LOW-DIMENSIONAL AND DISORDERED SYSTEMS

Features of the Shubnikov–de Haas oscillations of the conductivity of a high-mobility two-dimensional hole gas in a SiGe/Ge/SiGe quantum well

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The Shubnikov–de Haas oscillations in a two-dimensional hole gas in a quantum well of pure germanium in a SiGe/Ge/SiGe heterostructure with a hole concentration $p_H=5.68\times10^{11}$ cm^2 and mobility $\mu=4.68\times10^4$ cm^{2}V^{-1}s^{-1} are investigated in magnetic fields up to 15 T at temperatures from 40 mK to 4 K. The observed deviation from the known relation describing the conductivity oscillations in the Shubnikov–de Haas effect are explained by additional broadening of the Landau levels due to the existence of a nonuniform distribution of the concentration of charge carriers, and, accordingly, of their energy, in the plane of the two-dimensional gas. It is assumed that the latter is due to natural atomic-step variations of the well width. The effective hole mass ($m^*=0.112m_0$) is determined from the temperature dependence of the oscillation amplitude, and its dependence on magnetic field is used to determine the quantum scattering time and the value of the carrier concentration fluctuations. © *2006 American Institute of Physics*.
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The study of the Shubnikov–de Haas oscillations of the conductivity of a two-dimensional electron gas in semiconductor heterostructures can yield information about its characteristics (the concentration and effective mass of the of the charge carriers, their quantum scattering time, etc.). In addition, as will be shown below, the Shubnikov–de Haas oscillations permit one to draw conclusions about the structure of the quantum channel itself.

In the present study we investigate the Shubnikov–de Haas oscillations in a two-dimensional hole gas in a pure germanium quantum well in a SiGe/Ge/SiGe heterostructure. The heterostructure is obtained by low-energy plasma deposition (LEPECVD).[1] The quantum well is a thin layer of pure germanium 15 nm thick, sandwiched between two layers of $Si_{0.3}Ge_{0.7}$. A layer containing acceptor boron atoms is separated from the quantum well by a spacer 10 nm thick. The Hall concentration of holes in the structure studied was equal to $p_H=5.68\times10^{11}$ cm^{-2}, and the mobility $\mu=4.68\times10^4$ cm^{2}V^{-1}s^{-1}.

The experimental curves of the variation of the diagonal and off-diagonal components of the resistance of the structure in magnetic field exhibit pronounced Shubnikov–de Haas oscillations (Fig. 1) and the quantum Hall effect. The effective mass m^* and quantum scattering time τ_q are found from the variation of the oscillation amplitude ΔR with temperature and magnetic field. (By oscillation amplitude ΔR we mean the deviation of the resistance from the monotonic trend of the mean resistance R_0 at the maximum or minimum). The variation of the conductivity of a two-dimensional gas in the quantum region was considered theoretically in Refs. 2 and 3. According to the theory of Ref. 3, the variation of the resistance is described by the formula

$$\rho_{xx}=\frac{1}{\sigma_0}\left[1+4\sum_{s=1}^{\infty}\left(\frac{\Psi s}{\sinh\Psi s}\right)\exp\left(-\frac{\pi s}{\omega_c\tau_q}\right)\times\cos\left(\frac{2\pi s\varepsilon_F}{\hbar\omega_c}-\Phi\right)\right], \quad (1)$$

where $\Psi=2\pi^2k_BT/\hbar\omega_c$ determines the temperature and magnetic-field dependence of the oscillation amplitude, $\omega_c=eB/m^*$ is the cyclotron frequency, τ_q is the quantum (single-particle) relaxation time of the charge carriers, which

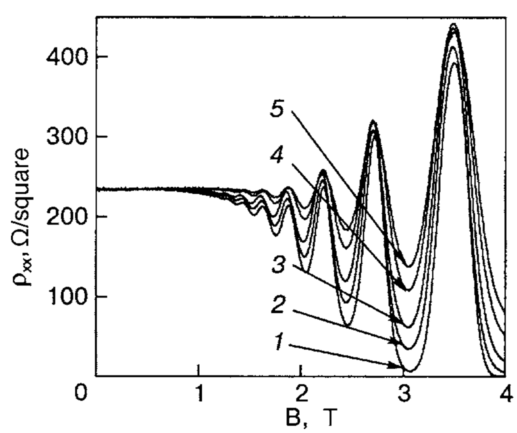

FIG. 1. Magnetic-field dependence of the resistance ρ_{xx} at $T=52$ mK (*1*), 0.5 K (*2*), 0.9 K (*3*), 2 K (*4*), and 3 K (*5*).

characterizes the collision width of the Landau levels, and Φ is the phase. The Fermi energy of a two-dimensional electron gas $\varepsilon_F=\pi\hbar^2 n/m^*$, where n is the concentration of electrons or holes. For the real situation it is sufficient to use the $s=1$ harmonic in formula (1). From the oscillation period in the inverse magnetic field one can determine the concentration of the carriers if their effective mass m^* is known.

