Current states in superconducting films: Numerical results

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We present numerical solutions of Aslamazov–Lempitskiy (AL) equations for distributions of the transport current density in thin superconducting films in the absence of external magnetic field, in both the Meissner and the vortex states. These solutions describe smooth transition between the regimes of a wide film and a narrow channel and enable us to find critical currents and current-voltage characteristics within a wide range of the film width and temperature. The normalized critical currents and the electric field were found to be universal functions of the relation between the film width and the magnetic field penetration depth. We calculate the fitting constants of the AL theory and propose approximating formulas for the current density distributions and critical currents. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4928918]

1. Introduction

The main distinguishing property of current states in wide superconducting films is an inhomogeneous distribution of the current density j across the film as a result of the Meissner screening of the current-induced magnetic field. It should be noted that the current state of a film is qualitatively different from the Meissner state of a bulk superconductor. Whereas the transport current I in the latter case flows basically within a surface layer with the thickness of the order of the London penetration depth λ, the current in a thin film with the thickness $d \ll \lambda$ is distributed over its width w according to the approximate power-like law,[1,2] $j \sim [(w/2)^2 - x^2]^{-1/2}$, where x is the transversal coordinate with the origin in the middle of the film. Thus, the characteristic length $\lambda_\perp(T) = 2\lambda^2(T)/d$,[3] which is commonly referred to as the penetration depth of the perpendicular magnetic field, is actually not the scale of the current decay, but rather plays the role of a "cutoff factor" in the above-mentioned law of the current distribution at the distances $\lambda_\perp$ from the film edges and thereby determines the magnitude of the edge current density. The latter was estimated in Ref. 1 as $j_e \approx I/d\sqrt{\pi w \lambda_\perp}$, assuming w to be larger than $\lambda_\perp$ and the coherence length ξ.

In such an inhomogeneous situation, the resistive transition of a wide film occurs[1–4] when j_e reaches the value close to the critical current density j_c^{GL} in the Ginzburg-Landau (GL) theory. This leads to the expression $I_c \approx j_c^{GL} d\sqrt{\pi w \lambda_\perp}$ for the critical current[1] which is widely used in analysis of experimental data (see, e.g., Refs. 5 and 6) and imposes a linear temperature dependence of the critical current $I_c(T) \propto 1 - T/T_c$ near the critical temperature T_c. The quantitative theory by Aslamazov and Lempitskiy (AL)[2] also predicts the linear dependence $I_c(T)$ but gives its magnitude numerically larger than the above estimate. This result has been confirmed in recent experiments.[7–9]

The instability of the current state at $I = I_c$ results in the entry of vortices into the film which leads to formation of the vortex part of its $I-V$ characteristic (IVC). The motion and annihilation of the vortices of opposite signs form a peak in the current density along the middle axis of the film. For certain current value I_m, the magnitude of this peak reaches j_c^{GL}, which causes instability of the stationary vortex flow.[2] Further behavior of the film depends on the conditions of the heat removal[10] and the quality of the films. In early experiments, an abrupt transition to the normal state has been usually observed at $I = I_m$, whereas in later researches, in which optimal heat compliance was provided, a step-like structure of the IVC, associated with the appearance of phase-slip lines, was observed at $I > I_m$.[7,11–14]

In the immediate vicinity of T_c where $\lambda_\perp(T)$ unlimitedly grows, any film reveals the features of a narrow channel at $\lambda_\perp \gg w$: the critical current in this case is due to uniform pair-breaking thus showing the temperature dependence of the GL pair-breaking current $I_c^{GL}(T) \propto (1 - T/T_c)^{3/2}$. As the temperature decreases, the film is expected to exhibit a crossover to the wide film regime at $\lambda_\perp \lesssim w$, when the vortex part of the IVC occurs and the temperature dependence $I_c(T)$ should be linear.[1,2] However, it was found[8,9] that in moderately wide films, the nonlinear temperature dependence of I_c holds down to low enough temperatures and transforms to the linear one only when $\lambda_\perp(T)$ becomes smaller than the film width by the factor of 10–20, although the vortex state already occurs at much larger values of $\lambda_\perp \sim (0.2 - 0.25)w$. Similar difficulties were met in the experiment[15] when trying to interpret the IVC measurements by using asymptotical results of the AL theory, because the strong condition $I_c \ll I_m$ used in Ref. 2 can be fulfilled only in extremely wide films whose width exceeds $\lambda_\perp(T)$ by several orders of magnitude. Thus, there exist a considerable intermediate region of the film widths and temperatures, where the assumptions and initial equations of the AL theory remain valid, but the asymptotic results cannot provide satisfactory agreement with the experimental data.

