Superposition of states in flux qubits with a Josephson junction of the ScS type (Review Article)

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(Submitted October 18, 2010)
Fiz. Nizk. Temp. **38**, 382–395 (April 2012)

The consequences of the transition to a quantum description of magnetic flux motion in the superconducting ring closed by an ScS type Josephson junction are considered. Here we review the principal results regarding macroscopic quantum tunneling (MQT) of Bose condensate consisting of a macroscopically large number of Cooper electron pairs. These phenomena are illustrated by the original data obtained from the study of MQT and coherent states in a modified flux qubit with energy level depletion $\Delta E_{01} \approx 2\cdot 10^{-23}$ J ($\Delta E_{01}/h \approx 30$ GHz). State superposition properties in a two-well potential and the issues associated with quantum measurements of local curvature of qubits' superposition energy levels are analyzed.
 [http://dx.doi.org/10.1063/1.3699781]

"… the problem of preserving the superposition states of the Schrödinger's cat type for mesoscopic systems is an important problem, and with its solution we will be able to speak about the many applications of quantum information"

S. Ya. Kilin, Usp. Fiz. Nauk **169**(5) (1999).

I. INTRODUCTION

Recently, due to the prospects expected from the implementation of quantum computing processes,[1–3] investigations of the coherence properties of macroscopic objects created on the basis of superconducting circuits with Josephson junctions[4] are of great interest both in terms of low temperature physics and quantum information technology.[5,6] Careful attention to the superconducting circuits with Josephson junctions is mostly because of the many physical systems proposed as the element base of quantum computers, at this time only these elements allow to create a large number of selectively managed, interconnected quantum bits (qubits), whose states are, in principle, measurable. The main advantage of a base of superconducting elements is the microscopic size of the elements, which allows through the means of modern technology to create a large number of qubits with similar characteristics. However, this is also a disadvantage. The fact is that in a real experiment it is very difficult to isolate a macroscopic quantum system from the surroundings, while in an open system due to the interactions with electromagnetic surroundings irreversible processes of decoherence and dissipation start to occur and rapidly transition the system into a mixed state.[2,5,6]

Depending on the ratio between the Josephson E_J and the charge E_C energies, three basic types of qubits can be identified[7–12]: charge, phase, and flux qubits. Currently the central problems in the study of phenomena associated with quantum fluctuations of the order parameter in qubits are the creation of conditions of their minimal interaction with the surroundings,[13,14] the study of the physical mechanisms of decoherence, the energy relaxation time T_1,[2,15–20] and the construction of quantum measurements of the coherent dynamics parameters of qubits.[21,22]

To minimize the interaction with electromagnetic environment various schemes of modified qubits were proposed, such as charge-phase qubits,[23–25] "quantronium,"[10] and "transmon."[26,27] In this context for flux qubits the choice of geometry (topology) of the quantization circuit is very important. As one of the best ways to protect the qubit from magnetic and electromagnetic fields is to reduce the area of its path, the flux qubit consisting of a superconducting loop of micron size wit three Josephson junctions[28,29] is most widely used for the demonstration of quantum algorithms.

With free evolution of flux ($E_J \gg E_C$), charge ($E_C \gg E_J$), and phase qubits, decoherence times T_2 were obtained in a few microseconds.[5,30] These results give rise to some hope for further increase of decoherence time, since in the first studies of quantum dynamics of charge qubits[7] values of T_2 were $\sim 10^{-9}$ s, and of charge-phase qubits—$3 \cdot 10^{-7}$ s.[25] Recently, the typical values of decoherence times for modified charge qubits achieved the microsecond range.[26] The answer to the question "what type of qubit is best" will depend on how much in the developed quantum schemotechnology we can suppress electrical charge fluctuations in the charge qubit ($E_C \gg E_J$), magnetic flux in the flux qubit ($E_J \gg E_C$), decrease the communication of the qubit with the electromagnetic environment, and create a qubit with large tunnel splitting of degenerate energy levels with a barrier sufficiently high with respect to thermal decay. To solve these problems and to carry out quantum measurements special powder broadband coolable filters[13,14] and electromagnetic screen systems were developed, and new circuits of signal recording are being created.[31,32] To increase decoherence times original modifications of qubits are under study and new types of weak bonds based on quantum phase-slip centers are offered.[33]

In this paper we consider the macroscopic quantum tunneling (MQT) and superposition states in a modified flux qubit.[34] Modification of the flux qubit is as follows: the qubit quantization circuit is done in a well-shielded from external environment toroidal cavity made of solid niobium; the weak atomic size bond with direct conductivity of the ScS type is used to improve the shape of the tunnel barrier in the φ-direction and as a Josephson junction[35,36]; to increase the energy gap in the banks of the Josephson junction Δ_0 the qubit is made of pure niobium with a single-crystal contact (Nb–Nb). Next we focus on the experimental problem of constructing an amplifier channel, cooled to 30–50 mK, for continuous fuzzy quantum measurements[60] of inductance L_Q of the main superposition state of the qubit.

II. MACROSCOPIC QUANTUM TUNNELING

Macroscopic quantum effects in superconducting systems with Josephson junctions are fundamentally due to superconductivity. A key property of the superconducting state, which allows to observe quantum effects at the macro level, is the phase coherence of a Bose-condensate consisting of a macroscopically large number of Cooper pairs.[37,38] Similarly to pure (coherent) quantum state in the simplest quantum systems, the Bose condensate state is described by a single function of the complex superconducting order parameter $\Delta(\mathbf{r}) = |\Delta(\mathbf{r})|\exp[i\varphi(\mathbf{r})]$, where $\varphi(\mathbf{r})$ is the order parameter phase. Superconducting current state in a magnetic field is characterized by the Cooper pair momentum in the Bose condensate $\mathbf{p}_s = \hbar\nabla\varphi(\mathbf{r}) - 2e\mathbf{A}(\mathbf{r})$, i.e. it is due to the gradient of the phase order parameter $\varphi(\mathbf{r})$ and the vector potential of magnetic field $\mathbf{A}(\mathbf{r})$. Due to this there is a well-known effect of fluxoid quantization (or, as a special case, the magnetic flux) is superconducting hollow cylinders. The module of the order parameter $|\Delta|$ in a spatially homogeneous superconductor describes the energy gap in the spectrum of its quasiparticle excitations, $\varepsilon_p = \sqrt{\xi_p^2 + |\Delta|^2}$, $\xi_p = p^2/2m - \mu$ (where p and m are the momentum and the mass of an electron, respectively, and μ is the chemical potential of the electronic system), and thus we see that the minimum excitation energy is $\varepsilon_p^{\min} = |\Delta|$. In the BSC model[39] at zero temperature the energy gap Δ_0 is 1.76 k_BT_c, where T_c is the critical temperature of the superconductor. At low temperature ($T \ll T_c$) for adiabatic dynamic processes in a superconductor with characteristic frequencies $\nu \ll \Delta_0/h$ the number of quasiparticles in the system is exponentially small, and dissipation processes for superconductors with large values of T_c may be significantly suppressed. It is therefore evident that the overall dynamic behavior of qubits based on niobium is substantially better than the characteristics of aluminum qubits, since $\Delta_0^{\rm Nb}/\Delta_0^{\rm Al} \approx 7$.