For determining the effective mass we use the dependence of $\ln[(\Delta R/R_0)\sinh(\Psi)/\Psi]$ on $1/(\omega_c\tau)$ or $1/(\mu B)$, where μ is the carrier mobility (the quantity in the argument of the exponential function of the oscillatory term in Eq. (1) is replaced beforehand by $-\pi\alpha/(\omega_c\tau)$, $\alpha=\tau/\tau_q$, where τ is the transport relaxation time). It is seen from formula (1) that for such a construction the points corresponding to extrema with different quantum numbers ν must lie on a single straight line with a gradient of $\pi\alpha$. The effective mass m^* in this case is a fitting parameter that is adjusted so as to line up the points belonging to different temperatures on a single curve (see Fig. 2a). This was achieved at a value $m^*=0.112m_0$, where m_0 is the free electron mass.

However, the single curve obtained in Fig. 2a is not a straight line. The reasons for this lie both in the peculiarities of the given sample and also in the fact that we have plotted data over a wide range of magnetic fields. If one uses data in a restricted interval of magnetic fields, as is done in many papers, a linear approximation gives very different values of α in different intervals. For example, a straight line drawn through the group of points in the low magnetic-field region corresponds to a value $\alpha=19.7$, for the group of points in the intermediate field region $\alpha=14$, and for the group of points in the high field region $\alpha=10.6$. The nonuniqueness of such determinations is clearly seen in Fig. 2a.

The deviation of the experimental data from formula (1) is well seen in a plot of $\ln(\Delta R/R_0)$ versus $\ln(\Psi/\sinh\Psi)-(\pi\alpha/\omega_c\tau)$ (Fig. 2b, $\alpha=10.6$). In those coordinates one should obtain coincident straight lines with gradient equal to unity. It turned out that those curves for different magnetic fields exhibit complex behavior and do not coincide at small values of the arguments.

The behavior of the curve in Fig. 2a in extremely high magnetic fields hypothetically can be imagined by making use of the assertion of Refs. 4–6 that as $1/\omega_c$ goes to zero, the curve should approach the value ln 4, i.e., 1.386. Indeed, as $1/\omega_c\to 0$ the ratio $\Psi s/\sinh\Psi s$ approaches unity, and the oscillatory component in formula (1) has a coefficient of 4 (it was pointed out in Ref. 3 that an error had been made in Ref. 2, where this coefficient was given as 2). This means that the maximum value of the ratio of the oscillation amplitude to the mean value of the resistance is equal to 4. We note that the examples of linear approximations shown in Fig. 2a do not meet this requirement. For example, the straight line in the high-field region for $1/\omega_c\to 0$ tends toward a value of 2, which corresponds to $\Delta R/R_0=8$, while the straight line in the intermediate field region tends toward 3, which corresponds to $\Delta R/R_0=20$. The straight drawn through the group of points in the low-field region intercepts the vertical axis at a value of 4.7, which leads to the totally absurd value $\Delta R/R_0=110$. Thus a linear approximation of the dependence under discussion without taking into consideration the extrapolated value of the function for $1/\omega_c\to 0$ can lead to an erroneous