In order to obtain a quantitative theoretical description of the current states within a wide region of the ratio $w/\lambda_\perp$, we perform in this paper a numerical solution of the AL equations which describes smooth transition between the regimes of a wide film and a narrow channel. The critical currents I_c and I_m normalized on the GL critical current I_c^{GL}, as well as the specifically normalized IVC, were found to be universal functions of the ratio $w/\lambda_\perp$. We calculate the fitting constants in the asymptotic formulas of the theory[2] and propose approximating expressions for the current density distributions which are in rather good agreement with the results of the numerical computations.

We note that the AL model assumes rather weak pinning, taking the presence of defects into consideration only through the viscosity of the vortex fluid. The opposite case of strong pinning corresponds to the model of critical state with unmovable vortices, which results in quite different distributions of the current and magnetic field (see, e.g., the reviews 16 and 17).

2. Basic equations and asymptotic results of AL theory

A starting point of the AL theory are static GL equations for the dimensionless modulus F of the order parameter (normalized on its equilibrium value in the GL theory) and the gauge-invariant vector potential $\mathbf{Q} = \mathbf{A} - \kappa_{\rm eff}^{-1}\nabla\chi$,

$$\kappa_{\rm eff}^{-1}\nabla^2 F + F(1 - F^2 - Q^2) = 0, \tag{1}$$

$$\mathrm{rot\,rot}\,\mathbf{Q} = -F^2\mathbf{Q}\delta(z). \tag{2}$$

Here the electromagnetic vector potential $\mathbf{A}$ is measured in units of $\varphi_0/2\pi\xi$, φ_0 is the magnetic flux quantum, χ is the order parameter phase and $\kappa_{\rm eff} = \lambda_\perp/\xi$, is the effective GL parameter. The axis z is perpendicular to the film whose thickness is assumed to be infinitely small, and all distances are measured in units of $\lambda_\perp$.

Usually in thin films, the GL parameter is large, $\kappa_{\rm eff} \gg 1$ Assuming the film width much larger than $\xi(T)$, one thus can neglect the gradient term in Eq. (1) and use the local relation $F^2 = 1 - \mathbf{Q}^2$ between the order parameter and the vector potential. Inside the thin film, the latter has only one component $Q \equiv Q_y$ and can be found from equation

$$\frac{dQ}{dx} = -\frac{1}{2\pi}\int\limits_{-\tilde{w}/2}^{\tilde{w}/2}\frac{Q(x')\left[1 - Q^2(x')\right]}{x' - x}dx', \quad \tilde{w} = w/\lambda_\perp, \tag{3}$$

with the Biot-Savard integral which relates the magnetic field dQ/dx to the dimensionless density $j = Q(1 - Q^2)$ of the surface current. Equations (1)–(3) determine the stability threshold of the Meissner state, when the vortices begin to penetrate into the film, and the edge value of the vector potential appears to be close to its critical value $Q_c^{GL} = 1/\sqrt{3}$ in the GL theory for narrow channels.* The asymptotic value of the critical current at $w \gg \lambda_\perp$ has been calculated in Ref. 2 and then refined in Ref. 18:

$$I_c^{AL} = \sqrt{15/8}I_c^{GL}(\pi\lambda_\perp/w)^{1/2}. \tag{4}$$

The resistive vortex state of a wide film is described by including the contribution of the vortices $n\varphi_0(n(x)$ is the vortex density) to the net magnetic field induction in the hydrodynamic approximation.[1,2,19] Using the continuity equation for the flux density nv of the vortex fluid, expressing the vortex velocity v through the linear current density j and the viscosity coefficient η as[21]

$$v = -\eta^{-1}\varphi_0 j\,\mathrm{sign}\,x, \tag{5}$$

and the average electric field—through the flux density as $E = -nv\varphi_0$, one obtains equation

$$4\pi\frac{\lambda_\perp}{w}\frac{dj}{dx} + 2\int\limits_{-1}^{1}\frac{j(x')dx'}{x' - x} = -\frac{\eta c^3 E}{\varphi_0 j(x)}\mathrm{sign}\,x. \tag{6}$$

Here and below, the coordinate x is normalized on the film half-width $w/2$, and the expression sign x indicates the opposite direction of the vortex motion in different halves of the film.

