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Experimentally controlled stochastic resonance in a superconducting quantum interferometer

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The stochastic resonance effect is observed experimentally in a niobium superconducting quantum interferometer (RF SQUID loop) and manifests itself in a non-monotonic rise of the gain of a weak low-frequency harmonic signal which peaks at a certain level of Gaussian quasi-white noise flux inside the loop. It is shown experimentally that the gain of the weak harmonic signal can be varied and maximized when the noise flux intensity is insufficient to realize the SR condition by introducing a deterministic ac magnetic flux into the loop, the ac flux frequency highly exceeding the useful signal frequency.
Keywords: RF SQUID, stochastic resonance, controllable stochastic resonance, ScS contact, Josephson junction.

В одноконтактном ниобиевом сверхпроводящем квантовом интерферометре (кольце ВЧ СКВИДа) экспериментально наблюдается эффект стохастического резонанса, выражающийся в немонотонном росте усиления слабого низкочастотного гармонического сигнала, которое достигает максимума при определенном уровне квазибелого гауссова шумового потока, вносимого в кольцо. Экспериментально показано, что при интенсивности шумового потока, недостаточной для реализации условия стохастического резонанса, можно регулировать стохастическое усиление слабого гармонического сигнала и достичь его максимально возможного значения путем внесения в кольцо детерминированного переменного магнитного потока с частотой, значительно превышающей частоту усиливаемого полезного сигнала.
Ключевые слова: ВЧ СКВИД, стохастический резонанс, управляемый стохастический резонанс, ScS-контакт, контакт Джозефсона.

У одноконтактному ніобієвому надпровідному квантовому інтерферометрі (кільці ВЧ НКВІДу) експериментально спостерігається ефект стохастичного резонансу, що виражається в немонотонному зростанні посилення слабкого низькочастотного гармонійного сигналу, яке досягає максимуму при певному рівні квазібілого гаусового шумового потоку, що вноситься до кільця. Експериментально показано, що при інтенсивності шумового потоку, недостатній для реалізації умови стохастичного резонансу, можна регулювати стохастичне посилення слабкого гармонійного сигналу і досягти його максимально можливого значення шляхом внесення до кільця детермінованого змінного магнітного потоку з частотою, що значно перевищує частоту підсилюваного корисного сигналу.
Ключові слова: ВЧ НКВІД, стохастичний резонанс, керований стохастичний резонанс, ScS-контакт, контакт Джозефсона.

Introduction

The Superconducting Quantum Interference Devices (SQUIDs) based on low- and high-T_c superconductors are the key element in designing the most sensitive magnetometers widely used in laboratory setups, industry equipment, biomedical applications, geophysics, etc. The sensitivity of SQUIDs, usually degraded in a noisy environment, can, however, be enhanced [1-6] due to the same thermodynamic fluctuations and the external noise by using the stochastic resonance (SR) effect.

The SR conception was coined in the early 1980s [7,8]. The SR manifests itself in various ways, the most obvious one is a non-monotonic rise of the response of a non-linear system to a weak informational (often periodic) signal. As a result, the signal is amplified and peaks at a certain noise intensity. Other signal "quality characteristics" (e.g., signal-to-noise ratio) become better, too, at the system output. To make the SR possible in a specific system, the time duration for which the system exists in one of its metastable states (MS) (the residence time) must be a function of the noise intensity. The SR effect has been found in numerous natural and artificial systems, both classic and quantum. Till now, a lot of detailed analytical and experimental studies of the SR were performed, the criteria and quantifiers to estimate the noise-induced ordering were elaborated [9,10]. For the aperiodic systems with strong dissipation (which are mostly explored both theoretically and experimentally), the "stochastic filtration" (SF) is rather more correct term than the widely accepted "stochastic resonance" [11)].

Although a noticeable number of the theoretical and modeling studies of the SR in the superconducting loop are published, there is still a lack of the experimental investigations of the stochastic dynamics in SQUIDs [12] (e.g., [1-3]). Therefore some interesting challenges remain in this field, including possible practical applications. One of such problems is the issue of maximizing the signal gain while stochastically amplified at a non-optimal noise level.

