Quantum effects in hole-type Si/SiGe heterojunctions

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The temperature and magnetic-field dependences of the resistance of Si/SiGe heterojunctions with hole-type conductivity are investigated. It is shown that the features of these dependences are due to a manifestation of quantum interference effects — weak localization of the mobile charge carriers, and the hole–hole interaction in the two-dimensional electron system. On the basis of an analysis of the quantum interference effects, the temperature dependence of the dephasing time of the wave function of the charge carrier is determined: $\tau_\varphi = 6.6\times 10^{-12} T^{-1}$ s. This dependence $\tau_\varphi \propto T^{-1}$ must be regarded as a manifestation of hole–hole scattering processes in the two-dimensional electron system. The contribution to the magnetoresistance from the hole–hole interaction in the Cooper channel is extracted, and the corresponding interaction constant $\lambda_0^C \approx 0.5$ is found. © *2000 American Institute of Physics.* [S1063-777X(00)01208-1]

INTRODUCTION

The most important research area in solid state physics for the past two decades has been the physics of low-dimensional electron systems.[1] Progress in semiconductor technology, in particular, the development of molecular-beam epitaxy, has made it possible to create various semiconductor structures with a two-dimensional electron gas. These include metal–insulator–semiconductor (MIS) structures and inversion layers, delta layers, and n–i–p–i–n superlattices, single heterojunctions, and quantum wells (double heterojunctions). In all cases the mobile charge carriers (electrons or holes) occupy quantum levels in the corresponding potential well. The motion of the electrons along a certain direction (along the z axis) is restricted, while the motion in the xy plane remains free.

Heterojunctions are contacts between two semiconductors with slightly different band structures, a situation which is achieved by introducing a small amount of isovalent substitutional impurity atoms into the lattice. The discontinuity of the bands at the boundary and the internal field that arises cause bending of the bands near the boundary, and this gives rise to a potential well with discrete energy states. The diverse phenomena in the two-dimensional electron gas (Shubnikov–de Haas (SdH) oscillations, the quantum Hall effect, electronic phase transitions) have become objects of intensive study in recent times. The observation of SdH oscillations in heterojunctions (e.g., in GaAs/AlGaAs (Ref. 2) or Si/SiGe [(Ref. 3)] and the quantum Hall effect can occur only in modern structures with high values of the electron mobility. In addition, heterojunctions not exhibiting magnetoquantum effects have displayed quantum interference effects — weak localization of electrons (WL) and electron–electron interaction (EEI). These effects have been observed, e.g., in GaAs/AlGaAs heterojunctions[4–6] and a SiGe quantum well.[7] As we know, for the manifestation of quantum interference effects a high degree of disorder is required, i.e., the presence of perceptible elastic scattering of electrons.

It is of interest to ascertain whether both magnetoquantum and quantum interference effects can be investigated in a single object. Let us consider in more detail the conditions necessary for observation of these effects. The WL and EEI effects are manifested in a region of magnetic field values comparable in scale with the values of the characteristic fields for these effects, and at the same time such that the magnetic length L_H at these fields remains larger than the electron mean free path l. The magnetic length $L_H = (\hbar c/2eH)^{1/2}$, which characterizes the electron wave function in a magnetic field, is determined only by the magnetic field and does not depend on the kinetic properties of the electrons. The length L_H corresponds to the field value at which an area $2\pi L_H^2$ is threaded by one magnetic flux quantum $\Phi_0 = hc/2e$. Manifestation of quantum interference effects is possible under the condition $L_H > l$. If the opposite inequality holds, $L_H < l$, then magnetoquantum effects such as SdH oscillations can come into play. Consequently, these two types of quantum effects can be manifested at different values of the magnetic fields. This assertion is clearly illustrated by the experimental data presented below for the two Si/SiGe heterojunctions.

1. GENERAL CHARACTERISTICS OF THE SAMPLES

The samples studied were grown[1)] by molecular-beam epitaxy (MBE) from solid Si and Ge sources by means of electron-beam evaporation and are dislocation-free, fully strained heterostructures with modulated doping. Samples A and B differ by the percent Ge in the $Si_{1-x}Ge_x$ channels ($x = 0.36$ and 0.13, respectively) and by their thicknesses (8 nm and 30 nm) and also by the optimal temperatures of the

TABLE I. Characteristics of the samples.