An asymptotic analysis of Eq. (6) at $w \gg \lambda_\perp$ shows[2] that the IVC is linear in the vicinity of I_c, whereas at large currents, the voltage grows quadratically,

$$V = E_0 L\begin{cases}(I - I_c)/I_c, & I - I_c \ll I_c;\\ C(I/I_c)^2, & I \gg I_c,\end{cases} \quad E_0 = \frac{8\varphi_0 I_c^2}{\eta w^2 c^3}, \tag{7}$$

until the transport current reaches the threshold of stability of the vortex state,

$$I_m = C' I_c^{GL}\ln^{-1/2}(w/\lambda_\perp). \tag{8}$$

In Eqs. (7) and (8), L is the film length, I_c^{GL} is the GL critical current formally calculated for uniform current distribution and C, C' are fitting constants which cannot be determined by the asymptotic analysis.

We note that the validity of the AL theory for the analysis of the vortex state is confined by applicability of the static Ginzburg-Landau equations to the description of the vortex motion. With this approach, the order parameter relaxation time $\tau_\Delta \sim (T_c/\Delta)\tau_\varepsilon$ (τ_ε is the energy relaxation time) is assumed to be much smaller than other characteristic times of the system. In the opposite case, finiteness of τ_Δ results in a considerable deformation of the vortex core and in occurrence of a wake with suppressed order parameter behind the moving vortex. As shown by numerical simulation,[20] this may anomalously enhance the vortex velocity and lead to creation of rapidly moving chains of vortices treated in Ref. 21 as nuclei of the phase-slip lines.

3. Results of numerical calculations

In our calculations, we perform numerical solution of Eq. (6) with a certain modification. As is obvious, the left-hand side of Eq. (6) is the approximate form of Eq. (3), in which the vector potential Q in the gradient term is replaced by the current density j. Such an approximation corresponds to the linear London relation $\mathbf{j} \sim \mathbf{Q}$ between the current and the vector potential which assumes independence of Δ of the vector potential. For this reason, Eq. (6) is usually referred to as a generalized London equation.[1,2] This does not essentially affect the asymptotic results[2] because the gradient term is small at $w \gg \lambda_\perp$; however, in our calculations, we

*Actually, according to the definition of Q, the correct formula for the dimensionless current density must have the opposite sign, $j = -Q(1 - Q^2)$. Thus, to deal with positive values of the transport current, one should consider negative values of Q. To avoid this inconvenience, the authors of Refs. 1 and 2 define j as presented in the text and use positive Q values, that does not affect the results. In our paper, we also follow this convention.

will use the full nonlinearized version of Eq. (6) in a dimensionless form (see also Ref. 18):

$$\alpha\frac{dQ}{dx}+\frac{1}{4}\int_{-1}^{1}\frac{i(x')dx'}{x'-x}=-\frac{E'\,\mathrm{sign}\,x}{i(x)},\qquad(9)$$

where the following definitions are introduced,

$$i=Q\left(1-Q^2\right),\quad E=E'E_0,\quad j(x)=\frac{3\sqrt{3}}{2}\frac{I_c^{GL}}{w}i(x),$$
$$E_0=\frac{54\varphi_0}{\eta w^2c^3}\left(I_c^{GL}\right)^2,\quad \alpha=\frac{\pi\lambda_\perp}{2w}.\qquad(10)$$

The distribution of the vector potential is obviously symmetric, $Q(x)=Q(-x)$, which enables us to consider Eq. (9) only in the region $x>0$ and to reduce the integral in (9) to the region $x'>0$. After integration of Eq. (9) from the film edge to a given point x, we finally get**

$$\alpha[Q(x)-Q_e]=\frac{1}{4}\int_{0}^{0}i(x')\ln\left|\frac{x^2-x'^2}{1-x'^2}\right|dx'-E'\int_{1}^{x}\frac{dx'}{i(x')},\qquad(11)$$

where $Q_e\equiv Q(1)$ is the edge value of the vector potential. In these notations, the net current I is given by equation

$$I=w\int_{0}^{1}j(x)dx=\frac{3\sqrt{3}}{2}I_c^{GL}\int_{0}^{1}i(x)dx.\qquad(12)$$

At $I<I_c$, the quantity Q_e increases with the current and has to be determined self-consistently from Eqs. (11) and (12) at zero electric field; this procedure simultaneously gives the solution for the current distribution across the film. As noted above, the resistive state of a wide film at $I=I_c$ occurs when Q_e approaches the critical value $Q_c=1/\sqrt{3}$. Such a relation is also obviously valid for narrow channels, that makes it reasonable to extend it over the films of arbitrary width. In the resistive vortex state, $I>I_c$, the quantity Q_e holds its critical value Q_c, and Eqs. (11) and (12) determine the dependence $E'(I)$, i.e., the IVC, $V(I)=E'(I)E_0L$.

