Scattering of a single photon on a two-qubit structure with resonators

A. N. Sultanov[a)]

Novosibirsk State Technical University, pr. K. Marksa, 20, Novosibirsk 630073, Russia

D. S. Karpov[b)]

B.I. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, 47 Nauki Ave., Kharkov 61103, Ukraine

Y. S. Greenberg

Novosibirsk State Technical University, K. Marksa Ave., 20, Novosibirsk 630073, Russia

S. N. Shevchenko

B.I. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, 47 Nauki Ave., Kharkov 61103, Ukraine and V.N. Karazin Kharkov National University, Svobody sq., 4, Kharkov 61022, Ukraine

A. A. Shtygashev

Novosibirsk State Technical University, K. Marksa Ave., 20, Novosibirsk 630073, Russia

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In this paper, the scattering of a single photon in a waveguide–resonator–qubit system is studied. An open waveguide is connected to two resonators, located at an arbitrary distance from each other and containing a single qubit each. The scattering of a single photon makes it possible to describe the behavior of the system completely quantum mechanically. We show the existence of Fano resonance, which is a direct manifestation of the interference between the incident photon and virtual photons associated with transitions between the states of the system. The obtained expressions for the transmission coefficients allowed us to take into account the influence of the incident photon frequency on the resonances and their widths. *Published by AIP Publishing.*
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Introduction

In the last years, experimental progress has made it possible to study several quantum-optical effects in one-dimensional solid-state quantum structures: superradiance,[1–4] photon-assisted state transfer between qubits,[5–9] magnetically induced transmittance of a single photon,[10,11] etc. One of the promising directions in this field is the study of the interaction of an open waveguide and a qubit.[12–15] Most of the current work is focused on systems in which a single qubit is studied. The next step in the development is the study of systems containing two or more qubits.

We consider the photon interaction of a two-qubit system in an open waveguide. In the present paper we consider the passage of a single photon. This allows us to state that the observed effects are of a purely quantum mechanical nature. In the case of a one-dimensional waveguide, the distances between entangled objects play an important role, which brings attention to the specific aspects of the influence of this distance on the scattering processes of a single photon.

There is a vast body of theoretical[16–20] and experimental[21–23] studies of scattering of one or more photons in structures that represent an open 1D waveguide, at certain points of which artificial two-level systems are placed. We consider a system in which each qubit is placed in a photon resonator. This physically limits the number of modes with which it can interact and, thereby, increases the lifetime of the qubit.[24] In this case, the resonators exchange a photon directly with the waveguide, thereby making an indirect connection between the qubit and the waveguide.[25] We expect to see the previously observed effects[16,18–20] in a system with *a fortiori* long lifetime and taking into account the retardation effect,[26] which is an important parameter in the implementation of solid-state quantum circuits.

Description of the system

Let us consider scattering of a photon in an open waveguide (Fig. 1). The waveguide is directly connected to two resonators. In each of them there is one two-level system. Expressions for the transmission and reflection coefficients are sought by constructing the scattering matrix, based on the non-Hermitian Hamiltonian method.[18] Its application for calculating the transport of a single photon is described in detail in Ref. 16. In the framework of this method, it is necessary to partition the entire Hilbert space of states into two subspaces with the operators P and Q, which satisfy the following properties:

$$P + Q = 1; \quad PP = P; \quad QQ = Q; \quad PQ = QP = 0. \quad (1)$$

Let us divide the space so that only the states from the continuum enter the first subspace; they correspond to the operator P. The second subspace only contains discrete states, and the operator Q corresponds to them. All the states of Q that are coupled with states from the continuum become unstable and decay; the decay process is described by a non-Hermitian effective Hamiltonian

$$H_{\text{eff}}(E) = H_{QQ} + H_{QP}\frac{1}{E - H_{PP} + i\varepsilon}H_{PQ}, \tag{2}$$

where $H_{XY} = XHY$ and X and Y correspond to either Q or P and ε is an infinitesimal quantity introduced in order to get rid of the singularity at resonance.[16]