Parameter	Sample A	Sample B
$R_\square$, kΩ (at 2 K)	4.5	2.7
$n_H\times 10^{-11}$, cm^{-2}	6.0	1.9
$n_{\mathrm{SdH}}\times 10^{-11}$, cm^{-2}	6.7	2.0
μ_H, cm^2 V^{-1} s^{-1}	~2 300	~12 000
m^*/m_0	0.243	0.242
D, cm^2 s^{-1}	14	25

pseudomorphic growth of the $Si_{1-x}Ge_x$ channels (450 °C and 875 °C). First a silicon buffer layer 300 nm thick was grown on the *n*-Si (001) surface of the substrates. This was followed by the growth of a $Si_{1-x}Ge_x$ channel, an undoped Si spacer layer 20 nm thick, and an upper, boron-doped (2.5×10^{18} cm^{-3}) Si epitaxial layer 50 nm thick. The conducting region at the Si/SiGe boundary had a ''double cross'' configuration in the form of a narrow strip ~0.5 mm wide, ~4.5 mm long, and with a distance between the two pairs of narrow potential leads ~1.5–2.2 mm.

Table I shows the characteristics of two of the samples studied (A and B) as obtained from measurements of the conductance, magnetoresistance oscillations, and the Hall coefficient at temperatures of 0.335–2.2 K.

The mobile charge carriers in these samples are holes, but to simplify the terminology we shall by convention refer to them below as electrons. The value of the resistance per square $R_\square$ is given in the table for 2 K, since the minimum of the resistance for sample A is observed near that temperature. The character of the temperature dependence of the resistance of the samples below 4.2 K turns out to be different. The resistance $R_\square$ for sample A as the temperature is lowered passes through a minimum (near 2 K) and then increases somewhat (from 4.5 kΩ to 4.93 kΩ at 0.337 K). This clearly indicates a manifestation of quantum interference effects and the appearance of quantum corrections to the conductance. The resistance $R_\square$ for sample B decreases in this temperature interval (from 2.7 kΩ to 2.5 kΩ), i.e., it does not exhibit pronounced quantum interference effects. Apparently the quantum corrections arise against the background of a temperature-related change in the resistance due to other factors. In such a situation the quantum corrections to the temperature dependence of the resistance cannot be reliably extracted. Therefore, for analysis of quantum interference we predominantly use the corrections to the magnetic-field dependence of the resistance (see Sec. 3).

Figure 1 shows the dependence of the diagonal and off-diagonal (Hall) components of the resistance as a function of the magnetic field for samples B and A at a temperature of ~0.33 K. The curves exhibit SdH oscillations and steps which appear on account of the quantum Hall effect. The quantum numbers ν of the steps and the oscillatory extrema can be determined from the quantum Hall effect data, since, as is well known, $R_H=h/e^2\nu^{-1}$ for a two-dimensional electron gas in the quantum-Hall-effect regime, i.e., $R_H = 25813\ \nu^{-1}\ \Omega$. The values of R_H found experimentally are in satisfactory agreement. Sample B is more perfect and has a

