction channel. In the theory of Ref. 11 it is shown that because of the competition between the two types of contributions, the magnitude and even the sign of the variation of the correction with temperature can change. The overall picture of the interaction can be reduced to a triplet channel in which the interaction is characterized by a Fermi-liquid constant F_0^σ that reflects the intensity of the spin-exchange interaction. For $F_0^\sigma<0$ the interaction tends to align the spins in one direction, while at the value $F_0^\sigma=-1$ a Stoner ferromagnetic instability is reached.

A general theory of the interaction correction, applicable to the diffusive, intermediate, and ballistic regimes, is given in Ref. 12. In that paper the authors consider the magnetoresistance of a two-dimensional electron system in a strong $(\omega_c\tau>1)$ transverse magnetic field. The theory is constructed for the case of electron scattering on a point (short-range) potential and for the case of Coulomb interaction with a scatterer. In the first case the Fock exchange contribution to the interaction correction is determined by the function $G_0(k_BT\tau/\hbar)$, which has the asymptotic forms $G_0(x)\approx-\ln x-1.7$ for $x\ll1$ and $G_0(x)\approx0.276\cdot x^{-1/2}$ for $x>1$. The correction due to this contribution is negative, and it would make for a negative magnetoresistance, but when the Hartree contribution is taken into account the situation changes. The Hartree contribution is asymptotically analogous to G_0 but has the opposite sign and is twice as large in magnitude. In the case of Coulomb interaction of electrons with scatterers the Fock contribution to the interaction correction is described by the function $G_F(k_{BT\tau}/\hbar)$, which has the asymptotic forms $G_F(x)\approx-\ln x-1.6$ at $x\ll1$ and $G_F(x)\approx0.138x^{-1/2}$ at $x>1$, and it leads to negative magnetoresistance. The Hartree contribution G_H, the functional form of which is determined by the constant F_0^σ and is similar to G_F but with the opposite sign in the case $\kappa\ll k_F$ (κ is the inverse screening length), can, in combination with the exchange contribution, change the sign of the correction. The theory predicts the appearance of positive magnetoresistance at $T>T_H\sim(k_F/\kappa)^2(\hbar/k_B)\tau^{-1}$, the fact that at high temperatures the interaction effectively corresponds to the case of scattering on a point potential, and in this case the Hartree contribution exceeds the exchange contribution and gives a positive sign of the total correction and, accordingly, of the magnetoresistance. If κ/k_F is not small, then the Hartree contribution is subjected to a strong Fermi-liquid renormalization, and in a simplified picture it is similar to G_F but has a coefficient of 3 and also an additional factor of $F_0^\sigma/(1+F_0^\sigma)$ multiplying the Bessel functions appearing in G_F.

In comparing the experimental data obtained here with the theory of Ref. 12, one should apparently give preference to the case of Coulomb interaction of the holes with scattering centers, since the quantum well in the heterostructure does not contain impurity atoms, the boron acceptor atoms being located in a layer separated from the quantum well by an impurity-free spacer layer 10 nm thick. An estimate of the ratio κ/k_F can be made on the basis of the fact that the transition from negative to positive magnetoresistance occurs in the 10 K region. This means that the value of $k_BT\tau/\hbar$ is approximately equal to 4 and, hence, $k_F/\kappa\approx2$. Thus the ratio $\kappa/k_F\approx0.5$ is not small. According to Ref. 12, the relative change of the resistance in a magnetic field is described by the relation

$$\frac{\Delta\rho(B)}{\rho^0}=-\frac{(\omega_c\tau)^2}{\pi k_F l}\left|G_F(k_BT\tau/\hbar)-G_H(k_BT\tau/\hbar{:}F_0^\sigma)\right|. \qquad (9)$$

The analytical form of the functions $G_F(k_{BT\tau}/\hbar)$ and $G_H(k_BT\tau/\hbar,F_0^\sigma)$ is given in Ref. 12. We constructed the functional form of the total interaction correction in accordance with the aforementioned features of the use of the theory of Ref. 12 in the different cases and made a comparison with the experimental data (Fig. 5). The experimental values of the correction $\Delta\sigma_{xx}^{EEI}$ were obtained from relation (7). We note that when the temperature is raised from 52 mK to 9 K the value of the resistance in zero magnetic field, ρ_{xx}^0, which is the point of reference for the corresponding correction, changes from 245 to 238 Ω. In fitting the theoretical dependence $\Delta\sigma_{\text{th}}(T)$ with the temperature dependence of the correction $\Delta\sigma_{xx}$ found from the experimental data, the fitting parameter used is the interaction constant in the triplet channel, F_0^σ. A successful description of the experimental data (see Fig. 5) was obtained at a value $F_0^\sigma = -0.228$, but with a slight shift of the calculated curve by a constant amount $-0.045 \cdot e^2/h$. This shift refines the resistance level from which the correction is reckoned. An analogous comparison of the experimental data for $\Delta\sigma_{xx}$ with the theory of Ref. 12 for GaAs/AlGaAs heterostructures with n-type conductivity was carried out in Ref. 13, but only the exchange function G_F was used as the theoretical dependence. For coincidence of the experimental and theoretical curves a shift of $-0.07 e^2/h$ was necessary.

The value found for F_0^σ is close to the value obtained from the formula $F_0^\sigma = -\frac{1}{2} r_s/(r_s + \sqrt{2})$.[11] The parameter r_s characterizing the ratio of the Coulomb interaction energy to the kinetic energy can be obtained from the formula $r_s = 1/(\pi n)^{1/2} a_B$, where $a_B = (\hbar^2 \chi)/(me^2)$ is the Bohr radius, χ is the dielectric constant (for germanium $\chi = 15.4$). For the system investigated it turned out that $a_B = 72.3$ Å and $r_s = 1.024$, and the theoretical value $F_0^\sigma = -0.21$.

IV. CONCLUSION

We have studied the manifestation of quantum interference effects, weak localization and interaction of electrons, in a two-dimensional hole gas in a quantum well of pure germanium in a SiGe/Ge/SiGe heterostructure. This system exhibits Shubnikov-de Haas oscillations and the quantum Hall effect. Together with this, the monotonic component of the change of the resistance in a magnetic field demonstrates a manifestation of the weak localization effect in the region of very low magnetic fields and the interaction effect over a wide range of fields, including the region of pronounced magnetic quantization.

Strictly speaking, for the weak localization effect due to interference of electrons (or holes) on trajectories with self-crossing, it is desirable to satisfy the condition $L_H \geqslant l$, where l is the mean free path with respect to inelastic scattering, and $L_H = (\hbar/2eB)^{1/2}$ is the magnetic length, which corresponds to the value of the field at which an area equal to $2\pi L_H^2$ is penetrated by one quantum of magnetic flux, $\Phi_0 = h/2e$. It in fact turned out that the weak localization effect is manifested over a much broader range of magnetic fields than is implied by the condition given above; specifically, it is manifested all the way up to fields at which magnetic quantization effects begin to be manifested.

The interaction effects govern the trend of the monotonic component of the resistance in the region of magnetic fields both before and after the appearance of quantum oscillations. The quantum corrections to the conductivity due to the interaction of carriers lead to a negative magnetoresistance which is quadratic in the field. The behavior of the interaction-induced quantum correction is in good agreement with the predictions of the modern theory[11,12] for the diffusive, intermediate, and ballistic regimes; in particular, with increasing temperature one observes a transition from a logarithmic to a power-law dependence of the interaction correction on temperature and also a transition from negative to positive magnetoresistance.

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