If the barrier height is fixed, the optimal stochastic gain can be achieved by varying the noise intensity [9,10], but in most practical cases the noise intensity may be suboptimal while a change in the device temperature is undesirable. The SR gain can be controlled by changing the interferometer parameters (mainly the Josephson junction critical current; the loop inductance is hardly changeable) but this will alter the "operation point" of the device which incorporates the interferometer. Therefore, more convenient mechanisms controlling the stochastic signal amplification at suboptimal noise in the SQUID should be looked for. A noticeable number of methods to control the stochastic gain in various system including SQUIDs were suggested such as re-normalizing the potential barrier height in a single-junction interferometer by microwave field [13] (this effect was later [14] utilized to parametrically amplify a weak informational signal in an RF SQUID with microwave pumping), dynamic violation of symmetry of a model potential by mixing two harmonics with various amplitudes and initial phase shift [15], by changing the threshold of a Schmitt trigger with frequency of the input signal [16] and by system flip-over in a certain time with a pulse signal [17], etc. We should like to notice the theoretical work [18] suggested an approach to control the SR which is similar to that we realized experimentally; the substantial differences we will discuss below.

This work reports the experimental results demonstrating a possibility of controlling the stochastic amplification of a weak signal in an RF SQUID loop by introducing a periodic ac magnetic flux into the loop whose frequency is much higher than the signal frequency and amplitude is large enough to ensure an increase in the mean rate of transitions of the loop between its metastable current (magnetic) states. We found this effect earlier [19] by numerical simulation of the magnetic flux dynamics in an RF SQUID loop and called it "stochastic-parametric resonance".

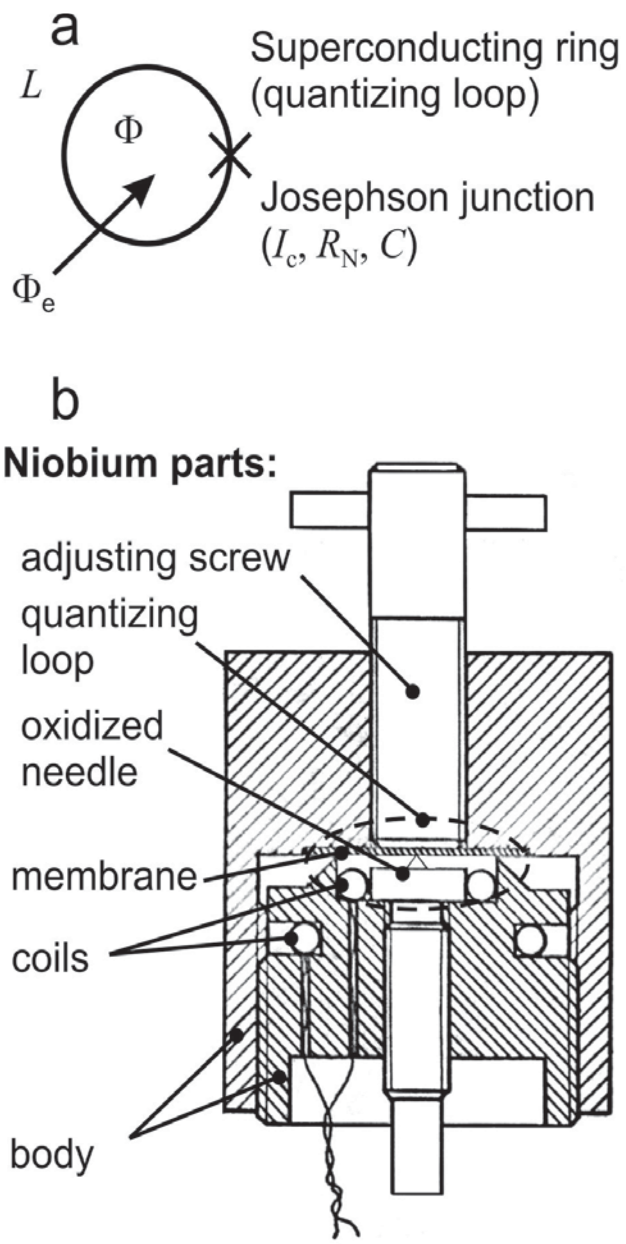

Fig. 1. (a) Principal schematic of the RF SQUID loop; the parameters are explained in the text. (b) Practical design of the RF SQUID loop as a 3D self-shielded toroidal construction with adjustable point Josephson contact, tank, transformer and input coils, all the parts are made of pure niobium except the tank and input copper-wire coils.