A fundamental role for the creation of quasi two-level macroscopic quantum systems (qubits) is that of the Josephson effects[4,37,38] that, like quantization of magnetic flux, are a manifestation of macroscopic phase coherence of the superconducting state, and physically are due to coherent tunneling of Cooper pairs in Josephson junctions. Due to the nonlinear nature of the dependence of current flowing through the Josephson junction on the difference between phases the superconducting loop closed by the junction becomes nonlinear, which is essential for creating qubits and measuring their quantum states.

The nature of the tunneling phenomena and properties of qubits depends on the effective potential for the collective variable system (phase, flux) and in many cases is a purely one-dimensional problem. This potential is determined by the current-phase relationship $I = I_c f(\varphi)$ for a Josephson junction, where I_c is the critical current of the junction, and $\varphi = \phi_1 - \phi_2$ is the phase difference of the order parameter in the banks of the contact. The current-phase dependences for ScS and SIS type contacts with normal resistance R_N in a superconductor with a gap $\Delta(T)$ have the respective forms,[4,35]

$$I_s = I_c \sin\frac{\varphi}{2}\,\mathrm{th}\frac{\Delta(T)\cos(\varphi/2)}{2k_BT}, \quad I_c(T) = \frac{\pi\Delta(T)}{eR_N}, \qquad (1a)$$

$$I_s = I_c^t \sin\varphi, \quad I_c^t(T) = \frac{\pi\Delta(T)}{2eR_N}. \qquad (1b)$$

When $T \ll T_c$ dependences Eqs. (1a) and (1b) differ substantially. This difference leads to different behavior of the loops with ScS and SIS type contacts in the quantum regime at low temperatures. In what follows we use the approximation of zero temperature for the potential of the ScS type contact as at $T \ll T_c$ the influence of final temperature on the shape of the potential in the Hamiltonian is irrelevant. As follows from the theory in Ref. 35, in the limit $T = 0$ the current-phase dependence of the ScS type contact Eq. (1a) has the form,

$$I_s = I_c \sin\frac{\varphi}{2}\,\mathrm{sgn}\left[\cos\frac{\varphi}{2}\right]. \qquad (2)$$

Using the relation $d\varphi/dt = (2e/\hbar)V$, which describes the AC Josephson effect (V is the potential difference on the banks of the contact), and the current-phase relations (1b) and (2) we get an expression for the dependence of the Josephson energy of the ScS and SIS type contacts on the phase difference φ, respectively,

$$U_J = -E_J\left|\cos\frac{\varphi}{2}\right|, \quad E_J = \frac{I_c\Phi_0}{\pi}, \qquad (3a)$$

$$U_J^t = -E_J^t \cos\varphi, \quad E_J^t = \frac{I_c^t \Phi_0}{2\pi}, \tag{3b}$$

where $\Phi_0 = h/2e$ is the magnetic flux quantum. Equations (3a) and (3b) describe one-dimensional barriers and one-dimensional wells in the φ-direction with the "singular" and "cosine" forms.

In addition to the energy of the Josephson contact U_J, potential energy U of the system contains the energy of the external circuit $U_{\rm circ}$, which includes a contact, $U = U_J + U_{\rm circ}$. For a flux qubit consisting of a superconducting ring closed at the Josephson junction the energy $U_{\rm circ}$ equals

$$U_{\rm circ} = \frac{LI_s^2}{2} = \frac{(\Phi - \Phi_e)^2}{2L}, \tag{4}$$

where Φ and Φ_e are, respectively, the full and the external magnetic flux, applied to the ring, L is the inductance of the ring, and $\Phi = \Phi_e + LI_s$. The value of the total flux Φ inside the ring is associated with the phase difference φ across the contact through the expression $\varphi + 2\pi\frac{\Phi}{\Phi_0} = 2\pi n$, where n is an integer. The phase difference of the order parameter φ on the banks of the weak-coupling superconductor is its macroscopic degree of freedom, which determines energy contributions to the Hamiltonian of the system.

For 20 years after the discovery of Josephson effects, experimentally observed phenomena in autonomous Josephson junctions and superconducting circuits, closed by the junction, were well described by the classical nonlinear equations of dynamics of variable φ with corresponding Hamiltonians. It is important to note that in these equations there is a viscosity term $\eta\, d\varphi/dt$, linear in derivative of the phase, which takes into account the finite flow of the quasiparticle current through the contact (the viscosity coefficient $\eta \sim 1/R$, where R is the resistance of the quasiparticle current contact).

A new stage in the physics of superconductivity was marked by the discovery of phenomena in which the coordinate φ of a superconducting macroscopic system demonstrated quantum behavior.[40] Quantum phenomena that occur in the low-capacity Josephson junctions at low temperatures and the associated quantum fluctuations of the order parameter were first considered in the theoretical paper.[41] In this paper the phase difference φ of the SIS type contact was considered as a quantum coordinate conjugate to the charge Q on the banks of the Josephson junction with capacitance C, $[\hat{Q}, \hat{\varphi}] = -2ei$, and the quantum Hamiltonian of the system is represented in the canonical form,

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \hat{U}(\varphi), \quad \hat{Q} = -2ei\partial/\partial\varphi, \tag{5}$$

where the first "kinetic" term is the electrostatic energy of the charge on the capacitance of the Josephson junction. With the help of the Hamiltonian Eq. (5) in Ref. 41 the theory of quantum decay of a metastable current state of the contact was constructed. This process of decay is due to tunneling in the coordinate φ from the metastable state of current to a more stable state with lower energy, and it has been called the effect of macroscopic quantum tunneling (MQT). The MQT process has incoherent single character, since in this case the system after the tunneling of the phase φ from metastable state with higher energy quickly loses energy and relaxes to the stable lower state with lower energy.