A specific property of these equations is that their solutions, i.e., the normalized current density distribution $i(x)$ and the electric field E', are universal functions of the parameters $w/\lambda_\perp$ and I/I_c^{GL}. This implies that the normalized critical current I_c/I_c^{GL} and the maximum current of existence of the vortex state, I_m/I_c^{GL}, as well as the normalized maximum electric field $E'_m=E(I_m)/E_0$, are universal functions of the parameter $w/\lambda_\perp$. Thus, the temperature dependencies of these quantities, being expressed through the variable $w/\lambda_\perp(T)$, must coincide for the films with different widths and thicknesses, which has been demonstrated in experiments.[8,9]

3.1. Solution in subcritical regime $I\le I_c$

Solution of Eqs. (11) and (12) can be found by iteration method, using Q_e as initial approximation for the function $Q(x)$. Although the iteration parameter $\alpha^{-1}\sim w/\lambda_\perp$ is large for a wide film, convergence of the iterations can be nevertheless provided by introducing certain weight factors for contributions of previous and current iterations. The result of numerical calculation of the reduced critical current, shown in Fig. 1, describes transition from the uniformly distributed GL depairing current $I_c^{GL}\propto(T_c-T)^{3/2}$ in a narrow channel to the critical current $I_c^{AL}\sim T_c-T$ for a wide film (4). As seen from Fig. 1, the asymptotic dependence (4) can be achieved with appropriate accuracy only at rather large ratio $w/\lambda_\perp>10-20$.

It should be noted that in some experiments,[7,9,23] the behavior of $I_C(T)$ at the beginning of transition to the wide film regime was found to be different from the smooth dependence following from the AL theory. Namely, when the temperature decreases and the ratio $w/\lambda_\perp$ exceeds 4–5, the critical current sharply falls to the value $I_c(T)\approx0.8I_c^{GL}(T)$ and holds this level until $w/\lambda_\perp\lesssim10$. Within this temperature interval, the film enters the vortex state at $I>I_c$, although the temperature dependence of I_c is similar to the case of a vortex-free narrow channel. Analogous behavior of the critical current in wide films has been registered in early experiments.[24,25] To explain such a specific dependence of $I_c(T)$, it was supposed[9] that the Pearl vortices[26,27] in moderately wide films may overcome the edge barrier at the edge current density $\sim(1-T/T_c)^2$ much smaller than the GL critical current density $\sim(1-T/T_c)^{3/2}$, possibly due to interaction with the opposite film edge.

In Fig. 2(a) we present the current density distribution across the film at the resistive transition point $I=I_c$ and different ratios $w/\lambda_\perp$. Interestingly, these distributions are well approximated by function

$$j_1(x)=j_e\frac{a}{\sqrt{1-(1-a^2)x^2}}.\qquad(13)$$

Equation (13) represents a modification of the asymptotic function[1,2,19] $j(x)=j(0)(1-x^2)^{-1/2}$ in with a regularization parameter $a=j_1(0)/j_e$ which provides finiteness of the approximated current density (13) at the film edges. As follows from its definition, this parameter characterizes suppression of the current in the middle of the film due to the Meissner screening. Substituting Eq. (13) with $j_e=j_c^{GL}\equiv I_c^{GL}/w$ into Eq. (12), we obtain equation for its value $a_c=\cos\varphi$ at the critical current,

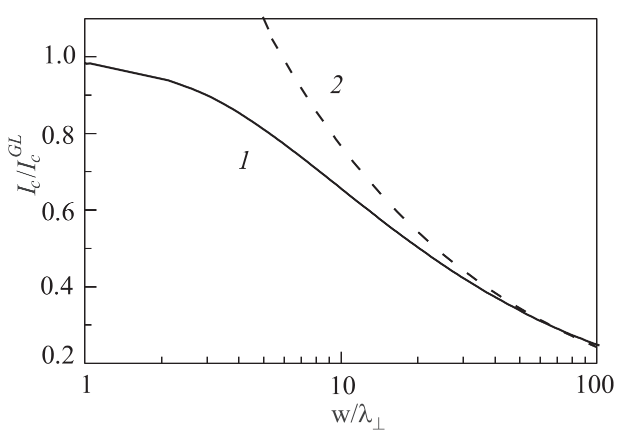

FIG. 1. Numerically calculated critical current (*1*) compared to the asymptotical estimate (4) (*2*).