RF SQUID dynamics and experimental

RF SQUID loop is the "heart" of RF SQUID magnetometers. It is a superconducting loop with inductance L interrupted by a Josephson junction with critical current I_c, normal resistance R and capacitance C (Fig. 1,a). Assuming sinusoidal current-phase relation $I_s(\varphi) = I_c \sin\varphi$ for the Josephson junction, the RF SQUID potential energy, which is the sum of the loop magnetic energy and the coupling energy the Josephson junction, in dimensionless units reads

$$u(x, x_e) = (x - x_e)^2 / 2 - \frac{\beta_L}{4\pi^2}\cos(2\pi x), \quad (1)$$

where $x = \Phi / \Phi_0$ and $x_e = \Phi_e / \Phi_0$ are dimensionless internal and external magnetic fluxes, correspondingly, $\Phi_0 \approx 2.07 \cdot 10^{-15}$ Wb is the magnetic flux quantum, $\beta_L = 2\pi L I_c / \Phi_0$ is dimensionless parameter of non-linearity; the energy is normalized to $\Phi_0^2 / 2L$. The β_L

parameter defines the number and depth of the local minima of the SQUID potential energy, the potential becomes two- or multi-well when $\beta_L > 1$.

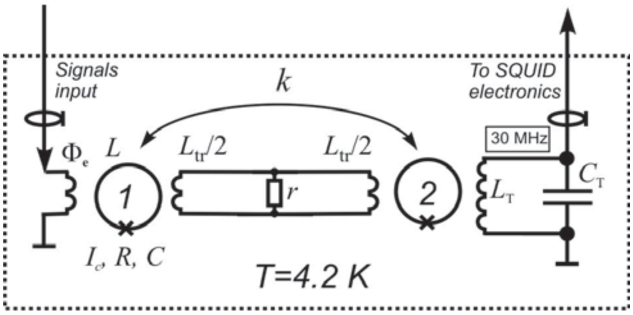

Fig. 2. The simplified diagram of the measurements. External magnetic flux Φ_e is applied by a coil to the loop of the interferometer-under-test (denoted by number 1). The in-loop flux is measured with RF SQUID magnetometer (denoted by number 2) via the superconducting magnetic flux transformer. The coupling coefficient between the loops is denoted by k. The dotted line indicates the superconducting led shield.

Real device, keeping topology of a ring, is usually a much more complicated unit than the schematic view of Fig. 1,a. Our interferometer-under-test was designed as the niobium 3D self-shielded toroidal construction with the adjusted point contact (Fig. 2,b). The device design is described in detail in [20]. In our experiments we tested the interferometers with $\beta_L \approx 4.7-5.4$ and low impedance ($R \sim 1\,\Omega$) Josephson junctions of the ScS (superconductor-constriction-superconductor) type having low intrinsic capacitance ($C \approx 3\cdot 10^{-15}$ F), the toroidal loop inductance being $L \approx 3\cdot 10^{-10}$ H. Such a parameter value set (the smallness of C and R) determines the overdamped regime of the SQUID as a stochastic oscillator and allowed us to neglect the second derivative in the flux motion equation [21] and reduce it to the form convenient for calculations and computer simulations [4-6]:

$$\frac{dx}{dt} = \frac{1}{\tau_L}\left[x_e(t) - x + \frac{\beta_L}{2\pi}\sin(2\pi x)\right], \qquad (2)$$

where $\tau_L = L/R$ is the loop flux decay time. As seen, the equation describes an aperiodic system. The external flux x_e is the sum of the fixed bias flux ($x_{dc} = 0.5$) symmetrizing the potential, the weak low-frequency signal $x_s = a\sin 2\pi f_s t$ (a<<1), uncorrelated (white) Gaussian noise $x_N = \xi(t)$, $\langle \xi(t)\cdot\xi(t')\rangle = 2D\delta(t-t')$, where D is the noise intensity (variance), and a high-frequency "pumping" $x_p = A\sin 2\pi f_p t$ with $f_p >> f_s$ and the amplitude A comparable with the noise mean-square amplitude $s = D^{1/2}$. In both in the calculations and the experiments the noise is frequency-band limited by a cut-off frequency f_c. To consider it practically uncorrelated in the context of discussed SR model, the cut-off frequency should sufficiently exceed the signal frequency: $f_c >> f_s$. In our experiments we chose $f_s = 37\ Hz$, $f_c = 50\ kHz$ and $f_p = 50\ kHz$. The Gaussian noise was generated by a real physical source (diode) and went through low-pass filters.