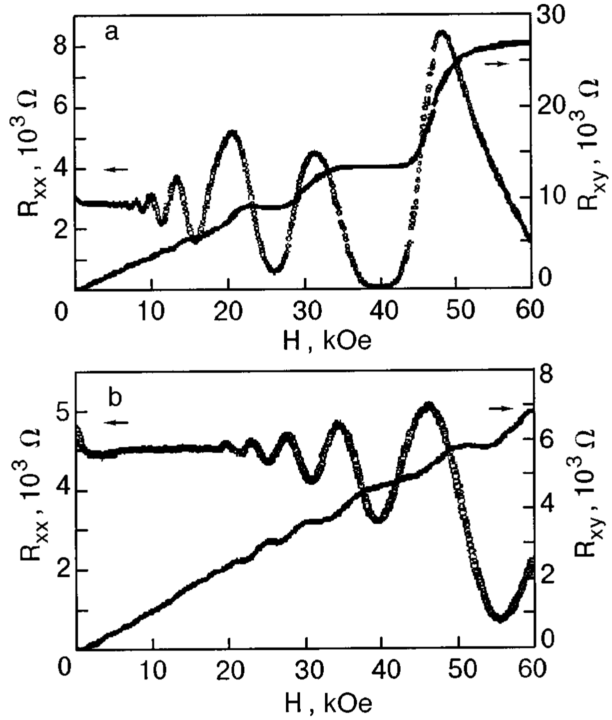

FIG. 1. Magnetic-field dependence of the diagonal component R_{xx} and off-diagonal (Hall) component R_{xy} of the resistance (per square) for samples B (a) and A (b) at a temperature of 0.33 K.

higher electron mobility, and the quantum-Hall-effect steps are more pronounced for it.

2. ANALYSIS OF THE SHUBNIKOV–DE HAAS OSCILLATIONS

The SdH oscillations are described by the relation

$$\frac{\Delta\rho_{xx}}{\rho^0_{xx}}=\frac{\Psi}{\sinh\Psi}\exp\left(-\frac{\pi a}{\omega_c\tau}\right)\cos\left(\frac{2\pi\varepsilon_F}{\hbar\omega_c}+\Phi\right), \tag{1}$$

where $\Psi=2\pi^2kT/(\hbar\omega_c)$; $\omega_c=eH/m^*$ is the cyclotron frequency, $\omega_c\tau\approx\mu H$, μ is the mobility, $\alpha=\tau/\tau_q$, τ is the transport time, τ_q is the quantum scattering time, ε_F is the Fermi energy, reckoned from the bottom of the first quantization band, and Φ is the phase. For a two-dimensional gas the Fermi energy is related to the electron concentration as

$$\varepsilon_F=\frac{\pi\hbar^2 n}{m^*}. \tag{2}$$

In relation (1) [upon substitution of (2)] the unknown parameters are the effective mass m^*, the concentration n, and α, where n appears in the last factor and the temperature appears only in the first factor, which governs the temperature-related damping of the SdH amplitude (Fig. 2). The desired quantity m^* can be found by methods which are well known in the literature. For example, if we take into account that $\omega_c\tau\approx\mu H$ and treat the mobility as known from the kinetic characteristics, then after representing the experimental data in the form of $\ln(\Delta\rho_{xx}/\rho^0)$ versus $\ln(\Psi/\sinh(\Psi))-\pi\alpha/\mu H$, one can find the value of m^* by fitting the data for the entire interval of magnets and temperatures studied to a single straight line. Another method[8] can also be used. By approximating $\sinh(\Psi)$ as $\exp(\Psi)/2$, one can repre-

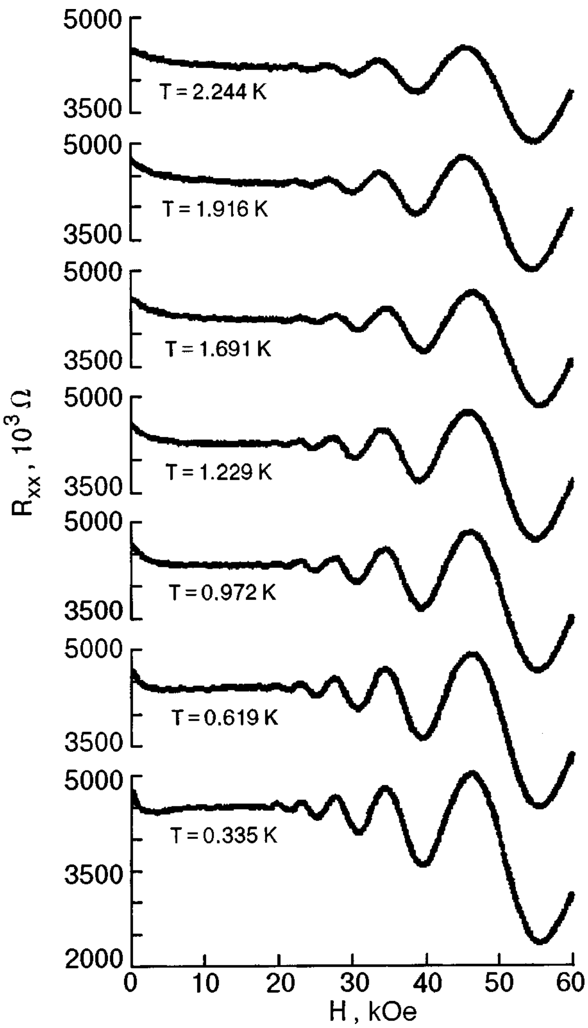

FIG. 2. Magnetic-field dependence of the diagonal component R_{xx} of the resistance (per square) for sample A at different temperatures.