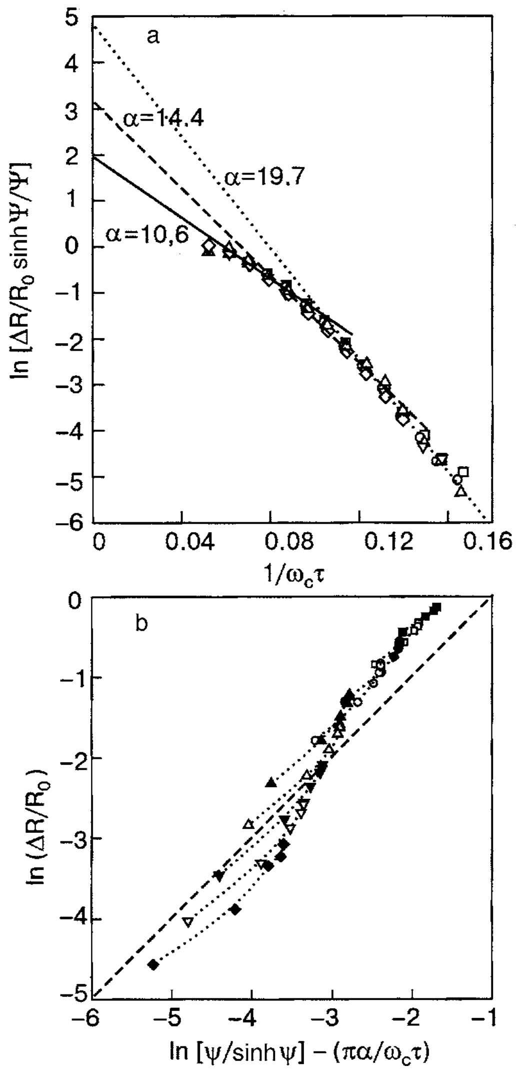

FIG. 2. Illustration of the numerical calculation of the parameters m^* and α for temperatures: 52 mK (□), 0.2 K (◯), 0.5 K (▽), 1.1 K (△), 2 K (◇), 3.55 (◁) (a) and magnetic fields B[T]: 3.48 (■), 3.06 (□), 2.72 (●), 2.45 (◯), 2.22 (▲), 2.02 (△), 1.88 (▼), 1.75 (▽), and 1.64 (◆) (b); the dashed line demonstrates a slope of 45°.

result. In Fig. 3a the experimental curve extrapolates to a value of ln 4 as $1/\omega_c\to 0$.

The nonlinearity of $\ln[(\Delta R/R_0)(\sinh\Psi/\Psi)]$ as a function of $1/(\omega_c\tau)$ in Fig. 2a and, hence, the deviation of the magnetic-field dependence of the oscillation amplitude from formula (1) is a symptom of the presence of a factor that affects the character of the Landau level broadening and, accordingly, the variation of the oscillation amplitude with magnetic field. The appearance of nonlinearity of $\ln(\Delta R)$ versus $1/B$ in certain cases (as a rule, in high-mobility systems with a two-dimensional gas of charge carriers) was pointed out in Refs. 7,8, for example. In Ref. 8 it was conjectured that this nonlinearity is due to spatial variation of the electron concentration and, hence, the Fermi energy, over the plane of existence of the two-dimensional gas. Because of this, in different regions of the sample the extrema of the oscillations on the magnetic-field scale do not coincide. In such a case the amplitude of the oscillations decreases in comparison with its value in a homogeneous sample, and this corresponds to additional effective Landau level broadening, which is called “inhomogeneous broadening.”

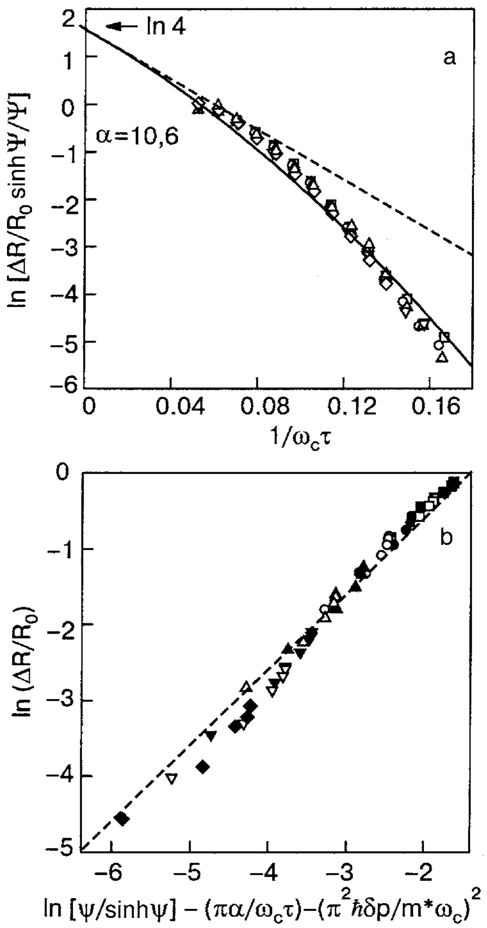

FIG. 3. Illustration of the procedure for numerical calculation of the parameters m^* and α with the use of the theoretical model of Ref. 8 for the temperatures: 52 mK (□), 0.2 K (◯), 0.5 K (▽), 1.1 K (△), 2 K (◇), 3.55 K (◁) (a); and magnetic fields B[T]: 3.48 (■), 3.06 (□), 2.72 (●), 2.45 (◯), 2.22 (▲), 2.02 (△), 1.88 (▼), 1.75 (▽), and 1.63 (◆). The dashed line in panel (a) was plotted according to relation (1), and the solid curve was calculated with allowance for an exponential factor that takes the "inhomogeneous broadening" of the Landau levels into account.[8] The dashed line in panel (b) demonstrates a slope of 45°.