The Hamiltonian (2) determines the resonant energy levels of the internal system, which arise due to the coupling with the P-subspace and lie in the lower part of the complex plane $E = E' - i\hbar\Gamma'$. The resonance parameters are determined from the following equation:

$$\det(E - H_{\text{eff}}) = 0. \tag{3}$$

where Γ' determines the decay rate of states from the Q-subspace. In the framework of this problem, the solution of the Schrödinger equation is written as

$$\begin{aligned} |\Psi\rangle &= |in\rangle + \frac{1}{\tilde{E} - H_{\text{eff}}}H_{QP}|in\rangle \\ &+ \frac{1}{\tilde{E} - H_{PP} + i\varepsilon}H_{PQ}\frac{1}{\tilde{E} - H_{\text{eff}}}H_{QP}|in\rangle, \end{aligned} \tag{4}$$

where $|in\rangle$ is the initial state of the system before scattering that satisfies the equation $H_{PP}|in\rangle = \tilde{E}|in\rangle$ and $\tilde{E}$ is the energy of the system in the initial state. It should be noted that in Eq. (4) the last term takes into account the evolution of the initial state over all orders of interaction between the P- and Q-subspaces.

The total Hamiltonian describing the system includes photonic resonators with fundamental frequencies ω_{c1} and ω_{c2} located at a distance d from each other in the waveguide, along which photons with an arbitrary frequency ω_k can propagate, as well as two qubits with eigenfrequencies Ω_1 and Ω_2, one in each resonator. The interaction parameters of the resonators with the waveguide are denoted as ξ_1 and ξ_2, respectively, and those of the resonators with the qubits as λ_1 and λ_2. We describe the qubit in the framework of the spin model and the photon in the waveguide in the Fock representation.[27] Then the total Hamiltonian of the system takes the form[16,24]

$$\begin{aligned} H &= \sum_{j=1}^{2}\frac{1}{2}\hbar\Omega_j\sigma_{zj} + \sum_{j=1}^{2}\hbar\omega_{cj}a_j^{\dagger}a_j + \sum_k \hbar\omega_k c_k^{\dagger}c_k \\ &+ \sum_{j=1}^{2}\sum_k \hbar\xi_j\left(c_k^{\dagger}a_j \mathrm{e}^{-ikx_j} + c_k a_j^{\dagger}\mathrm{e}^{ikx_j}\right) \\ &+ \sum_{j=1}^{2}\hbar\lambda_j\left(a_j^{\dagger} + a_j\right)\sigma_{xj}, \end{aligned} \tag{5}$$

where the first three terms describe the behavior of all the above elements (qubits, resonators, waveguide), and the last two describe the interaction between them. $\sigma_{zi} = |e\rangle\langle e| - |g\rangle\langle g|$ is the spin operator of the i-th qubit; $a_i^{\dagger}(c_k^{\dagger})$ and $a_i(c_k)$ are the boson operators for creation and annihilation of photons in the resonator (waveguide); and x_i is the coordinate of the i-th resonator along the x-axis in the one-dimensional waveguide and $|x_1 - x_2| = d$.

Let us assume that one photon enters the system and the resonator can exchange only one photon with the waveguide. Then we can restrict the Hilbert space of the states of the system to the following set of vectors:

$$\begin{aligned} |K\rangle &= |1_w\rangle \otimes |g_1, g_2, 0_{c1}, 0_{c2}\rangle, \\ |1\rangle &= |0_w\rangle \otimes |g_1, g_2, 1_{c1}, 0_{c2}\rangle, \\ |2\rangle &= |0_w\rangle \otimes |g_1, g_2, 0_{c1}, 1_{c2}\rangle, \\ |3\rangle &= |0_w\rangle \otimes |e_1, g_2, 0_{c1}, 0_{c2}\rangle, \\ |4\rangle &= |0_w\rangle \otimes |g_1, e_2, 0_{c1}, 0_{c2}\rangle, \end{aligned} \tag{6}$$

where $g(e)$ is the ground (excited) state of qubits; $1_w(0_w)$ is the presence (absence) of a photon with an arbitrary wave vector k in the waveguide; $1_{ci}(0_{ci})$ is the presence (absence) of a photon in the resonator. The constant exchange of a photon between a qubit and its resonator leads to splitting of the levels[16,24,25,28,29]

$$\Omega_{Rj} = \sqrt{(\Omega_j + i\Gamma_j - \omega_{cj})^2 + 4\lambda_j^2}, \tag{7}$$

where Γ_j are the decay rates of the photon into the waveguide from the j-th resonator, which are formally equal to the half-width of the Lorentzian corresponding to the amplitude-frequency characteristic of the resonator. For the states (6), the projection operators take the following form:

$$\begin{aligned} Q &= \sum_{n=1}^{4}|n\rangle\langle n|, \\ P &= \sum_{k_w}|K\rangle\langle K|. \end{aligned} \tag{8}$$