**Similar method has been applied to the problem of critical magnetic field of a wide film at I = 0.[22]

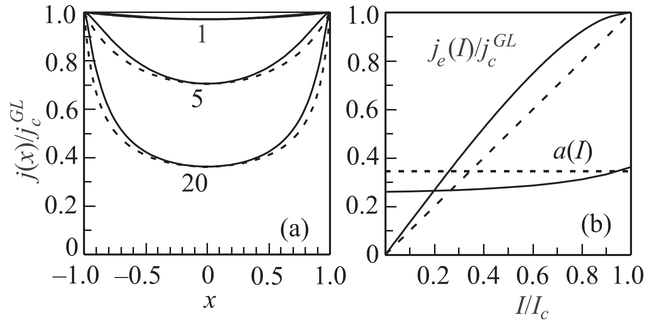

FIG. 2. Current distributions over the film width at the resistive transition point, $I - I_c$, numerically calculated for different values $w/\lambda_\perp = 1$, 5, and 20 (solid lines). Dashed lines show the approximating dependence (13) with the critical value of the coefficient $a = a_c$ found from Eq. (14) (a); numerically calculated dependencies of the edge current density j_e and the current suppression coefficient in the middle of the film, $a = j(0)/j_e$, on the transport current I for the wide film $w/\lambda_\perp = 20$ (solid lines). For comparison, the values (15) and (16) found from the generalized London's equation (6) are shown by the dashed lines (b).

$$I_c/I_c^{GL} = \varphi/\tan\varphi. \tag{14}$$

In the case of a wide film, $w \gg \lambda_\perp$, the coefficient a_c is small, $a_c \ll 1$, and it can be estimated by using the asymptotic value (4) of the critical current as

$$a_c = 2.74(\lambda_\perp/\pi w)^{1/2}. \tag{15}$$

Within the framework of the generalized London's equation (6) in which the effect of the current on the order parameter is neglected, the current distribution is determined only by geometric factors, therefore the coefficient a is independent of the current and holds a constant value a_c. The edge current density in this approximation varies linearly with the transport current, that reproduces the result of Ref. 1,

$$j_e = (I/I_c)j_c^{GL} \tag{16}$$

(see dashed lines in Fig. 2(b)).*** In the general case described by Eq. (9), the dependence $j_e(I)$ appears to be nonlinear, and the coefficient a increases with the current and approaches a maximum value a_c at $I = I_c$ (solid lines in Fig. 2(b)). Physically, this is due to suppression of the order parameter by the transport current, that weakens the screening effect while the current increases.

3.2. Solution in the vortex state, $I_c < I \leq I_m$

In the region of the vortex resistivity, the distribution of the screening current is superimposed by the distribution associated with the vortex motion and having a peak at the middle of the film, as shown in Fig. 3. The logarithmic feature $\sim\ln^{1/2}(w/\lambda_\perp)$ of this peak predicted in Ref. 2 appears to be rather weak and remains visible only for a certain intermediate current value; at $I \to I_m$, this feature practically vanishes. Such an inhomogeneous current distribution with three maxima in the vortex state of wide films has been visualized experimentally[29] by using the laser scanning microscope.

***Numerical solutions of (6) have been obtained in Ref. 28 and used for determination of the penetration depth.

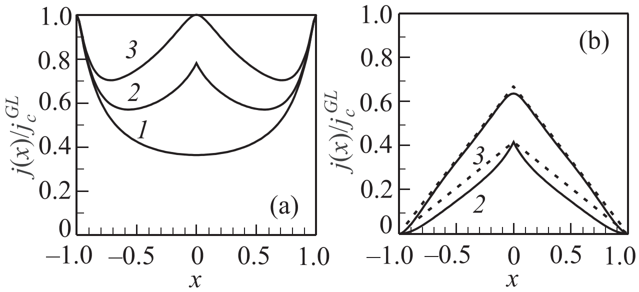

FIG. 3. Solid lines—distributions of the net current density (a) and the vortex contribution (b) in the vortex state at $w/\lambda_\perp = 20$ numerically calculated at different values of the transport current: $I = I_c$ (*1*), $I = 0.5(I_c + I_m)$ (*2*), $I = I_m$ (*3*). The approximating distributions (17) are shown by dashed lines.

