SHORT NOTES

Experimental observation of induced stochastic transitions in a multiwall potential of an rf-SQUID loop

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(Submitted July 4, 2014)
Fiz. Nizk. Temp. **40**, 1312–1316 (November 2014)

Experimental observation of a weak low-frequency harmonic signal in a point (superconductor-constriction-superconductor, ScS) contact superconducting quantum interference device (rf-SQUID loop), being amplified due to the stochastic transitions between two or more metastable states of the loop, under the influence of applied noise flux of varying intensity (the effect of stochastic resonance, SR). In addition to the usual SR effect found in a bi-stable system with Gaussian noise, there were observed transitions between several metastable states of the multiwell SQUID loop potential, due to the influence of binary noise, which can be interpreted as a kind of noise "spectroscopy" of the loop's metastable states, with varying values of trapped magnetic flux. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4901990]

1. Introduction

Superconducting quantum interference devices (SQUIDs) are the basis of the most sensitive magnetometers today, which are widely used in physics experiments, and applied fields, such as medicine, geophysics, and industries. The sensitivity of low-temperature SQUIDs and their quantum analogues (SQUBIDs, super-conducting qubit detectors) have almost reached the quantum limit.[1–3] However, with an increase in temperature T, and inductance of the quantization loop, L, there is an increase in the uncertainty of the noise flux, thus decreasing the energy resolution (sensitivity) of the magnetometers. This problem is especially dire for nitrogen-cooled HTS SQUIDs, for which it is necessary to significantly reduce the inductance of the quantization loop, and, respectively, the magnetic field sensitivity. Nonetheless, as shown in Refs. 4–9, thanks to the same thermodynamic fluctuations and external noise, the sensitivity of the magnetometers can be significantly improved by the use of the effect of stochastic resonance (SR).

The SR phenomenon, discovered in the 1980s,[10,11] is manifested via the non-monotonic amplification of the response from a non-linear, often bi-stable system, to a weak informational periodic signal, the properties of which are significantly improved at a certain optimal level of noise, at the system's output. In fact, in order for SR to exist in a particular system, it is sufficient for the residence time of the system in its metastable states (MS), to somehow depend on the noise intensity. The SR effect is found in many natural and artificial systems, classical and quantum, and analytical approaches and quantifying criteria for the estimation of the ordering due to the noise impact, have been developed and described.[12,13] For aperiodic systems with strong damping (which are most often examined theoretically and investigated experimentally) the established term "stochastic resonance" is not entirely correct, and it is better to discuss "stochastic filtering" (SF).[14] Practically all HTS SQUIDs refer specifically to such systems.

Despite of the considerable number of theoretical and modeling studies of SR in a superconducting loop, there are not many experimental studies of the stochastic dynamics in SQUIDs (for example, Refs. 4–6).[15] Therefore, interesting questions remain in this area, from a practical point of view. For example, in the field of intense fluctuations, $L \geq L_F = \Phi^2{}_0/(4\ \pi^2 k_B T)$, is it possible to significantly increase the HTS SQUID sensitivity using SF, for certain choice of parameters? Here, L_F is the inductance fluctuation, $L_F \sim 10^{-10}$ H at $T = 77$ K, $\Phi_0 = 2.07 \times 10^{-15}$ Wb is the superconducting magnetic flux quantum, and k_B is the Boltzmann constant. In addition, we will note that even if the system has a lot of states, almost always, in the context of SF and SR, only transitions between two MS are examined. The type of noise and its statistics may also contribute its specifics to the stochastic dynamics of the interferometer.

In this study, we experimentally observed the amplification of a weak, low-frequency harmonic signal in an rf-SQUID loop, due to the stochastic transitions between two or more MS loops under the influence of Gaussian and binary noises, of varying intensity (stochastic filtration effect).

2. Problem statement and experimental method

In the absence of fluctuations, the number of local minima of potential energy of the loop quantization

$$u(x, x_e) = (x - x_e)^2/2 - \frac{\beta_L}{4\pi^2}\cos(2\pi x),$$

(number of MS rings) is determined by a dimensionless parameter of nonlinearity $\beta_L = 2\ \pi L I_C/\Phi_0$: $n \approx 2\beta_L \pi$. Here, $x = \Phi/\Phi_0$ and $x_e = \Phi_e/\Phi_0$, are dimensionless internal and external magnetic fluxes, respectively, the energy is normalized to $\Phi_0{}^2/2L$, and I_C is the contact critical current.

Our experiments were conducted on low resistance ($R \sim 1\,\Omega$) Josephson-junction ScS interferometers (superconductor-constriction-superconductor) with a low self-capacitance ($C \approx 3 \times 10^{-15}$ Φ) for large $\beta_L \approx$ 7–10. At such parameters, the case of large damping is realized, which is characteristic of the majority of HTS SQUIDs.