The possibility of the appearance of the MQT effect for the magnetic flux effect Φ in a macroscopic superconducting ring, closed by a Josephson junction, is considered in Refs. 42 and 43. In this case the operators of flow Φ and of the canonically conjugate charge $Q = C\dot{\Phi}$ on the banks of the Josephson junction satisfies the standard commutation relation $[\hat{Q}, \hat{\Phi}] = -i\hbar$. The canonical Hamiltonian of the system, similar to Hamiltonian Eq. (5), has the form,

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \hat{U}(\Phi), \quad \hat{Q} = -i\hbar\partial/\partial\Phi, \tag{6}$$

and describes the quantum mechanical behavior of a macroscopic magnitude of flow Φ in the superconducting ring in potential $U(\Phi) = U_J + U_{\rm circ}$, the form of which Eqs. (3a), (3b), and (4) depends on the external flow Φ_e. Fig. 1 schematically shows construction of the superconducting quantum interference device (SQUID) with a pure metallic contact of atomic dimensions[35,36] (elastic and inelastic mean free paths of electrons inside the contact are much larger than its size) and a diagram of $U(\Phi)$ for those values of parameters (capacitance $C = 3.77$ fF, parameter $\beta_L = 0.8$, the external flow $\Phi_e = 0.52\ \Phi_0 < \Phi_{ec} = 0.62\ \Phi_0$), under which MQT is realized in a double-well potential. The meaning of Φ_{ec} is that when $\Phi_e = \Phi_{ec}$ the potential barrier vanishes. For $L = (2\text{–}3)\cdot 10^{-10}$ H the chosen value of $\beta_L = 0.8$ corresponds to the triatomic contact with normal resistance $R_N \approx 4.3$ kΩ $\approx R_Q/3$, where $R_Q = h/2e^2 = 12.9$ kΩ is the quantum resistance of the single atom channel.

The first observations of MQT showed[44–46] that upon decreasing the temperature near the characteristic value of $T_0 = \hbar\omega_p/2\pi k_B$ (ω_p is the plasma frequency of the Josephson junction[40]) the mechanism of decay of a metastable current state changes from classical thermally activated to quantum tunneling state, and the lifetime of the metastable state at $T < T_0$ becomes independent of temperature.

Microscopic theories of MQT for the decay of metastable states in Josephson junctions, constructed in Refs. 47–50, inspired new experiments. Detailed studies[51–53] for the probability density of the decay of metastable current states have yielded good agreement with the MQT theory for both the SIS (Refs. 47–49, 51 and 53) and the ScS type contacts.[50,52]

Let us consider the characteristics of tunneling in SQUIDs with pure ScS type contacts. Previously[54] it was noted that the use of the resistive model with a cosine potential for the description of MQT in such samples (Figs. 1(a) and 1(b)) leads to a significant (almost an order of magnitude) discrepancy between theoretical and experimental results for the tunneling rate. The unusually high rate of tunneling decay of metastable states in pure ScS type contacts were explained in the microscopic theory constructed in Ref. 50, and the unusual shape of the potential barrier for such contacts, resulting from the current-phase relation (2), was later confirmed experimentally.[55]

Differentiation of the high-frequency current-voltage characteristic (HF CVC) of the SQUID provides a value convenient for comparison with theory: $\Delta\zeta(\Phi_e) \sim dV_T/dI_0$—

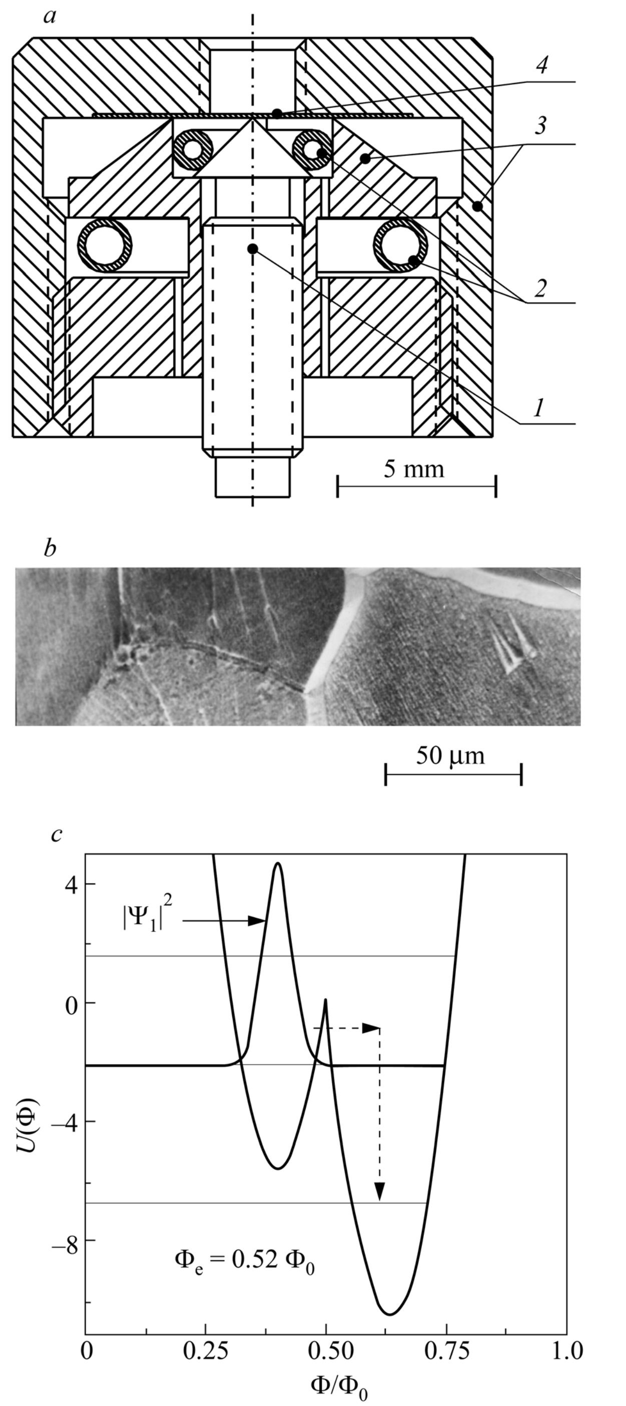

FIG. 1. The design of an RF SQUID with a pure ScS type contact: *1*–Nb (99.999%) needle, *2*–resonant circuit coil, *3*–body, *4*–Nb membrane (a). Micrograph of the membrane surface (mosaic crystal) (b). Potential U of the superconducting ring, closed by pure ScS type contact, depending on the magnetic flux Φ in the ring when external flux is $\Phi_e = 0.52\Phi_0$; parameter $\beta_L = 0.8$, contact capacitance $C = 3.77$ fF. Magnetic flux, represented by the square of the wave function $|\Psi_1|^2$, tunnels into the right well, then the state relaxes to a lower energy level (MQT phenomenon, the process is shown by dashed lines with arrows) (c).