In the chosen basis of states, the effective Hamiltonian is a 4×4 matrix with the following components:

$$\begin{aligned} \langle 1|H_{\text{eff}}|1\rangle &= -\frac{1}{2}\hbar\Omega_1 - \frac{1}{2}\hbar\Omega_2 + \hbar\omega_{c1} - i\hbar\Gamma_1, \\ \langle 2|H_{\text{eff}}|2\rangle &= -\frac{1}{2}\hbar\Omega_1 - \frac{1}{2}\hbar\Omega_2 + \hbar\omega_{c2} - i\hbar\Gamma_2, \\ \langle 3|H_{\text{eff}}|3\rangle &= -\frac{1}{2}\hbar\Omega_1 - \frac{1}{2}\hbar\Omega_2, \\ \langle 4|H_{\text{eff}}|4\rangle &= -\frac{1}{2}\hbar\Omega_1 + \frac{1}{2}\hbar\Omega_2, \\ \langle 1|H_{\text{eff}}|2\rangle &= \langle 2|H_{\text{eff}}|1\rangle = \hbar^2\sqrt{\Gamma_1\Gamma_2}\mathrm{e}^{ikd}, \\ \langle 1|H_{\text{eff}}|3\rangle &= \langle 3|H_{\text{eff}}|1\rangle = \hbar\lambda_1, \\ \langle 1|H_{\text{eff}}|4\rangle &= \langle 4|H_{\text{eff}}|1\rangle = 0, \\ \langle 2|H_{\text{eff}}|3\rangle &= \langle 3|H_{\text{eff}}|2\rangle = 0, \\ \langle 2|H_{\text{eff}}|4\rangle &= \langle 4|H_{\text{eff}}|2\rangle = \hbar\lambda_2, \\ \langle 3|H_{\text{eff}}|4\rangle &= \langle 4|H_{\text{eff}}|3\rangle = 0. \end{aligned} \tag{9}$$

In this expression, the elements $\langle 1|H_{eff}|2\rangle$ and $\langle 2|H_{eff}|1\rangle$ contain the dependence on the momentum of the photon. The structure of the wave function is such that its momentum is equal to the photon momentum in the initial state of the system.

We are interested in the probability of detecting a photon in the waveguide after interacting with the system, and this probability can be easily determined by changing the coordinate representation of the wave function (4), $\langle x|\Psi\rangle = \Psi_x$, where $|x\rangle = |x_w\rangle \otimes |g_1, g_2, 0_{c1}, 0_{c2}\rangle$ In this case, the wave function in the coordinate representation is written as:

$$\Psi_x = e^{ikx} - i\hbar\Gamma_1 e^{ik|x-x_1|}e^{ikx_1}R_{11} - i\hbar\Gamma_2 e^{ik|x-x_2|}e^{ikx_2}R_{22} - i\hbar\sqrt{\Gamma_1\Gamma_2}\left(e^{ik|x-x_1|}e^{ikx_2}R_{12} - e^{ik|x-x_2|}e^{ikx_1}R_{21}\right), \quad (10)$$

where $R_{i,j} = (\frac{1}{E-H_{eff}})_{i,j}$. We provide only the expressions for the elements of the matrix $R_{i,j}$, which are necessary for computing Eq. (8). We take into account that the energy of the state $|K\rangle$ is equal to $E = \hbar(\omega - \frac{1}{2}\Omega_1 - \frac{1}{2}\Omega_2)$

$$\begin{aligned} R_{11} &= \frac{(\omega-\Omega_1)(\omega-\tilde{\omega}_2^-)(\omega-\tilde{\omega}_2^+)}{D(\omega)}, \\ R_{22} &= \frac{(\omega-\Omega_2)(\omega-\tilde{\omega}_1^-)(\omega-\tilde{\omega}_1^+)}{D(\omega)}, \\ R_{12} = R_{21} &= \frac{-i\sqrt{\Gamma_1\Gamma_2}e^{ikd}(\omega-\Omega_1)(\omega-\Omega_2)}{D(\omega)}, \\ \tilde{\omega}_j^{\pm} &= \frac{\Omega_j + \omega_{cj} - i\Gamma_j \pm \Omega_{Rj}}{2}, \end{aligned} \quad (11)$$

where the determinant of the matrix R can be represented in the general form as

$$D(\omega) = (\omega-\omega_{1+})(\omega-\omega_{1-})(\omega-\omega_{2+})(\omega-\omega_{2-}). \quad (12)$$