The interferometer-under-test (denoted by 1 in Fig. 2) was coupled to an instrumental RF SQUID magnetometer (denoted by 2 in Fig. 2) via the superconducting magnetic flux transformer L_{tr} with the interferometer loop-to-loop flux coupling coefficient $k = 0.05$. The resistor $r = 0.3\,\Omega$ shunted the transformer forming a low-pass filter with the cut-off frequency r/L_{tr}. It eliminated the influence of the RF (30 MHz) pumping oscillations in the instrumental SQUID tank circuit $L_T C_T$ onto the interferometer-under-test. The spectral density of the magnetic flux noise (the sensitivity) of the magnetometer was $S_\Phi^{1/2} \approx 2\cdot 10^{-4}\ \Phi_0 / Hz^{1/2}$ in the operation frequency band of 2 to 200 Hz. The coupling coefficients, the fluxes and the coil RF currents were determined from the measurements of the amplitude-frequency and the amplitude-flux characteristics of the interferometer-under-test while changing the loop flux within $\pm 5\Phi_0$. The experimental setup is similar ideologically to that reported in [2] and will be described elsewhere. The measurements were taken at temperature 4.2 K inside a superconducting shield. The cryostat was placed into a three-layer mu-metal shield. The output signal was fed to the spectrum analyzer Brüel&Kjær model 2033. The number of the instrumentally averaged spectra was 16.

Results and discussion

The numerical calculations [1,2,4-6] showed that the spectrum density of the internal flux in the SQUID loop at the frequency of the useful signal rapidly rises, peaks and then slowly decreases with the increase of the Gaussian noise intensity D, in accordance with the theory [9,10].

Fig 3 displays the experimentally obtained amplitude spectral density $S_\Phi^{1/2}(f_s)$ of the flux Φ inside the interferometer loop at the information signal frequency f_s as a function of mean-square amplitude of the Gaussian noise $D^{1/2}$. The amplitude of the harmonic information signal inside the interferometer-under-test was $a = 0.05$ in Φ_0 units, $\beta_L = 4.71$. The interferometer behavior is typical for the scenario of SR (or SF) in a bi-stable system. The maximum gain of about 10 dB was obtained in this

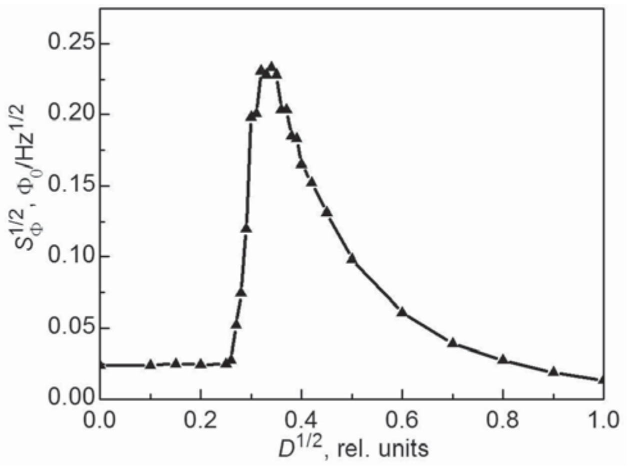

Fig. 3. The amplitude spectral density $S_\Phi^{1/2}(f_s)$ of the magnetic flux inside the RF SQUID loop at the signal frequency f_s as a function of the mean-square amplitude $D^{1/2}$ of the Gaussian noise. The signal amplitude $a = 0.05\ \Phi_0$, the signal frequency $f_s = 37\ Hz$, the noise cut-off frequency $f_c = 50\ kHz$, the non-linearity parameter $\beta_L = 4.71$, the temperature $T = 4.2\ K$.

experiment. It is interesting to note that the exact shape of this "classical" SR curve turns out to be fairly sensitive to the specific potential relief $U(\Phi)$. Verifying the curve by a numerical simulation, we found that the best fit corresponded to the model of ScS Josephson contact at a finite temperature [22] rather than traditionally used tunnel junction model (1) or even ScS contact model at zero temperature [6]. Although the niobium needle is thermally oxidized (Fig. 1,b) and the critical current calculated from the expression for β_L was small enough ($I_c = 5.2\ \mu A$), the real structure of the point contact may involve both tunnel and direct conductivity in various proportions making difficult to formulate an exact adequate model for its description. The curve comparison with various models and the details of the fitting procedure will be discussed in further papers.