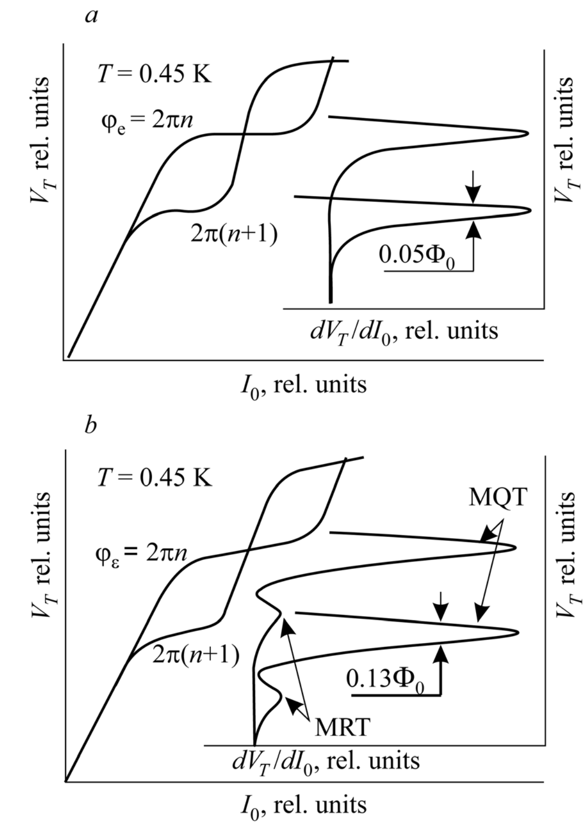

FIG. 2. The current-voltage characteristics $VT(I0)$ of an RF SQUID and their derivatives $dV_T/dI_0(V_T)$ at $T = 0.5$ K. An RF SQUID with an oxidized ScS type contact (a). An RF SQUID with a pure ScS type contact (b). Additional peaks on the derivative of HF CVC SQUID, shown with arrows, correspond to the macroscopic resonance tunneling around degeneracy of energy levels (see Ref. 56) of a quantum oscillator.

width of the probability distribution of the decay of a metastable state with respect to Φ_e, the external magnetic flux applied to the ring. At $T = 0.4$–0.5 K characteristic experimental values of $\Delta\zeta(\Phi_e)$ are around $(0.13$–$0.15)\Phi_0$ for high-resistivity ($R_N > 1$ kΩ) contacts, made from a specially purified niobium, which quantitatively agrees with the microscopic theory[50] for values of the contact capacitance $C \approx (3$–$4)\cdot 10^{-15}$ F. The dissipation in the theory is nonlinear and its influence on the probability of MQT is small subject to the general adiabatic conditions $(LC^*)^{-1/2}$, $(R_N C^*)^{-1} \ll \Delta_0/h$. These conditions are well satisfied for high-resistance Nb–Nb contacts with a small renormalization of the capacitance $C^* \cong C$.[50] The experimental values of $\Delta\zeta(\Phi_e) = (0.13$–$0.15)\Phi_0$ are very different from the characteristic values ($\Delta\zeta(\Phi_e) \approx 0.05\Phi_0$) following from the theory valid for tunneling transitions with a normal shunt,[49] even in the most favorable case of weak damping and extremely low capacitance ($C \approx 3$ fF).

In experiments with insufficient thermalization of the measured macroscopic quantum system the decay rate Γ of current states can be determined by the excess noise temperature setting T_N: $\Gamma = (\omega P/2\pi)\exp\{-\Delta U/k_B(T + T_N)\}$, where ΔU is the potential barrier separating two wells. However, the observation in our experiments[54] of more narrow distribution of the probability density decay $\Delta\zeta(\Phi_e) = (0.03$–$0.05)\Phi_0$ for Josephson junctions Nb–Nb_xO_y–Nb was a strong argument in favor of high speed of MQT in pure ScS type contacts. Fig. 2 shows typical values for the width $\Delta\zeta(\Phi_e)$, obtained in a high-frequency SQUID with oxidized (Fig. 2(a)) and pure (Fig. 2(b)) ScS type contacts. Detection[52] of macroscopic resonance tunneling of magnetic flux (MRT) in the superconducting ring closed by the ScS type contact, and demonstration of quantum energy levels of the Josephson oscillator in experiments with MQT transitions induced by an electromagnetic field[53] can be regarded as direct evidence that the low temperature behavior of Josephson systems becomes quantum-mechanical.

The increase in the tunneling rate in pure ScS type contacts is important for both the observation of macroscopic resonant tunneling[56] and for the creation of superposition states in the double-well potential, which underlie the construction of qubits. The high tunneling barrier permeability in the ring closed by the pure ScS type contact is primarily related to modification of its shape, resulting from the current-phase relation for contacts with direct conductivity[55] and the lack of dissipation in the adiabatic limit in the system.[50]

III. SUPERPOSITION OF TWO INDEPENDENT STATES OF MAGNETIC FLUX IN SUPERCONDUCTORS

In the early 80s a superconducting ring closed with an SIS type contact was proposed as a physical system in which it is possible to observe quantum coherent superposition of two classically different macroscopic states of magnetic flux Φ in a symmetrical double-well potential.[57,58] Such potentials with degenerate energy levels occur when $\Phi_e = \Phi_0/2$ (Fig. 3(a)). As a result of tunneling transitions the degenerate levels of independent wells are split by a certain small amount $\Delta E_{01} = \hbar\Omega \ll \Delta_0$, which is determined by the rate of energy exchange between two wells. If at the initial time $t = 0$ the wave function is concentrated in the left well then for such a coherent process the probability of finding the system in this state can be expressed as[43]

$$P_L(t) = \frac{1}{2}(1 + \cos\Omega t). \quad (7)$$

Of fundamental importance is the fact that with phase coherence in flux qubits with a frequency Ω the macroscopic magnetic moment (pseudospin) changes: $\mu_s \approx I_s S \sim 10^{10} \mu_B \approx 10^{-13}$ J/T ($I_s \sim 10^{-6}$ A is the supercurrent in the loop, $S \sim 10^{-7}$ m^2 is its area, $\mu_B = 0.93 \cdot 10^{-23}$ J/T is the Bohr magneton). However, the quantum superposition state, or "Schrödinger's cat state," are rapidly destroyed, for example, because of the strong coupling of the flux qubit to the electromagnetic environment. Some of the decoherence processes, which lead dephasing in a typical time T_2, will be discussed below. The criterion for the existence of a flux qubit with the superposition of two qubit states is the condition $\Omega T_2 \gg 1$. In other words, the superposition state in the circuit of a qubit should be established much faster ($\sim 2\pi/\Omega$) than the process of dephasing with a characteristic time of decoherence T_2.