The expression $D(\omega) = 0$ is a transcendental equation of the fourth degree. Its analytical solution for identical qubit-resonator couples ($\Omega_1 = \Omega_2$, $\omega_{c1} = \omega_{c2}$) has the following form:

$$\begin{aligned} \omega_{1\pm} &= \frac{1}{2}\left(\Omega + \omega_c - i\Gamma + i\Gamma e^{i\frac{\omega}{\omega_c}k_{c1}d}\right) \\ &\pm \frac{1}{2}\left[\left(\Omega - \omega_c + i\Gamma - i\Gamma e^{i\frac{\omega}{\omega_c}k_{c1}d}\right)^2 + 4\lambda^2\right]^{1/2}, \\ \omega_{2\pm} &= \frac{1}{2}\left(\Omega + \omega_c - i\Gamma - i\Gamma e^{i\frac{\omega}{\omega_c}k_{c1}d}\right) \\ &\pm \frac{1}{2}\left[\left(\Omega - \omega_c + i\Gamma + i\Gamma e^{i\frac{\omega}{\omega_c}k_{c1}d}\right)^2 + 4\lambda^2\right]^{1/2}. \end{aligned} \quad (13)$$

In Eq. (13), the following substitution was introduced, $kd = \frac{\omega}{\nu_g}d = \frac{\omega}{\nu_g}\frac{\omega_{c1}}{\omega_{c1}}d = \frac{\omega}{\omega_c}k_{c1}d$, where k_{c1} is the photon wave vector in the first resonator (normalization can be done to the photon wave vector in any resonator). It can be seen from Eq. (13) that the resonance energy and its width depend on the frequency of the incident photon. This fact is a manifestation of the retardation effect: the photon in the waveguide propagates with a finite velocity, and it takes a certain time to reach the second qubit-resonator couple of the system.[19] At distances much shorter than the wavelength, the energy of the resonance does not depend on the frequency of the incident photon: it "instantly" affects both qubit-resonator couples. It is due to the retardation effect, which is formally intrinsic to both virtual and real photons, the interference occurs. Expressions (13) allow us to find the dependences of the real resonances of the system and their widths on the parameters of the system. These dependences on the distance

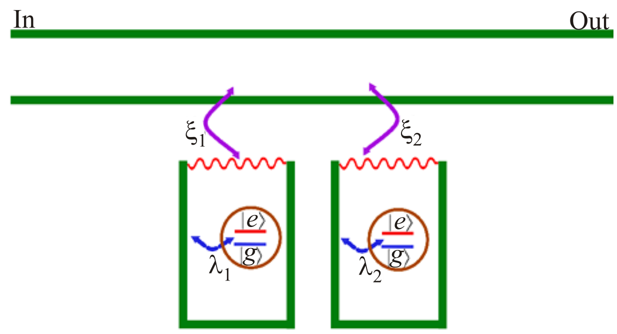

Fig. 1. Schematic representation of the waveguide-qubit-resonator system. The blue dashed arrows show the coupling between the resonator and the two-level system. The magenta solid arrows represent the coupling between the resonator and the waveguide. The two-level system is coupled to the waveguide through the resonator. The photon propagates in the waveguide from left to right.

between the qubits are shown in Figs. 2 and 3. The graphs are constructed for identical qubits and resonators. The following parameters were selected: $\Omega_1/2\pi = \omega_{c1}/2\pi = 3$ GHz at the frequency of the incident photon $\omega = \omega_{c1}$.