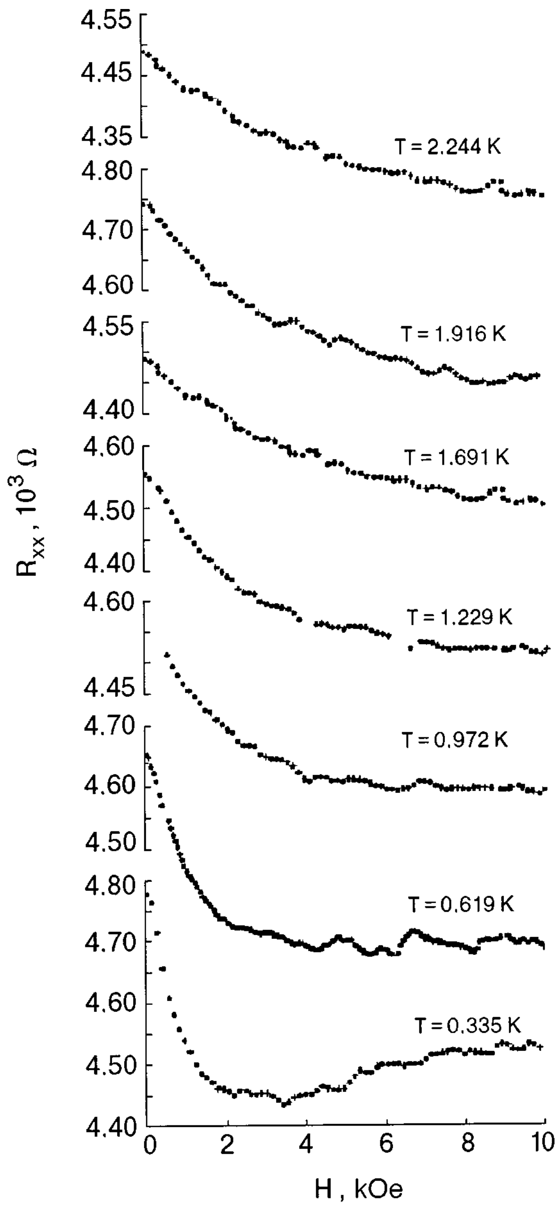

FIG. 3. Magnetoresistance of sample A in low magnetic fields at various temperatures.

sent the experimental data for the amplitudes of the SdH oscillations in the form of linear relations $\ln(A/T)\propto C - 2\pi^2 km^*T/(e\hbar H)$, where C is a temperature-independent constant. The slope of the straight lines at a fixed magnetic field is determined the quantity m^* that we seek. If the effective mass has been determined, then an analysis of the magnetic-field dependence of the amplitude of the SdH oscillations can yield the value of n. The value of the charge carrier concentration found from analysis of the period of the SdH oscillations in high fields under the assumption of a quadratic dispersion relation has turned out to be extremely close to the value found from Hall measurements in low fields (see Table I).

In the band structure of bulk samples of undeformed silicon the two degenerate maxima in the valence band at the point $\mathbf{k}=0$ correspond to hole valleys with effective masses $m^*=0.5m_0$ (heavy holes) and $m^*=0.15m_0$ (light holes).[9] The concentration of light holes is very small compared to that of the heavy holes, but they have a substantially higher mobility than do the heavy holes. From the SdH oscillations we have found for the first time the values of the effective masses of holes in fully strained pseudomorphic Si/SiGe heterostructures (see Table I). We see that, because of the complete lifting of the degeneracy, only one type of hole appears — heavy holes with an effective mass $m^*=(0.24\pm 0.01)m_0$. It is this value of the effective mass which we shall use below in an analysis of the quantum corrections to the investigated hole-type Si/SiGe heterojunctions.