It should be emphasized that the discussed model (6) of quantum dynamics of flow Φ in the ring with a Josephson

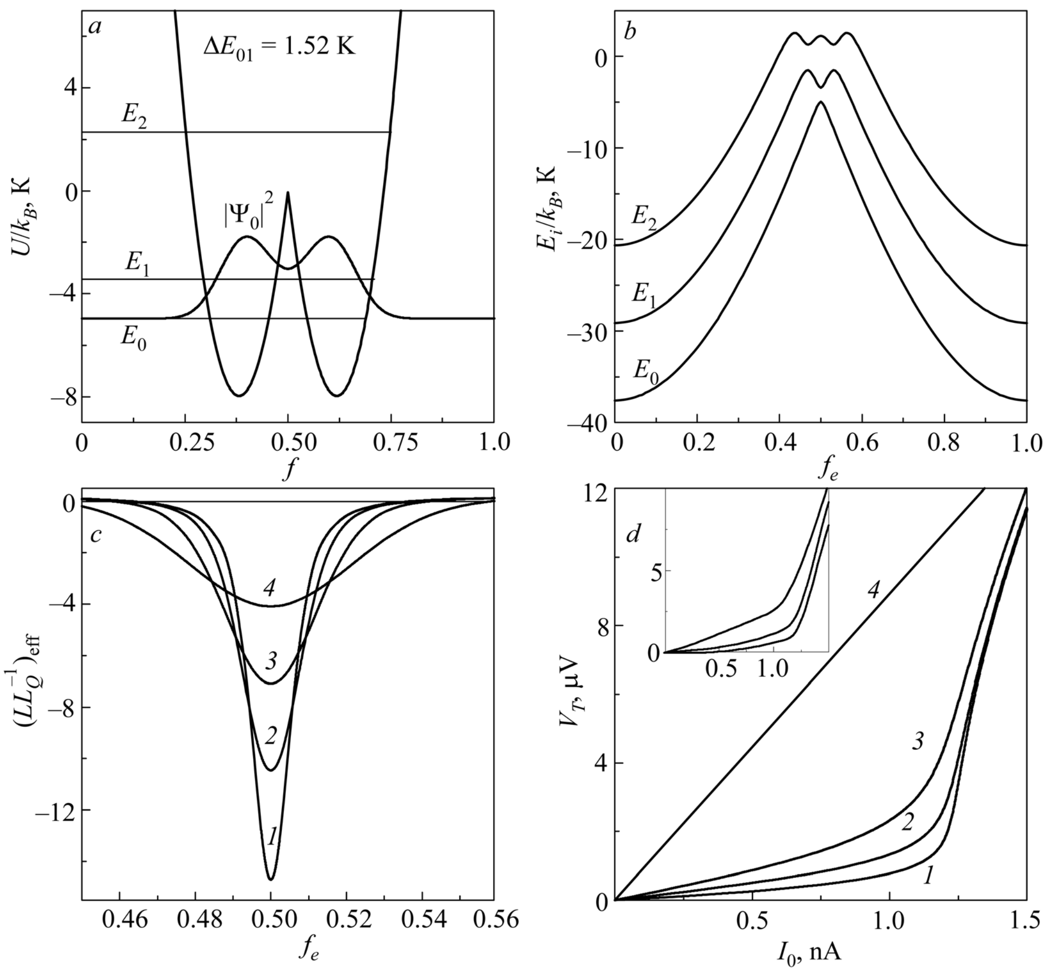

FIG. 3. Superposition of states in a flux qubit, calculated for a three-atom point contact. The calculation was performed for parameters $C = 3.77$ fF, $\beta_L = 0.8$. Potential $U/k_B(f)$, expressed in units of temperature, for $\Phi_e = \Phi_0/2$ with tunneling splitting of the energy levels $\Delta E_{01}/k_B = 1.52$ K; the square of the wave function for the ground state is shown schematically (a). Dependences $E_i/k_B(f_e)$ of the energy levels E_0, E_1, and E_2 on the external magnetic flux, expressed in units of temperature (b). Effective quantum inductance as a function of the reduced external magnetic flux $(LL_Q^{-1})_{\text{eff}}(f_e)$ for different values of noise variance σ; parameter $\sigma^{1/2}$ for curves *1–4* equals, respectively, 0, 0.005, 0.01, and 0.02 (c). Family of HF CVCs $V_T(I_0)$ near low currents of excitation I_0 for $\Phi_{dc} = \Phi_0/2$. Parameter $\sigma^{1/2}$ for curves *1–3* equals, respectively, 0, 0.01, and 0.02. Curve *4* corresponds to the values $\Phi_{dc} = \Phi_0$, $\sigma^{1/2} = 0$ (d).

junction describes pure states of the quantum system. Taking into account Eqs. (3a), (4), and (6) the Hamiltonian for the dissipationless superconducting ring with inductance L, closed by an ScS type contact with critical current I_c and low capacitance C, at zero temperature has the form,

$$\hat{H}_q = \frac{\hat{P}^2}{2M} + \hat{U}(f) = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial f^2} + \left(\frac{\Phi_0 I_c}{2\pi}\right)\left[-2|\cos(\pi f)| + \frac{2\pi^2(f-f_e)^2}{\beta_L}\right],$$
$$M = \Phi_0^2 C, \quad I_c(0) = \frac{\pi\Delta_0}{eR_N}, \quad \beta_L = \frac{2\pi L I_c}{\Phi_0}, \tag{8}$$

where $f = \Phi/\Phi_0$ and $f_e = \Phi_e/\Phi_0$ are dimensionless variables of internal Φ and external Φ_e magnetic fluxes in the ring, respectively. The obvious advantage of such a Hamiltonian for a qubit creation is the singular potential resulting from the current-phase relation for the ScS type contact. Consideration of such a Hamiltonian is based on the fact that only the model with the potential corresponding to the current-phase relation of the ScS type contact and with dissipation vanishing near zero temperature can satisfactorily describe the results of experiments on macroscopic quantum tunneling in a ring with a pure high-impedance ScS type contact.[54]

Solutions of the stationary Schrödinger equation,

$$\hat{H}_q(f; f_e)\Psi(f) = E(f_e)\Psi(f), \tag{9}$$

with Hamiltonian Eq. (8) give wave functions $\Psi(f)$ and energies $E(f_e)$ of the stationary states of a superconducting loop with the ScS type contact at the specified value of external magnetic flux f_e. Consider the superconducting loop with $\beta_L = 0.8$, in which a double-well potential appears around $f_e = 1/2$. Choosing the values of quantization circuit inductance $L = 3\cdot 10^{-10}$ H and contact capacitance $C = 3.77\cdot 10^{-15}$ F, close to our experimental parameters we obtain (Fig. 3(a)) the splitting of degenerate energy levels of the qubit into two, $E_0(f_e)$ and $E_1(f_e)$, with $\Delta E_{01}/k_B \cong 1.5$ K at a sufficiently high for the thermodynamic decay barrier $\Delta U/k_B \cong 3.5$. The dependence of the energy levels on external magnetic flux Φ_e is shown in Fig. 3(b), where for the sake of completeness the next, non-superposition level $E_2(f_e)$ is given. The distance to that level around $\Phi_e = \Phi_0/2$ (Fig. 3(a)) is significantly greater than between the split levels, which justifies the applicability of the two-level approximation when considering the coherent dynamics of a flux cubit based on an ScS type contact.