Figures 2 and 3 show that in the case of weak coupling ($\lambda \ll \Gamma$), it is possible to observe the effects of increasing and decreasing of the photon emission rate from the resonator into the waveguide. This phenomenon is due to the fact that by placing the qubit-resonator couples at different distances that are multiples of the wavelength at the fundamental frequency of the resonator, we thereby enforce constructive or destructive interference of the wave functions of each couple.[19]

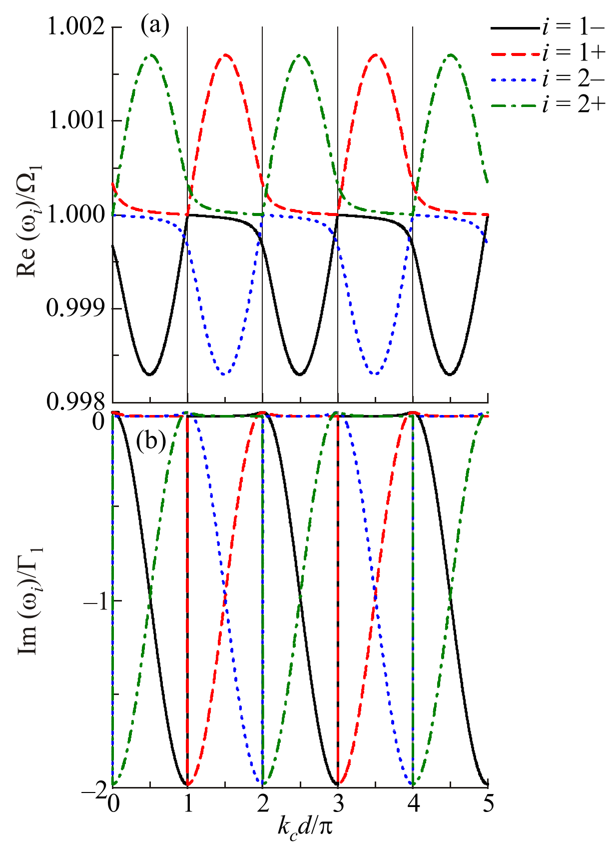

Fig. 2. Dependence of the resonance energy (a) and its width (b) on the distance between the resonators with qubits in the weak dispersion regime ($\lambda/2\pi = 1$ MHz, $\Gamma/2\pi = 5$ MHz). The black solid, red dashed, blue-dotted, and green dash-dotted lines correspond to the roots ω_{1-}, ω_{1+}, ω_{2-}, and ω_{2+}, respectively.

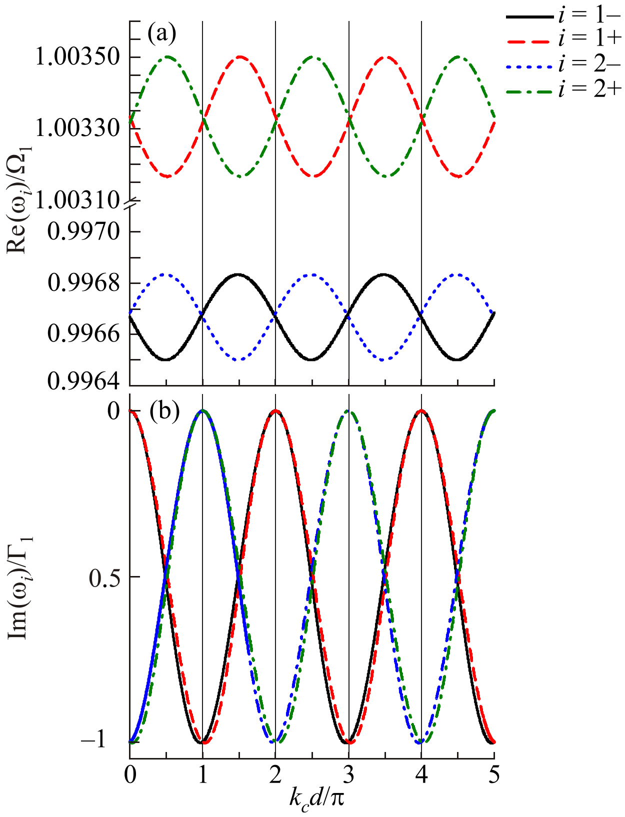

Fig. 3. Dependence of the resonance energy (a) and its width (b) on the distance between resonators with qubits in the strong resonance regime ($\lambda/2\pi = 10$ MHz, $\Gamma/2\pi = 1$ MHz). The black solid, red dashed, blue-dotted, and green dash-dotted lines correspond to the roots ω_{1-}, ω_{1+}, ω_{2-}, and ω_{2+}, respectively.