Given the smallness of C and R, we can neglect the second derivative of the flux equations[16] and bring it to the form that is convenient for the calculations[7–9]

$$\frac{dx}{dt} = \frac{1}{\tau_L}\left[x_e(t) - x + \frac{\beta_L}{2\pi}\sin(2\pi x)\right], \quad (1)$$

where $\tau_L = L/R$ is the flux damping time in the loop. As can be seen, the equation describes an aperiodic system. The external flux x_e is the sum of the permanent flux displacement ($x_{dc} = 0.5$) for the symmetrization of potential, weak low-frequency signal $x_s = a\sin 2\pi f_S t$ ($a \ll 1$), and uncorrelated (white) Gaussian noise $x_N = \xi(t)$, $\langle \xi(t)\cdot\xi(t')\rangle = 2D\delta(t-t')$, where D is the noise intensity. However, both in calculation and experiment, the noise has a top limit on the frequency. For it to be virtually uncorrelated, the cutoff frequency f_C must be considerably larger than the frequency of the signal $f_C \gg f_S$. In our experiments, we selected $f_S = 37$ Hz and $f_C = 50$ kHz. The generator allowed us to get the noise with Gaussian distribution, as well as the telegraph noise from the primary real source (diode).

The test interferometer (*1* on Fig. 1) was made of niobium with a 3D self-shielded toroidal design having adjustable point contacts, and was connected via magnetometer to the measuring rf-SQUID (*2* on Fig. 1) through a superconducting magnetic flux transformer (the flux coupling coefficient between the interferometer loops is $k = 0.05$). The design of the sensor is described in detail in Ref. 2. The noise spectral density of the magnetic flux (sensitivity) of the magnetometer was $S_\Phi^{1/2} \approx 2 \times 10^{-4}\,\Phi_0/\mathrm{Hz}^{1/2}$ in the measured band of frequency 2–200 Hz. The calibration of the coupling coefficients, fluxes, and currents of the coils, was performed by measuring the amplitude-frequency and signal properties of the test interferometer, when the flux was changed by $\pm 5\Phi_0$. The experimental set up is ideologically similar to the one published in Ref. 5, and will be described in another piece. Measurements were conducted at a temperature of 4.2 K inside a three-layer Permalloy and a superconducting shield. The output signal of the magnetometer was fed to the spectrum analyzer. The number of apparatus-averaged Fourier spectra in each measurement was 16. The measurements were done twice, both with the informational signal, and without, and the difference between the two obtained spectra was used as the result.

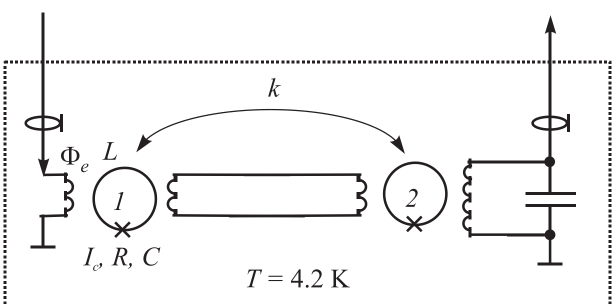

FIG. 1. A simplified diagram of the measurements. An external magnetic flux, Φ_e, is created by a coil, and acts on the loop, 1, of the test interferometer. The flux in the loop is measured by the rf-SQUID magnetometer, designated by the number 2, through the superconducting magnetic flux transformer. The flux coupling coefficient between the loops is k. The dotted line shows the superconducting lead shield.

3. Results and discussion

Numerical calculations[4,5,7–9] show that by increasing the intensity, D, of the Gaussian noise, the spectral flux density in the SQUID loop at the frequency of the useful signal increases rapidly, reaches a maximum, and then decreases slowly in full accordance with the theory.[12,13]

Fig. 2(a) shows the numerically-calculated dependence, (calculated using Eq. (1)), of the spectral flux (amplitude) density inside the interferometer loop, at the frequency of the informational signal $S_\Phi^{1/2}(f_s)$, on the RMS amplitude of the Gaussian noise $D^{1/2}$ (solid line), as well as the experimentally obtained points. The amplitude of the informational harmonic signal inside the test interferometer was $a = 0.015$ in Φ_0 units. The interferometer behavior was typical for an SR effect (more specifically, SF) in a bi-stable system. On the inset in Fig. 2(a) we see the spectrum of magnetic flux within the loop, corresponding to the maximum signal increase. At this point, the average frequency of MS transitions (Kramers rate[17]) caused by the noise, is $r_K \approx 2f_S$, and