For moderately wide films, in which the above-mentioned logarithmic factor is of the order of unity, the vortex contribution can be approximated by a piecewise-linear function

$$j_2(x) = j_c^{GL} b(1 - |x|) \tag{17}$$

depicted in Fig. 3(b) by dashed lines. As follows from (17), the parameter $b = j_2(0)/j_c^{GL}$ represents the relative (in units of j_c^{GL}) current density created by vortices in the middle of the film. Within such an approximation, this parameter linearly depends on the transport current,

$$b(I) = 2(I - I_c)/I_c^{GL}. \tag{18}$$

According to Ref. 2, the vortex state becomes unstable when the height of the central peak of the current distribution approaches j_c^{GL}. Using this condition and solving Eqs. (9) and (11) at the critical edge value of the vector potential, $Q_e = Q_c$, we determine the maximum current of existence of the vortex state I_m and the normalized maximum electric field $E'_m = E'(I_m)$. The results of numerical calculation compared to the asymptotic results of the AL theory are presented in Fig. 4 by solid and dashed lines, respectively. At large enough values of $w/\lambda_\perp \gtrsim 20 - 30$, the asymptotic dependencies[2]

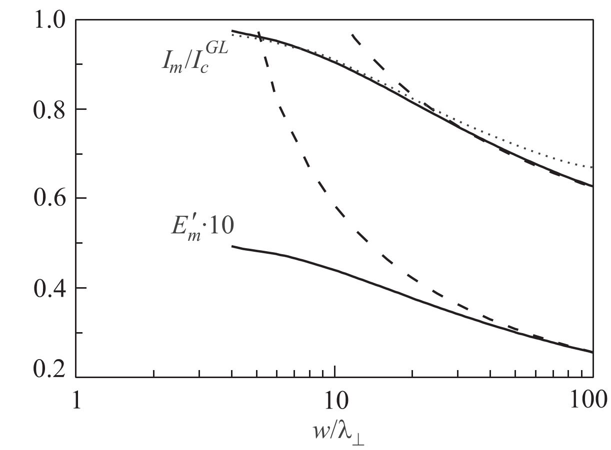

FIG. 4. Dependencies of the maximum current of existence of the vortex state I_m and the normalized maximum electric field E'_m on the parameter $w/\lambda_\perp$ (solid lines). Dashed lines show their asymptotic behavior (19) in the AL theory with the fitting constants $C_1 = 1.2$, $C_2 = 0.4$, $C_3 = 0.062$. Dotted line depicts the approximating dependence (20) of I_m, in which the result of numerical calculation of I_c (see Fig. 1) and the formula (15) for the parameter a_c were used.

$$I_m/I_c^{GL} = C_1 \ln^{-1/2}(C_2 w/\lambda_\perp), \quad E'_m = C_3 (I_m/I_c^{GL})^2, \quad (19)$$

can be fitted to the numerical results by an appropriate choice of the fitting constants of the AL theory (shown in the caption of Fig. 4) which cannot be evaluated within the framework of the asymptotical analysis. In order to obtain a satisfactory agreement, one has to introduce an additional constant C_2 into the argument of the logarithm, since the formulas (19) were derived in Ref. 2 with logarithmic accuracy. At smaller $w/\lambda_\perp \lesssim 20$, the asymptotic results (19) considerably overestimate the numerically obtained values of I_m and E'_m.

Another useful expression for I_m suitable for a rather wide range of film widths can be obtained from the approximating current distributions (13) and (17). At the stability threshold of the vortex state, where $j(0) = j_1(0) + j_2(0) = j_c^{GL}$, the relation $b = 1 - a_c$ is fulfilled which leads to equation

$$I_m = I_c + 0.5 I_c^{GL}(1 - a_c). \quad (20)$$

As seen from Fig. 4, this approximation (dotted line) rather well reproduces the result of numerical calculations of I_m, up to the point of nucleation of the vortex resistivity at $w/\lambda_\perp \approx 4$ as 4. For extremely wide films, in which $I_m \gg I_c$ and the logarithmic peak of the current is well pronounced, the AL asymptotic expression (19) for I_m with numerically calculated fitting constants is more preferable.