Moreover, in the weak dispersion regime, different roots of the determinant contribute to these effects at different distances. In the strong resonance regime, a strong splitting of the resonance energies is observed. In this case, the contribution of the Rabi splitting for the qubit-resonator system is greater. For the distances between qubit-resonators that are multiples of $\pi/2$, the effect of lowering the emission rate is observed. In the strong resonance regime, this effect is stronger (see Figs. 2 and 3). This phenomenon can be explained as follows: after a photon is emitted by the first resonator, it can be (re-)reflected from the second resonator and re-absorbed by the first. By varying the distance between the resonators, we change the probability of such repetitive excitations and emission. It is also assumed that the qubits are repetitively excited. This leads, as in the case without resonators,[19] to an increase in the lifetime of the first qubit. In other words, with increasing the distance, the mutual contribution to interference from the couples drops due to a decrease in the amplitudes of the wave function.

In our problem, the attenuation in the waveguide is not taken into account. However, we assume that taking it into account would lead to a decrease in the amplitude of the detected signal, as well as to weakening the effect of the distance between the qubit-resonator couples on the photon emission rate into the waveguide, since the latter is associated with the interference of wave functions. In other words, as the distance increases, the mutual contribution to interference from both couples drops since the amplitudes of the wave functions decrease.

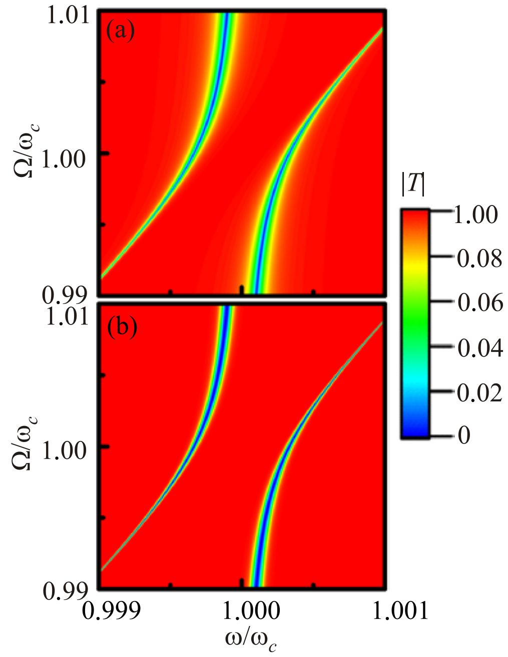

Fig. 4. Dependence of the magnitude of the transmission coefficient T on the frequency of the incident photon and the distance between the qubit levels for small $k_c d < 1$ (a) and large $k_c d = \pi/2$ (b) distances between the resonators in the strong resonance regime ($\lambda \gg \Gamma$, $\Gamma/2\pi = 1$ MHz, $\lambda/2\pi = 10$ MHz).

Let us assume that the photon moves from the side of the first resonator with the coordinate x_1. Figures 4 and 5 show the amplitude-frequency characteristics for identical qubit-resonator couples in the strong resonance (Fig. 4) and weak dispersion (Fig. 5) regimes considered earlier. From Eq. (10) we obtain the transmission ($x > x_2$) and reflections