3. QUANTUM INTERFERENCE EFFECTS

The initial parts of the curves of the resistance of the samples versus magnetic demonstrate a negative magnetoresistance effect (Fig. 3), which falls off noticeably in amplitude as the temperature is raised. This is just how the quantum correction to the resistance from the WL effect behaves in the case of weak spin–orbit scattering. The manifestation of the WL effect in small fields and the SdH quantum-oscillation effect in strong fields in the same sample is possible, as we have said, if there exists a region of magnetic fields for which the magnetic length L_H remains larger than the electron mean free path l. An estimate of the mean free path l and the characteristic transport elastic time time τ can be made by using the expression $R_{\square}^{-1}=ne^2\tau/m^* =ne^2 l/v_F m^*$ and the value $v_F=(2\pi n)^{1/2}\hbar/m^*$ for a two-dimensional electron gas. For samples A and B we have obtained the following formulas: $v_F=9.78\times10^6$ cm/s, $\tau=2.86\times10^{-13}$ s, and $l\approx2.8\times10^{-6}$ cm for sample A, and $v_F=5.37\times10^6$ cm/s, $\tau=1.7\times10^{-13}$ s, and $l\approx9\times10^{-6}$ cm for sample B. It follows that quantum interference effects can be observed in sample A in magnetic fields up to 4.5 kOe and in sample B up to 0.5 kOe. We devote most of our attention in the analysis of the quantum interference contribution to the magnetoresistance for sample A.

In the manifestation of quantum interference effects — the weak localization of electrons[10–15] and the electron–electron interaction[12–14,16,17] — analysis of the behavior of the quantum corrections to the conductance in a magnetic field yields information about the most important characteristics of the relaxation and interaction of electrons in the investigated two-dimensional electron system: the dephasing time τ_φ of the electron wave function, its change with temperature, and the electron–electron interaction parameters λ.

3.1. Determination of the temperature dependence of τ_φ

In a two-dimensional electron system in a perpendicular magnetic field the change in conductance due to the WL effect is described in the general case by the expression[13,14]

$$\Delta\sigma_H^L(H)=\frac{e^2}{2\pi^2\hbar}\left[\frac{3}{2}f_2\left(\frac{4eHD\tau_\varphi^*}{\hbar c}\right)-\frac{1}{2}f_2\left(\frac{4eHD\tau_\varphi}{\hbar c}\right)\right], \quad (3)$$

where $f_2(x)=\ln x+\Psi(1/2+1/x)$, Ψ is the logarithmic derivative of the Γ function, $\tau_\varphi^{-1}=\tau_{\varphi0}^{-1}+2\tau_s^{-1}$, $(\tau_\varphi^*)^{-1}=\tau_{\varphi0}^{-1}+(4/3)\tau_{so}^{-1}+(2/3)\tau_s^{-1}$, $\tau_{\varphi0}$ being the phase relaxation time due to inelastic scattering processes, τ_{so} the spin–orbit scattering time, and τ_s the spin–spin scattering time for scattering on magnetic impurities (in the absence of which this time can be left out), and D is the electron diffusion coefficient. The first term in (3) corresponds to the interference of the wave functions of electrons found in the triplet spin state, and the second to those in the singlet spin state. In the case of strong spin–orbit scattering ($\tau_\varphi\gg\tau_{so}$) by virtue of the inequality $\tau_\varphi\gg\tau_\varphi^*$ the change in conductance is determined by the second term, which corresponds to a positive magnetoresistance. For $\tau_\varphi\ll\tau_{so}$ the magnetoresistance is negative, and the field dependence $\Delta\sigma_H^L(H)$ is described by the expression

$$\Delta\sigma_H^L(H)=\frac{e^2}{2\pi^2\hbar}f_2\left(\frac{4eHD\tau_\varphi}{\hbar c}\right). \quad (4)$$

The function $f_2(x)$ has the form $\frac{1}{24}x^2$ at small x, i.e., in low magnetic fields, and ln(x/7.12) in high fields. The characteristic field corresponding to the region of strong variation of this function ($H_0^L=\hbar c/(4eD\tau_\varphi)$) is usually of the order of $\sim$0.1 kOe.