We showed earlier [19] by a computer simulation that the SR signal gain can be maximized at an insufficient noise level by introducing a high-frequency field into the interferometer. We called this cooperative effect "stochastic-parametric resonance" because the high-frequency field affects the Josephson inductance as a device parameter. However, many various signal combinations were proposed to control the SR gain (see Introduction) that way or another changing the potential. Particularly, the difference may lie in the auxiliary signal frequency: if it is higher than the loop response time L/R then it works much like the temperature, really re-normalizing the barrier height [23]. Our case is adiabatic one, f_p<<L/R. To distinguish the effect discussed here, it is probably better to call it "deterministically-assisted stochastic resonance" (DASR). In the work [18] we found theoretical elaboration of a similar idea but the authors added an ac field with a frequency only 2-3 times higher than the weak signal frequency and analyzed both commensurate and incommensurate cases. We propose [19] "noise-substituting" periodic oscillations with a frequency that substantially exceeds the weak signal frequency, $f_p > 10 f_s$; practically, $f_p \approx 1000 f_s$.

Figure 4 shows the experimental curves of the amplitude spectral density $S_\Phi^{1/2}(f_s)$ of the internal magnetic flux Φ at the signal frequency f_s vs. amplitude A of the ac magnetic flux (high-frequency pumping) at

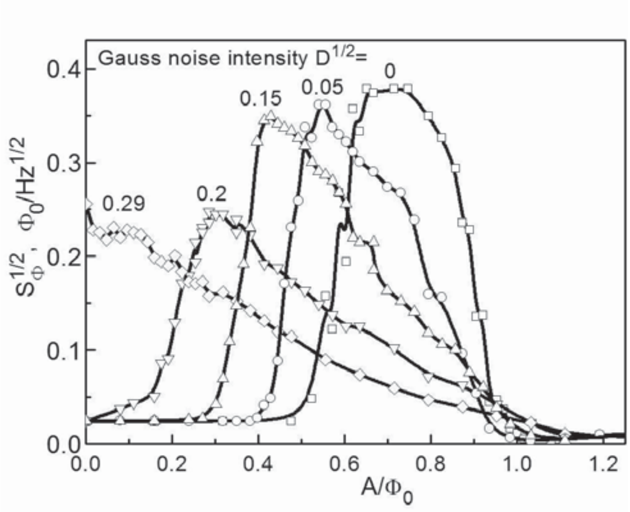

Fig. 4. The amplitude spectral density $S_\Phi^{1/2}(f_s)$ of the magnetic flux inside the RF SQUID loop at the signal frequency f_s as a function of the high-frequency ac magnetic flux A expressed in Φ_0 units at various sub-optimal mean-square amplitudes $D^{1/2}$ of the Gaussian noise indicated neae each curve). The signal amplitude $a = 0.05\ \Phi_0$, the signal frequency $f_s = 37\ Hz$, the noise cut-off frequency $f_c = 50\ kHz$, the sinusoidal ac flux frequency $f_p = 50\ kHz$, the non-linearity parameter $\beta_L = 4.71$, the temperature $T = 4.2\ K$.

various noise levels $D^{1/2}$. It is seen that if the noise intensity is lower than the optimal value necessary for obtaining maximum stochastic amplification (compare with Fig. 3), then the maximal gain can be reached through additional high-frequency pumping. Since f_p>>f_s, there is no experimental difference between frequency-commensurate and incommensurate modes. All the unwanted intermodulation products are far-spaced in the frequency domain and can be easily filtered out. Thus, this effect can be used in SQUID-based instruments (and other devices because of its universality) for fine tuning the stochastic amplification gain. Further investigation may include a change in signal-to-noise ratio, non-linear distortion, amplified frequency band, etc., as compared to classical SR.

Conclusion

1. The stochastic resonance effect in a single-junction superconducting quantum interferometer (RF SQUID) manifesting itself in an amplification of a weak harmonic low-frequency signal is experimentally observed.

2. The SR-like effect of a weak signal amplification, that we found earlier by numerical modeling, but caused, unlike SR, by the action of a periodic (deterministic) high-frequency field and the cooperative action of both this field and the noise flux inside the system is experimentally proved. We suggest designating it as "deterministically-assisted stochastic resonance" (DASR).

3. A possibility of controlling the stochastic amplification of a weak harmonic signal and maximizing the signal gain at a suboptimal noise level using the DASR effect is experimentally demonstrated.

4. A statement concerning the most adequate model of the Josephson contact incorporated in the interferometer-under-test is made which requires a further numerical analysis.

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