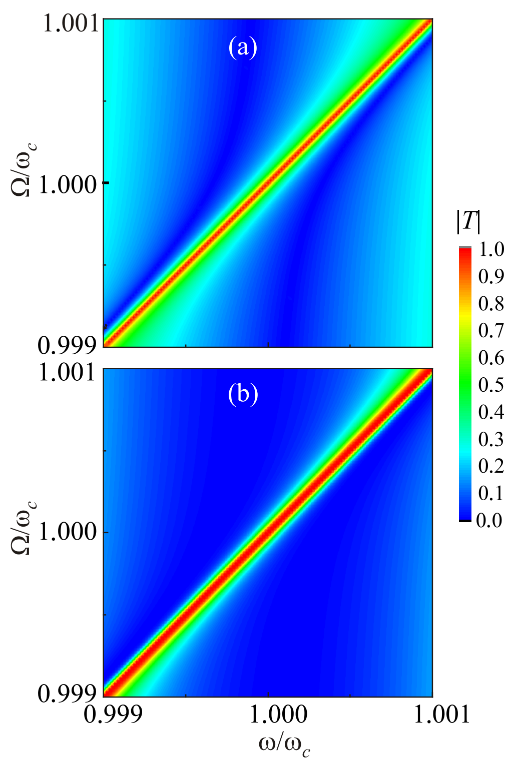

Fig. 5. Dependence of the magnitude of the transmission coefficient T on the frequency of the incident photon and the distance between the qubit levels for small $k_c d < 1$ (a) and large $k_c d = \pi/2$ (b) distances between the resonators in the weak dispersion regime ($\Gamma/2\pi = 5$ MHz, $\lambda/2\pi = 1$ MHz).

$(x < x_1)$ coefficients, assuming propagation of plane waves in the waveguide[16]

$$T = 1 - i\hbar\Gamma_1 R_{11} - i\hbar\Gamma_2 R_{22} - i\hbar\sqrt{\Gamma_1\Gamma_2}\left(e^{ikd}R_{12} + e^{-ikd}R_{21}\right), \quad (14)$$

$$R = -i\hbar\Gamma_1 R_{11} e^{-ikd} - i\hbar\Gamma_2 R_{22} e^{ikd} - i\hbar\sqrt{\Gamma_1\Gamma_2}(R_{12} + R_{21}). \quad (15)$$

In the case of weak coupling, we observe a clear manifestation of interference effects in the lines corresponding to anti-crossing, i.e., associated with the non-resonant interaction of the resonator mode with the qubit. At distances that are multiples of π, we observe patterns identical to Fig. 5(a), and at distances that are multiples of $\pi/2$, the patterns are as in Fig. 5(b), which differ in the widths of the resonances. It is important to note that the maximum transmission coefficient corresponds to the case when the energy of an external photon matches the energy of the qubit and the resonator (the considered frequency band belongs to the resonator band). Moreover, increasing the distance leads to an increase in the transparency band of the system in the case when the qubit and the resonator are detuned from each other. The cross-sections of the surfaces in Fig. 5, for the case of matching frequencies of the qubit and the resonator, are shown in Fig. 6.

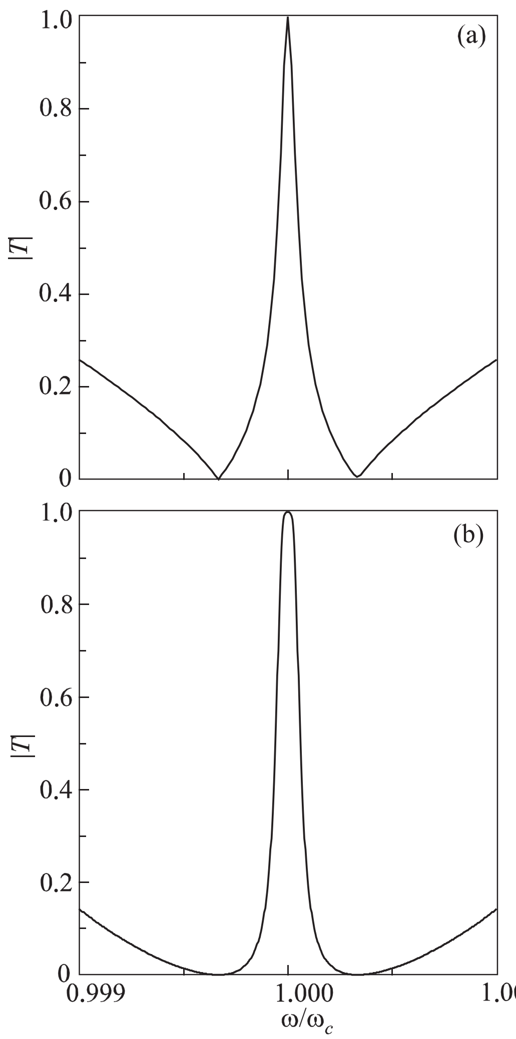

Fig. 6. Amplitude-frequency characteristics in the weak dispersion regime for small (a) and large (b) ($k_c d = \pi/2$) distances between the resonators. The frequency of the qubits matches to the frequency of the resonators $\Omega = \omega_c$.