At small values of the magnetoresistance one can use the relation $-\Delta\sigma_H^L(H)=[R(H)-R(0)]/(R(H)R_{\square}(0))$, and here the field dependence of $-\Delta\sigma_H^L(H)$ reflects the trend of the magnetoresistance. To fit the $\Delta\sigma_H^L(H)$ curves to relation (3) and thus to obtain the desired value of τ_φ requires knowledge of the electron diffusion coefficient D, which is determined from the formula for a two-dimensional electron gas: $D=(1/2)v_F^2\tau$.

Analysis of the experimental curves for the magnetoresistance, replotted in the form of the $\Delta\sigma_H^L(H)$ curves in accordance with (3) showed that the quantum correction due to the WL effect gives a good description of only the initial part of the $\Delta\sigma_H^L(H)$ curves (here the results of the fitting to relations (3) and (4) are no different, since these objects have weak spin–orbit scattering). As the magnetic field increases, at $H\sim0.2$ kOe a magnetoresistance component of the opposite sign appears, its amplitude falling off with increasing temperature in the interval 0.335–2 K. The assumption that this component is due to the ordinary magnetoresistance of the form $\Delta\rho/\rho\propto H^2$ does not hold up, since the change in mobility in this temperature interval is insignificant. We have arrived at the conclusion that this component is a quantum correction due to the electron–electron interaction. Several forms of this correction are known. Manifestation of the quantum correction due to the EEI in the diffusion channel is unlikely, since it is due to disruption of the interaction in the spin subbands as a result of Zeeman splitting and becomes substantial at rather high magnetic fields ($H>H_0^D=\pi kT/(g\mu_B)$, where g is the Landé factor and μ_B is the Bohr magneton). The Maki–Thompson correction, which is due to a fluctuation process, has the same functional form as the localization correction and cannot alter the shape of the magnetoresistance curves (see Fig. 3). The most likely candidate is the quantum correction due to the EEI in the Cooper channel. The latter correction is described by the expression:[13,14,17]

$$\Delta\sigma_H^C=-\frac{e^2}{2\pi^2\hbar}\lambda_H^C\varphi_2(\alpha); \quad \alpha=\frac{2eDH}{\pi ckT}. \quad (5)$$

The function φ_2 is similar to the function f_2, but the characteristic field $H_0^C=\pi ckT/(2eD)$ is considerably higher than H_0^L, as a rule. In low magnetic fields ($H<H_0^C$) we have $\varphi_2(\alpha)\approx0.3\alpha^2$, so that one may use this approximation in our case.

As we see from Eq. (5), the Cooper quantum correction varies with temperature as T^{-2}, which agrees well with the variation of the positive component of the magnetoresistance. The sign of the quantum correction $\Delta\sigma_H^C$ (and, accordingly, the sign of the magnetoresistance) is determined by the sign of the interaction constant λ_H^C: in the case of repulsion of the quasiparticles one has $\lambda_H^C>0$, giving a positive magnetoresistance. The interaction constant λ_H^C is the parameter to be extracted from a fitting of the experimental curves to expression (5). Here, depending on the form of the curves, expression (3) or (4) is used, with τ_φ as the adjustable parameter.

As a result of the calculations, in which a good description of the experiment was achieved, we obtained the temperature dependence of the electron dephasing time τ_φ (the unfilled symbols in Fig. 4). It is approximated by a power-law function $\tau_\varphi=6.6\times10^{-12}T^{-1}$.

For sample B a negative magnetoresistance is also observed in low fields, but it is very weakly expressed, and, furthermore, as we have mentioned, it can be analyzed in terms of the concepts of quantum interference only in fields