The formation of Shubnikov–de Haas oscillations in the case when the potential, electron concentration, and Fermi energy undergo large-scale fluctuations, described by a Gaussian distribution, in the plane of the two-dimensional gas was investigated theoretically by Shik,[8] who showed that an additional exponential cofactor appears in the expression for the oscillation amplitude (1), with an exponent $-(\pi\delta\varepsilon_F/\hbar\omega_c)^2$ (the Shik term). Then the exponential cofactor in formula (1) becomes

$$\exp\left[-\frac{\pi}{\omega_c\tau_q}-\left(\frac{\pi^2\hbar\,\delta n}{m^*\omega_c}\right)^2\right]. \tag{2}$$

The first term in the argument of the exponential function describes collision broadening of the Landau levels and is inversely proportional to the magnetic field, while the second term takes the "homogeneous broadening" of the Landau levels into account and is inversely proportional to the square of the field. Thus the influence of the second term is diminished at very high magnetic fields. On the other hand, an increase of the carrier mobility increases the relative influence of the Shik term.

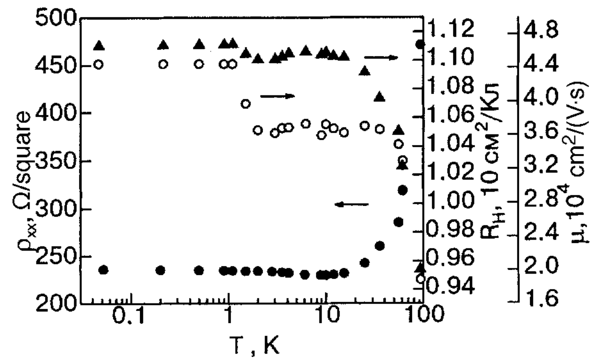

FIG. 4. Temperature dependence of the resistance ρ_{xx} (●), the Hall coefficient R_H (◯), and the mobility μ (▲).

Figure 3a demonstrates the possibility of describing the experimental curves of $\ln[(\Delta R/R_0)(\sinh\Psi/\Psi)]$ versus $1/(\omega_c\tau)$ by a second-degree polynomial of the form $Y=-a_1X-a_2X^2+\mathrm{const}$, in which, in accordance with formulas (1) and (2), $a_1=\pi\tau/\tau_q$, $a_2=(\pi^2\hbar\tau\delta p/m^*)^2$. In this approximation one obtains $\alpha=8.3$ from the linear term (the dashed curve in Fig. 3a), while the quadratic term gives a value of the concentration fluctuation $\delta p=2.8\times10^{10}$ cm^{-2}, which amounts to 4.9% of the mean carrier concentration $p_H=5.68\times10^{11}$ cm^{-2} obtained from measurements of the Hall coefficient. The strong difference of the quantum time τ_q from the transport scattering time τ, e.g., for $\alpha\geqslant10$, is usually attributed to a contribution to the scattering from the long-range potential of ionized impurities far away from the quantum well. In the case of scattering on impurities in the quantum well itself or on irregularities (roughness) of the heterointerface, the difference of τ_q from τ is small, i.e., $\alpha\sim1$. The value found for α indicates that the scattering of holes on the long-range potential is predominant in our heterostructure, but that does not rule out a contribution to the scattering on rare natural irregularities of the interface.

The successful use of the concepts of "inhomogeneous broadening" of the Landau levels for describing the magnetic-field dependence of the oscillation amplitude is attested by Fig. 3b, which presents curves similar to those in Fig. 2b but with the Shik term included: all the points for different temperatures and magnetic fields lie around a single straight line with a gradient close to unity.

Here it is pertinent to consider the experimental variation of the carrier concentration as the temperature is increased in the region 1–2 K. The striking and nontrivial effect is that the Hall coefficient, which remains constant to good accuracy as the temperature changes from 50 mK to ∼1 K, decreases sharply in the interval 1–2 K (Fig. 4), and then comes out onto a new level, on which it remains practically unchanging to 30 K. At temperatures of 1–2 K the mobility also undergoes a slight change (to a value $\mu=4.47\times10^4$ cm^{2}V^{-1}s^{-1}), the resistance of the sample decreases more intensely for $T>1$ K than for $T<1$ K and has a minimum at ∼10 K.