In Fig. 5 a normalized IVC per unit length of a wide film ($w/\lambda_\perp = 20$) is shown by the curve *1* within the region of the stable vortex state $I_c < I < I_m \approx 1.7 I_c$. Its initial part coincides with the linear AL asymptotic (line *2*) at $I - I_c \ll I_c$,

$$E'(I) = \frac{4}{27}\left(\frac{I_c}{I_c^{GL}}\right)^2 \left(\frac{I}{I_c} - 1\right) \approx 0.873 \frac{\lambda_\perp}{w}\left(\frac{I}{I_c} - 1\right), \quad (21)$$

obtained by using Eq. (4) for I_c. At $I > 1.4 I_c$, the IVC is well described by the modified AL asymptotic for $I \gg I_c$:

$$E'(I) = C_1 \frac{4}{27}\left(\frac{I_c}{I_c^{GL}}\right)^2 \left(\frac{I}{I_c} - C_2\right)^2$$
$$\approx 0.873 C_1 \frac{\lambda_\perp}{w}\left(\frac{I}{I_c} - C_2\right)^2 \quad (22)$$

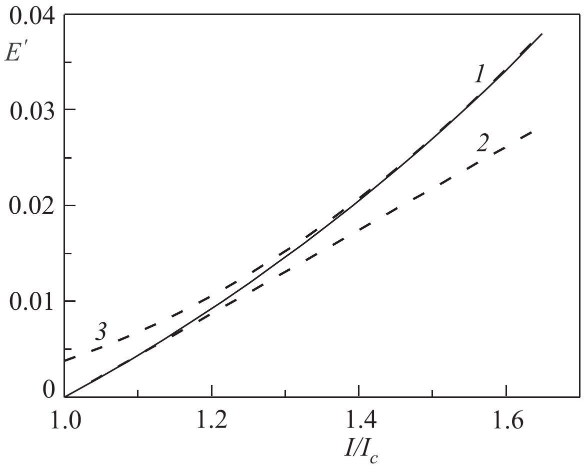

FIG. 5. Numerically calculated IVC of the superconducting film in the vortex state at $w/\lambda_\perp = 20$ (solid line *1*). Dashed straight line *2* is the linear AL asymptotic for $I - I_c \ll I_c$ (21), and the parabola *3* is the shifted AL asymptotic for $I_c \gg I_c$ (22).

with the fitting constants $C_1 = 0.97$ and $C_2 = 0.7$. Introduction of an additional constant C_2, which shifts the original AL parabola, enables us to generalize the result obtained in Ref. 2 for the case of large supercriticality, $I \gg I_c$, to the region of currents comparable with I_c. Such a modification of the AL asymptotic formulas has been successfully used for fitting of the parabolic part of the IVC.[15] In experiments with films of relatively small width (in which the vortex state nevertheless exists), the region of vortex resistivity is rather narrow, $I_m - I_c \ll I_c$, and only a linear part of the IVC is observed.[7,15,23]

Similar current distributions and IVCs were obtained by numerical simulation of the vortex motion in an infinitely long and thick superconducting slab.[20] Although these results cannot be quantitatively applied to the thin film because of essential difference between Abrikosov vortices in a bulk slab and Pearl vortices in a thin film,[26,27] they give an additional theoretical evidence of intrinsic nonlinearity of the IVCs in the vortex state, which is often ascribed to the flux creep or to the nonequilibrium state of quasiparticles in the vortex core.[30]

4. Summary

We studied distributions of the transport current density in thin superconducting films in zero external magnetic field within a wide range of the film widths w and temperatures, using numerical solutions of the integro-differential equations for the gauge-invariant vector potential. We found that these solutions can be approximated by rather simple analytical formulas, the parameters of which have a clear physical meaning and can be relatively easily calculated.

We found that the reduced critical current I_c and the reduced maximum current of existence of the vortex state I_m (both normalized on the Ginzburg-Landau critical current in a uniform current state), as well as the reduced maximum electric field in the vortex state are universal functions of the parameter $w/\lambda_\perp$; this has been confirmed in the experiment. We calculated numerically the current-voltage characteristic of a wide film in the vortex state and propose a modification of the asymptotic results[2] which provides much better fitting with the experimental data. For wide enough films, $w/\lambda_\perp \gtrsim 20 - 30$, our results coincide with the asymptotic dependencies[2,18] with properly chosen fitting constants.

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