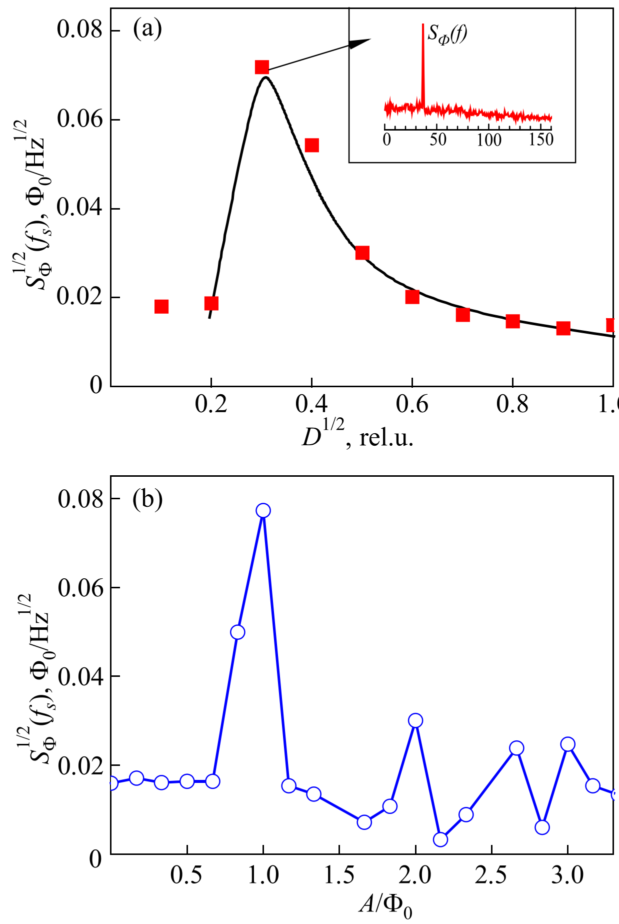

FIG. 2. The amplitude spectral density $S_\Phi^{1/2}$ of the magnetic flux in the rf-SQUID loop, at the frequency f_S, for the amplified weak harmonic signal, dependent on the RMS amplitude $D^{1/2}$ of Gaussian noise (solid line—numerical calculations, (■)—experimental results) (a) and normalized amplitude A/Φ_0 of the binary noise (b). Inset in (a): experimental spectrum of the output signal at the maximum point of maximum signal amplification.

the flux spectrum in the loop acquires 1/*f* properties. For the selected, and fairly large, β_L, the maximum increase obtained in this experiment, was about 10 dB s.

The system demonstrates another behavior in the presence of binary (telegraph) noise, the amplitude of which is fixed, and the phase is random. The residence time of the system in one of its MS in the case of Gaussian noise, changes smoothly with an increase in the intensity of sound. In case of binary noise, the system will stay in one determined MS, until the amplitude of the noise reaches sufficient levels to induce a transition to another MS. "Stochastic synchronization" in this situation occurs between the useful signal phase and the accidental phase of the binary noise.

As noted earlier, the SR phenomenon is usually examined under the assumption of two MS, divided by a potential barrier of a small amplitude, in order to get large amplification coefficients. For a low-temperature SQUID, this means that the magnitude β_L must be in the interval of 1–1.5. However, nitrogen-cooled HTS SQUIDs, in which the use of SR may be appropriate, retain the behavior characteristic of a hysteresis-free regime, up until $\beta_L \approx 3.5$[18–21] due to the large dispersion of the fluctuation of the magnetic flux. In this region of intense fluctuations, the number of MS can be measured with the help of SR.

A study of the SF effect in the interferometer with multiple MS, dependent on the amplitude of the external telegraph noise, is conducted. Fig. 2(b) shows the amplitude spectral density $S_\Phi^{1/2}$ of the magnetic flux, inside the loop at the useful signal frequency f_S, dependent on amplitude A of the binary noise in the loop, expressed in units of Φ_0. The signal parameters are the same as those of the previous case: the frequency $f_S = 37$ Hz, the amplitude of flux changes in the loop $a = 0.015$ (Φ_0). The amplitude of the magnetic flux noise inside the loops changed from 0 to ~3.5 Φ_0. From the result shown in Fig. 2(b), it is clear that with an increasing noise amplitude, we observe additional maximums in signal strengthening, which correspond to the stochastic transitions between several MS of the loop, with different values of the trapped flux. A good definition of the amplitude of telegraph noise allowed us to produce a kind of "spectroscopy" of MS, for a superconducting interferometer loop, within the limits of intense fluctuations. The estimated value β_L for this interferometer, according to measurements of the frequency response without external noise, was about 10.

4. Conclusion

The SR effect (or, in the case of an aperiodic system with strong damping, SF) allows us to amplify the weak periodic useful signal via stochastic "synchronization," caused by the noise of the transitions between two or more metastable states, with the aforementioned useful signal, as observed in the experiment with superconducting quantum interferometers. To cause the amplification (for example, in Ref. 5) predicted by the theory (see links to Refs. 4, 5, and 9) or numerical calculations,[8,9] it is necessary to optimize the parameters of the SQUID, in particular, to use the interferometers with small $\beta_L \geq 1$. For an HTS rf-SQUIDs, in the region of intense fluctuations, the effective dual-well potential and maximum amplification of the properties of weak informational signals on account of SR, can be observed in the region where $\beta_L \approx 4$.

The stochastic amplification of a weak regular signal is observed via noise-induced transfers, that occur not only between two neighboring states, but between several metastable current states of the superconducting interferometer loop. This is especially pronounced in the case of binary noise, which allows us to interpret the resulting picture as a kind of "noise spectroscopy" for the metastable states of the system.

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Translated by A. Bronskaya