Figure 5a shows the temperature dependence of the carrier concentration obtained from measurements of the Hall coefficient. The concentration jump is of a threshold character; the carrier concentration increases from $p_H=5.68\times10^{11}$ cm^{-2} to $p_H=5.93\times10^{11}$ cm^{-2}, i.e., by 4.4%.

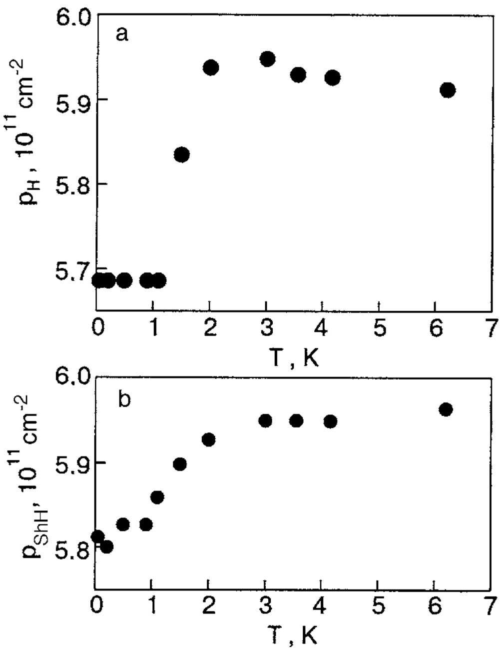

FIG. 5. Temperature dependence of the concentrations p_H found from the Hall coefficient (a) and p_{ShH} found from the period of the Shubnikov–de Haas oscillations (b).

Analysis of the Shubnikov–de Haas oscillations above and below the threshold temperature confirms the existence of a concentration jump at 1–2 K (Fig. 5b). The carrier concentration calculated from the oscillation period increased from $p_{ShH}=5.81\times10^{11}$ cm^{-2} to $p_{ShH}=5.93\times10^{11}$ cm^{-2}, i.e., by 2%. Although the two methods of determining the carrier concentration are in some disagreement as to the absolute numbers for the initial value of p, they both attest to the existence of a jump in the carrier concentration at 1–2 K.

The jump in carrier concentration at 1–2 K is apparently of an activational character and involves the transition of some of the holes to quantum levels close to the ground level in the quantum well. The origin of these "split-off" quantum levels can be linked to variation of the thickness of the germanium layer forming the quantum well due to natural irregularities of the heterointerface. Since the layer thickness varies *discretely*, namely by the thickness of an atomic monolayer, the corresponding quantum levels are separate from the ground quantum level for the mean thickness of the layer. Let us make some estimates in this regard. For simplicity we shall assume that the quantum well is a square well, i.e., we neglect the configuration of the bottom of the well (or, in our case, the top of the well) under the influence of the asymmetric distribution of charges around the well. The energy of the quantum levels in a square quantum well is given by the expression[9]

$$E_n=\frac{\pi^2\hbar^2 n^2}{2m^* L^2}, \tag{3}$$

where L is the width of the quantum well, and n is the number of the level. For the designed thickness of the quantum well in our heterostructure, $L=150$ Å, the energy of the first level is $E_1=13.44$ meV, that of the second $E_2=53.77$ meV, that of the third $E_3=120.99$ meV, and so on. Only the first quantum level is occupied, as an estimate of the Fermi energy corresponding to the carrier concentration found gives a value $\varepsilon_F=12.52$ meV.

The height of an atomic step on the (100) face of germanium is 1.4Å. If the germanium layer has extended regions in which the thickness is less than that value, i.e., $L=148.6$ Å, then the energy of the first quantum level is $E_1=13.7$ meV, i.e., the quantum level lies 0.26 meV higher than for the mean thickness. The relative variation of the energy and, hence, of the carrier concentration is 1.9%. As a result, there can be an activational redistribution of carriers in the layer, both over energy and in coordinate space. The states that are emptied in the redistribution are occupied as a result of the appearance of additional carriers activated by the impurity atoms.

Thus the proposed scheme explains not only the observed jump of the carrier concentration at 1–2 K but also provides a picture of the the appearance of fluctuations of the carrier concentration in the plane of the quantum well, leading to the "inhomogeneous broadening" of the Landau levels that was discussed above.

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