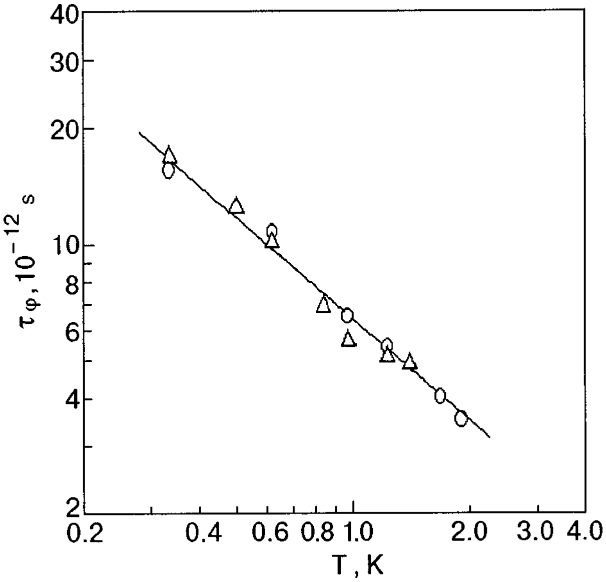

FIG. 4. Dephasing time versus temperature; the data were obtained from the weak localization and electron interaction effects for samples A (○) and B (△).

less than 0.5 kOe. The EEI contribution is not manifested in such fields. On the basis of an analysis of the initial parts of the magnetoresistance curves with the use of relation (4), we found that τ_φ has the same dependence for sample B (the triangles in Fig. 4) as for sample A (of course, the error with which τ_φ is determined is substantially larger for sample B than for sample A).

A dependence of the form obtained here, $\tau_\varphi \propto T^{-1}$, describes electron–electron scattering processes in two-dimensional systems.[17] The electron–electron scattering time was calculated in Ref. 18 for the case of collisions involving small changes in the energies and momenta of the electrons:

$$\tau_{ee}^{-1}=\frac{kT}{2\pi\hbar^2\nu_{ds}D}\ln(\pi\hbar\nu_{ds}D), \tag{6}$$

where ν_{ds} is the electron density of states. Using in (6) for the case of sample A the value found for D and the calculated value $\nu_{ds}=m^*/(\pi\hbar^2)$ (for a 2D electron system), we obtain the result $\tau_{ee}=7.39\times10^{-11}T^{-1}$. The values of τ_{ee} calculated from (6) differ from the experimental values of τ_φ by an order of magnitude, but such a disagreement is completely acceptable in view of the estimates used for ν_{ds}, D, etc.

3.2. Interaction constant λ_H^C

The temperature dependence of λ_H^C (Fig. 5) for sample A agrees well with the theoretical prediction:[14,17]

$$(\lambda_H^C)^{-1}=-\ln\left(\frac{T}{T_c}\right). \tag{7}$$

In relation (7) for superconductors (in the case of attraction $\lambda_H^C<0$), T_c has the well-known form

$$kT_c=k\theta_D\exp\left(\frac{1}{\lambda_0}\right), \tag{8}$$

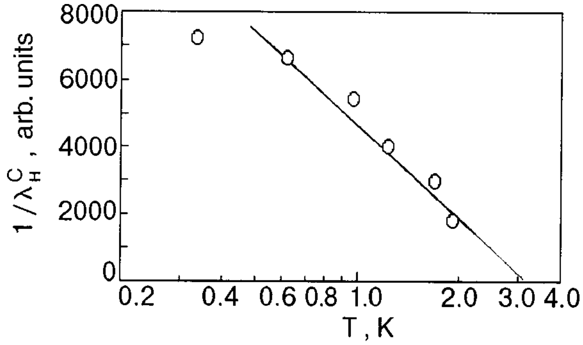

FIG. 5. Temperature dependence of the interaction parameter obtained from the weak localization and electron interaction effects for sample A.

where λ_0 is the interaction constant in the BCS theory. However, as was shown in Ref. 19, even in the case of repulsion of the electrons at small distances ($\lambda_H^C>0$) for the EEI effects, formula (7) remains valid at low magnetic fields, but the temperature T_c takes on a formal meaning:

$$kT_c=\varepsilon_F\exp\left(\frac{1}{\lambda_0^C}\right). \tag{9}$$

In Fig. 5 it is easy to determine this characteristic temperature T_c (it is equal to 3.2 K) and then to find the bare value of the interaction constant, $\lambda_0^C=0.5$.