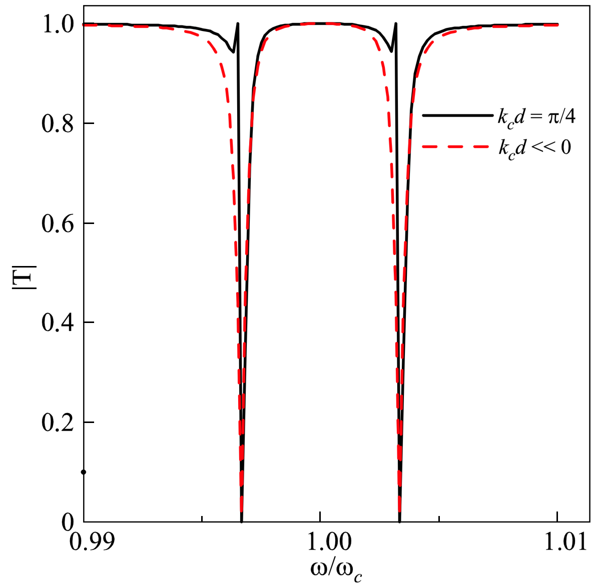

Fig. 7. Amplitude-frequency characteristic for the case of resonance $\Omega_1 = \omega_{c1}$ in the strong resonance regime.

At distances shorter than $\pi/2$ and comparable with the wavelength of the resonator fundamental mode, a resonance with an asymmetric profile occurs (Fig. 7). Two wave processes compete in the system. There is an interference of wave functions associated with each qubit-resonator couple, and Fano resonance is observed.[30]

The considered model makes it possible to estimate the entanglement of qubits in the system. We have evaluated the dependence of the concordance C[31–33] on the frequency of the incident photon under the weak dispersion regime. The maximum of concordance corresponds to the frequency of the incident photon $\omega = \pm\lambda_{1,2}$ (see Fig. 8).

Thus, the model proposed by us describes previously known effects, such as the dependence of the photon emission rate into the resonator cavity on the distance between qubits,[16]

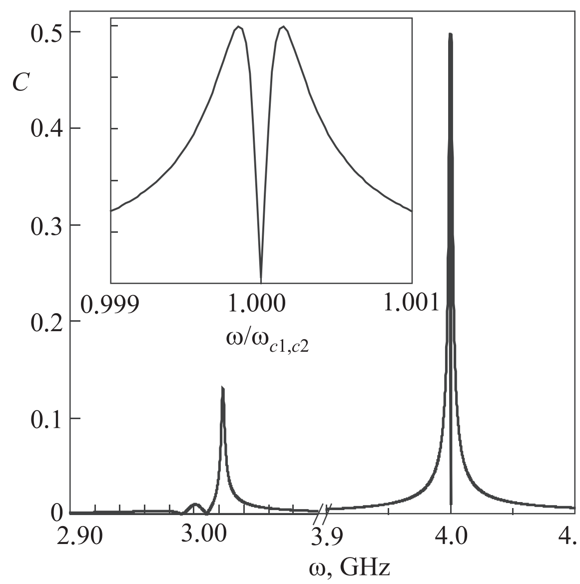

Fig. 8. Concordance of the system as a function of the frequency of an incident photon. The following parameters were used: $\Omega_1/2\pi = 3$ GHz, $\omega_{c1}/2\pi = 3$ GHz, $\lambda_1/2\pi = 1$ MHz, $k_c d < 1$, $\Omega_2/2\pi = 4$ GHz, $\omega_{c2}/2\pi = 4$ GHz, $\Gamma_{1,2} \gg \lambda_{1,2}$. At a photon frequency $\omega \pm \lambda_{1,2}$, the concordance of the system becomes nonzero.

the appearance of Fano resonances at the frequencies with the Rabi splitting taken into account in the pure quantum case. This shows that the observed effects occur even in the case of the propagation of a single photon in the system.[30] The calculation also takes into account the effect of non-instantaneous interaction of a photon with two resonators, which leads to the appearance of interference of the wave functions of virtual (undetectable) photons. It is shown that the internal resonances of the system depend not only on its configuration but also on the frequency of the incident photon.

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[a]Email: sultanov.aydar@ngs.ru

[b]Email: karpov@ilt.kharkov.ua

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