The main superposition state $|\Psi_0\rangle$ of a cubit (in a double-well potential) with minimum energy $E_0(f_e)$ will play a major role in construction of a new superconducting quantum magnetic flux detector, by analogy with the detector based on a qutrit.[59] In this paper a qutrit is a system of superimposed levels in a triple-well potential. Fig. 3(b) shows that the basic superposition level $E_0(f_e)$ has a substantial local nonlinearity, which can be expressed through quantum inductance $L_Q(f_e)$ (Ref. 24),

$$L_Q^{-1}(f_e) = \frac{\partial^2 E_0(\Phi_e)}{\partial \Phi_e^2} = \frac{1}{\Phi_0^2}\frac{\partial^2 E_0(f_e)}{\partial f_e^2}, \tag{10}$$

whose characteristic dependence on the magnetic flux in a pure quantum state for the qubit with parameters given in Fig. 3(a) is shown in Fig. 3(c).

From this relation it is clear that the superposition of states occurs in a relatively narrow ($\Phi_e \approx 10^{-2}\Phi_0$) typical range, and, as a result, with the splitting of the levels $\Delta E_{01}/k_B \cong 1.5$K the flux qubit is very sensitive to fluctuations of the magnetic flux. If the levels fluctuate, then upon interaction of the qubit with electromagnetic field fluctuations will be observed in the Rabi frequency. At the point of degeneracy $\Phi_e = \Phi_0/2$ the effect of fluctuations can be significantly reduced ($\partial L_Q/\partial \Phi_e = 0$), yet thorough filtration and magnetic shielding are needed. However, this can not be done with regard to noise generated by the measuring tract. For example, using the scheme for continuous fuzzy quantum measurements[60–63] the flux qubit is inductively ($M = k(L_T L)^{1/2}$) coupled with a high-quality ($Q \gg 1$) resonant circuit with frequency $\omega_T = (L_T C_T)^{-1/2}$, in which the signal is magnified by a cooled amplifier.[25,29,52] In this scheme the electromagnetic noise of the circuit and the transistor will determine the degree of "Reverse action" on the qubit. Possibilities of reducing the adverse reaction of the measuring scheme (in the form of noise flux, induced in the qubit) on experimental results will be discussed below. For now, assume that the characteristic frequencies ω_i of noise, acting on the qubit through the measuring path, are small compared to the splitting frequency Ω, but are large relative to ω_T. In this case the effective value of the quantum inductance of the ground state $(LL_Q^{-1})_{\text{eff}}(f_e)$ can be found using the technique of averaging over thermodynamic (quasi-stationary) fluctuations.[64] In the approximation of the Gaussian distribution of noise for the quantum inductance, we obtain

$$(LL_Q^{-1})_{\text{eff}}(f_e) = \frac{1}{\sqrt{2\pi Q}}\int df' \exp\left(-\frac{f'^2}{2\sigma}\right)(LL_Q^{-1})(f_e + f'), \tag{11}$$

where $\sigma = \langle \delta f_e^2 \rangle$ is the variance of the noise flux acting on the qubit from the measuring circuit. Fig. 3(c) shows the dependence of $(LL_Q^{-1})_{\text{eff}}(f_e)$ for several values of the standard deviation of noise flux $\sigma^{1/2}$.

We emphasize that the investigated in this paper problem of adverse influence of the measuring tract on the qubit, leading to homogenization of the quantum inductance of the ground state of the qubit by noise, is fundamentally different from the known effect of temperature.[29,65] Averaging of $(LL_Q^{-1})(f_e)$ by temperature of the qubit is due to the finite population of the upper level and the formation of $\langle (LL_Q^{-1})(f_e) \rangle$ as the average over the equilibrium density matrix of the system.

From Eq. (11) it follows that for the observation of dependence, differing by no more than 15% at the extremum point, from $\sigma^{1/2} = 0$ the value $\sigma^{1/2} \le 0.003$ is required, which implies the creation of amplifiers with ultralow power consumption. Indeed, the noise flux in the qubit from the resonant circuit is determined by its noise temperature $\delta\Phi_N \approx (k^2 k_B T_T L)^{1/2}$. Given that $k_2 \le 10^{-2}$, for the value of $\sigma^{1/2} \le 0.003$ we get $T_T \le 30$ mK, i.e. the first stage of the cooled amplifier must operate around refrigeration temperatures of $\sim$30 mK. More recently we proposed a general concept to solve this problem.[66] The created single-stage HEMT amplifier produces gain of 10 dB at 0.5 GHz (with a strip of 10%) at a power consumption of less than 10^{-6} W, which makes it possible to have it at such temperatures.

Substituting the expression for $(LL_Q^{-1})_{\rm eff}\ (f_e)$ into the usual Eq. (12) for the current-voltage characteristics of the RF SQUID we obtain the HF current-voltage characteristics for a qubit in a pure state, and depending on the variance of the noise contour (see Fig. 3(d)),

$$V_T = \frac{\omega_T L_T Q I_0}{\sqrt{1+4\xi(V_T,\Phi_{dc})^2 Q^2}},$$

$$\xi(V_T,\Phi_{dc}) = -k^2\frac{1}{2\pi}\int_0^{2\pi}(LL_Q^{-1})_{\rm eff}\left[\Phi_{dc}+\frac{MV_T}{\omega L_T}\sin\tau\right]\cos^2\tau d\tau, \quad (12)$$

which can be solved numerically.

Fig. 3(d) shows the branches of the high-frequency current-voltage characteristics obtained for the ground superimposed state of the qubit with parameters given in Fig. 3(a), when the value of external magnetic flux is $\Phi_e = \Phi_0/2$. As expected, the magnetic flux noise smoothes the effect of quantum inductance of the ground superimposed level in high-frequency current-voltage characteristics of the flux qubit. At low currents I_0 of excitement the high-frequency current-voltage characteristics of a qubit are formally similar to the corresponding characteristics of the high-frequency SQUID in a hysteresis-free regime ($\beta_L = 2\pi L I_C/\Phi_0 < 1$) with the classical Josephson inductance (see inset in Fig. 3(d)). However, the situation changes dramatically when the nonlinearity of the principal superimposed energy level in a three-well potential is considered. In this case, the main peak of the quantum inductance is shifted into the region $\Phi_e \approx \Phi_0$, and not to $\Phi_e \approx \Phi_0/2$, and direct evidence of superposition state can be obtained from the high-frequency current-voltage characteristics and signal characteristics of the qutrit without placing it in an external electromagnetic field.[59]

In this regard, note that in 1985 the averaged by the noise characteristics of a qubit were apparently observed by one of the authors of Refs. 67 and 68. In these studies it was suggested that the observed at low temperature (anomalous) dependences of an RF SQUID with an ScS type contact are associated with the coherent superposition of states. However, theoretical evidence of a large tunneling rate for ScS type contacts and calculations of high-frequency current-voltage characteristics of a qubit under the influence of noise from the measurement channel were absent at the time.