The interaction constant found from the quantum corrections is usually written in terms of the universal constant F — the angle-averaged interaction amplitude of the electrons at small momentum transfers. In the presence of screening of the Coulomb type the constant F takes on values from zero in the absence of screening (the ''bare'' interaction) to unity in the case of complete screening. The functional form of F is different for the interaction constants found from the temperature and magnetic-field dependence of the quantum corrections, in the regions of weak and strong magnetic fields, and for weak and strong spin–orbit interaction. In the case considered, that of weak spin–orbit interaction, one should take $\lambda_0^C=1-F$ for the interaction constant found from the magnetic-field dependence of the quantum correction. Thus $F=0.5$, which is a completely reasonable value.[2)]

The value we have found for F is confirmed by an analysis of the change in resistance of sample A at temperatures below the resistance minimum. For example, in the region 0.3–0.8 K the temperature dependence of the resistance is described well by a straight line in the coordinates $R_\square - \ln(T)$ (Fig. 6) and can be represented by the temperature dependence predicted by the theory of WL and EEI:[10,12]

$$\Delta\sigma=\frac{e^2}{2\pi^2\hbar}a_T\ln(T), \tag{10}$$

where $a_T=p+\lambda_T$ in the case of weak spin–orbit interaction ($\tau_\varphi<\tau_{s0}$) and $a_T=-1/2p+\lambda_T$ in the case of strong spin–orbit interaction ($\tau_\varphi>\tau_{so}$), with p being the exponent of the power-law dependence $\tau_\varphi\propto T^{-p}$.

For sample A we obtained a value $a_T=1.2(\pm0.01)$. Since in our case $a_T=p+\lambda_T$ and $p=1$, we obtain $\lambda_T\simeq0.2$. For weak spin–orbit interaction the constant λ_T in zero or low magnetic field has the form[14,17]

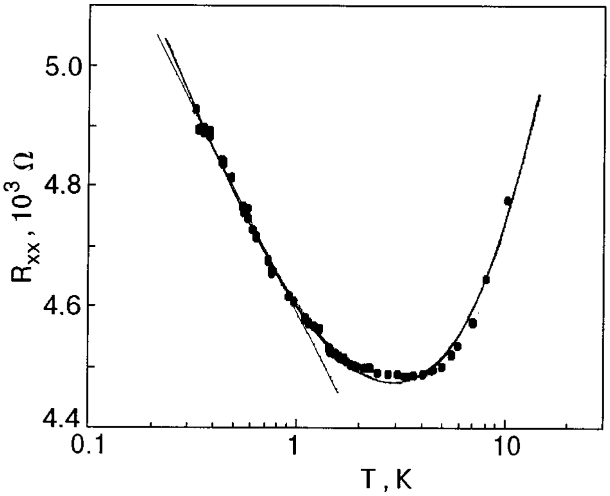

FIG. 6. Temperature dependence of the resistance R_{xx} of sample A.

$$\lambda_T = 1 - \frac{3}{2} F. \tag{11}$$

From Eq. (11) for $\lambda_T \simeq 0.2$ we get $F = 0.53$.

CONCLUSION

In summary, systems containing a two-dimensional gas of holes and having a certain relationship between the elastic and inelastic relaxation times can manifest effects of weak localization and interaction of holes (in the magnetoresistance and in the temperature dependence of the resistance) in low magnetic fields, and magnetoquantum effects (Shubnikov–de Haas oscillations and the quantum Hall effect) in high fields. Analysis of the quantum interference effects has yielded the value and temperature dependence of the dephasing time τ_φ of the wave function of the mobile charge carriers in the Si/SiGe heterojunctions studied here. It was found that this temperature dependence has the form $\tau_\varphi \propto T^{-1}$ and describes hole–hole scattering processes in a two-dimensional conducting system. Information was also obtained on the temperature-dependent interaction constant λ_T^C in the Cooper channel.

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1) The samples were prepared in the Advanced Semiconductors Group, University of Warwick, Conventry, England.

2) We note that in Ref. 7 for a p-$Si_{0.88}Ge_{0.12}$ system (quantum well) the deviation of the magnetoresistance curves from the calculated form of the localization correction was interpreted as being due to the contribution of the interaction in the diffusion channel due to Zeeman splitting, and as a result, the unrealistic value $F=2.45$ was obtained, which the authors of Ref. 7 were at a loss to explain.

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