Let us now consider the effect of capacitance (mass) on quantum inductance of the ground superposition level and of the HF CVC qubit. Fig. 4(a) shows superposition levels resulting in a double-well potential of a qubit with a pure ScS type contact, for $C = 9.42\cdot 10^{-15}$ F and $\beta_L = 0.8$. With such a capacitance deep-seated levels of unconnected wells become split at point $\Phi_e = \Phi_0/2$ by $\Delta E_{01}/k_B = 0.36$ K. These splitting values are typical for the best flux qubits with SIS type contacts, but only at much lower heights of the potential barrier ΔU. It follows that a flux qubit with an ScS type contact is characterized by significantly lower rates of thermal decay $\sim\exp(-\Delta U/k_B T)$, i.e., by fewer errors due to hopping over the barrier ("leakage to non computational states"). As can be seen in Fig. 4(c), the dependence of quantum inductance of a pure state ($\sigma^{1/2} = 0$) on Φ_e narrows sharply with increasing capacitance, and peak amplitude increases by about a factor of five compared with the previous case. However, such narrow peaks are even more sensitive to fluctuations and are quickly washed out by the noise of the measuring scheme. After averaging the noise with variances $\sigma^{1/2} = 0.01$ and $\sigma^{1/2} = 0.02$ (see Fig. 4(c)) the characteristic dependences of $(LL_Q^{-1})_{\rm eff}(f_e)$ and of HF CVC qubits (Figs. 3(d) and 4(d)) for the two capacitance values are almost identical. Consequently, the results of experiments involving inductances of superposition energy levels as a quantitative method for determining the characteristics of qubits, should be approached with caution. Assuming that thermalization chain shift in the magnetic flux and electromagnetic field is done to $T \approx 10$ mK, consider one of the possible schemes of quantum measurements with a minimum variance of noise acting on the qubit.

IV. THE MODIFIED SCHEME OF SIGNAL AMPLIFICATION DURING THE CONTINUOUS FUZZY MEASUREMENT OF QUANTUM SUPERPOSITION STATES OF A FLUX QUBIT

Creation of amplification path with minimal adverse influence on the measured quantum system (on the qubit) is a difficult task of experimental physics. Some general problems of constructing quantum measurements, for example, carrying out nonperturbing operations that do not require entry into the "subquantum" level of noise, are discussed in Ref. 70. Here we focus on the experimental aspect of continuous fuzzy quantum measurements of local curvature of superposition energy levels and the dynamics of qubits. In quantum measurements the energy radiated by the transistor in the direction of the qubit is determined by the brightness temperature T_b at the transistor input. The temperature T_b depends on the physical temperature of the crystal lattice of the transistor, on the effective temperature of the conduction electrons of the channel, and on the frequency-dependent reflection coefficient at the input, i.e. "emissivity." The integral value of T_b is close to the physical temperature of the transistor and significantly exceeds the noise temperature T_n. Because in the registration scheme due to the large power dissipation in DC ($P = 0.1$–1 mW) the first stage of the amplifier is at $T \approx 1$ K, it is the Planck radiation at temperature T_b that usually has a major influence on the rate of decoherence of a qubit by the measuring tract. The matching element (resonant circuit), which has a galvanic contact with the transistor, can have noise temperature $T_T \approx 1$ K or even higher due to the high brightness temperature of the transistor $T_b \sim P$.

Fig. 5 shows a typical example of noise smoothing of an additional step (shown by arrow), appearing due to superposition of three states in flux qutrite with a pure ScS type contact. These results were obtained at transistor temperature of 1.5 K. Carrying out quantum measurements it is appropriate to decrease T_b by deep cooling of the transistor, and at low frequencies to apply powder filters, which effectively suppress broadband Planck emission.[13,14] If P and T_b are sufficiently decreased, and the amplifier is set up with refrigeration set to $T \le 30$ mK, then the variance of the noise flux induced in the qubit from the resonant circuit, all elements of which are at $T \le 30$ mK, is $\sigma^{1/2} \le 10^{-3}$, i.e. quite small compared to the typical width $(LL_Q^{-1})_{\rm eff}\ (f_e)$ for the

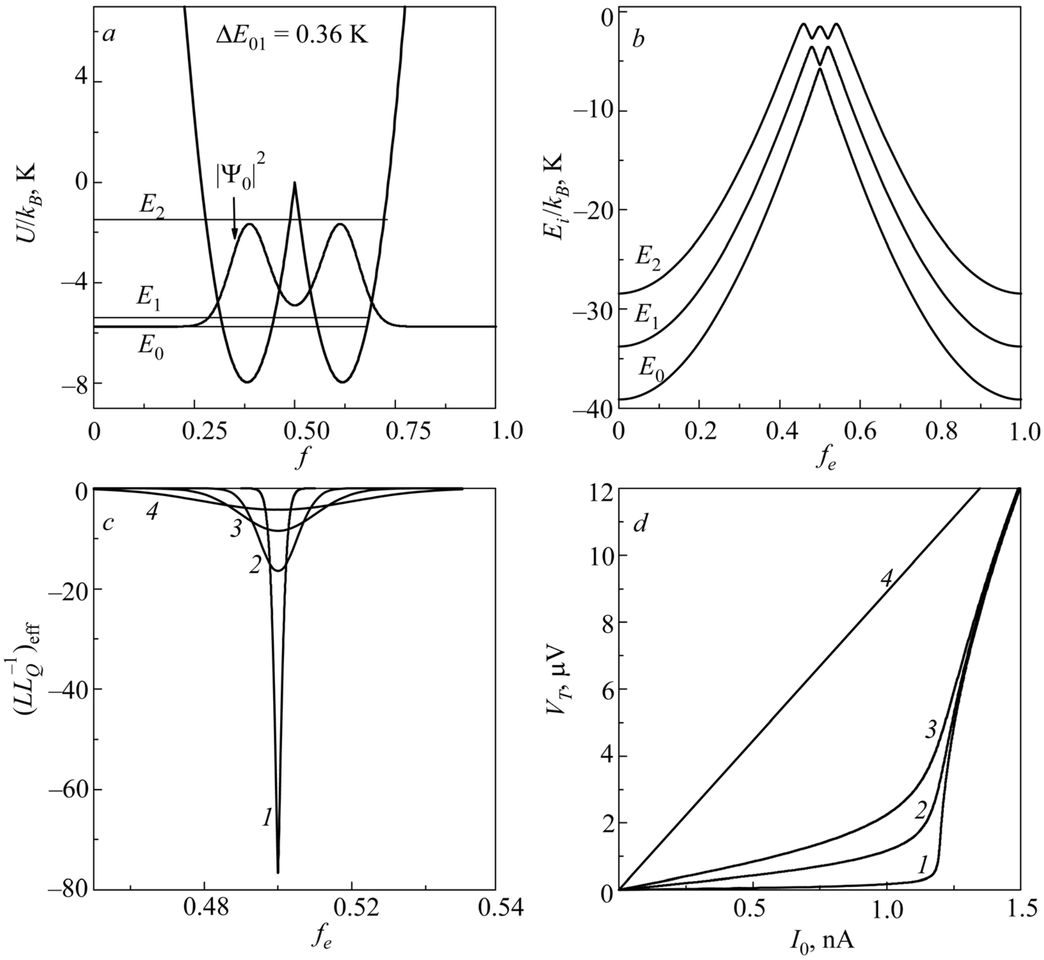

FIG. 4. The superposition of flux qubit states, designed for a three-atom point contact of increased capacity $C = 9.42$ fF. Potential $U/k_B(f)$, expressed in temperature units, for $\Phi_e = \Phi_0/2$ and $\beta_L = 0.8$ with a tunnel splitting $\Delta E_{01} = 0.36$ K; the square of the wave function for the ground level is shown schematically (a). Dependences $E_i/k_B(f_e)$ of the energy levels E_0, E_1, and E_2 on the external magnetic flux, expressed in units of temperature (b). Effective quantum inductance as a function of the reduced external magnetic flux $(LL_Q^{-1})_{\text{eff}}(f_e)$ for different values of noise variance σ; parameter $\sigma^{1/2}$ for curves *1–4* equals, respectively, 0, 0.005, 0.01, and 0.02 (c). Family of HF CVCs $V_T(I_P)$ near low currents of excitation for $\Phi_{dc} = \Phi_0/2$; parameter $\sigma^{1/2}$ for curves *1–3* equals, respectively, 0, 0.01, and 0.02. Curve *4* corresponds to the values $\Phi_{dc} = \Phi_0$, $\sigma^{1/2} = 0$ (d).

considered qubit with level splitting $\Delta E_{01}/k_B \approx 1.5$ K (see Fig. 3). To solve this problem in Ref. 66 a principle of single-stage amplifiers with a submicron DC power consumption, and values $P_{10} = 0.95$ µW at $\nu = 0.5$ GHz (P_{10}—power consumption of the amplifier with gain $G = 10$ dB) were employed. In such an unsaturated DC mode the two-stage amplifier with serial FET Agilent ATF-36077 yielded $G = 45$ dB at 5nW power consumption and $\nu = 0.5$ GHz. There is every reason to believe that the transition to InP-, InAs-, and especially to InNb-HEMT, P_{10} in the unsaturated mode can be reduced down to several nanowatts and/or the operating frequency and be increased. As noted by the authors of Ref. 66, cooling HEMT amplifiers operating in this mode from 4.2 K down to 300 mK improves their basic characteristics.

Modern solid-state amplifiers designed to operate at low temperatures are made on field heterostructural high electron mobility transistors (HEMT). The activation energy of donor impurity is so small that the restriction of the operating temperature of these transistors is only the power they dissipate P (G, ν). Numerical estimates show that when using the unsaturated mode the values of $\sigma^{1/2}$ can be decreased 30–40 times. Such values of $\sigma^{1/2}$ are quite applicable for the registration of the fine structure of the local curvature of the ground superposition level of a qubit with an ScS type contact and its dynamics in electromagnetic field with characteristic frequencies of Rabi oscillations $\Omega_R \approx 1$ GHz for the splitting of the levels $\Delta E_{01}/h \approx (30–35)$ GHz.

Flux qubits with SIS type contacts have essentially smaller values of degenerate level splitting $\Delta E_{01}/h \approx (3–7)$ GHz (see Refs. 9, 29, 33 and 71 and references therein). For flux qubits with SIS type contacts even a small inverse influence of the measuring scheme on the qubit ($\sigma^{1/2} \approx 10^{-3}$) significantly smoothes out the narrow peak of the effective quantum inductance (see Fig. 4). Therefore, to observe the fine structure of the qubit, the resonant circuit and the first stage of the amplifier should be at $T \leq 10$ mK. Studies conducted in the recent years let us hope for a positive solution to this problem.

Good results of the measurements of flux qubits using amplifiers on pseudomorphic GaAs–HEMT were obtained in Ref. 29. Upon cooling of such an amplifier to $T \approx 1$ K at $\nu = 100$ MHz power consumption P_{10} is on the order of 100 mW.[72] To reduce T_n and T_b for the HF range specialized HEMTs are developed that at low currents preserve a large slope.[73] For higher-frequency cooled amplifiers in the normal mode, including InP, power dissipation in the amplification of 10 dB increases to $P_{10} \approx 10^{-3}$ W.[74] With such a large power dissipation the active zone of the cooled transistor is heated to

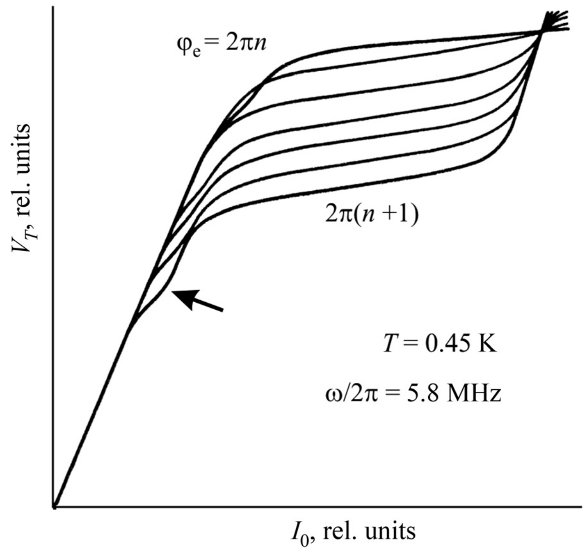

FIG. 5. A family of current-voltage characteristics $V_T(I_0)$ of a flux qutrit with a superposition step. The family parameter is the value of the external magnetic flux $\Delta\Phi_e \approx \Phi_0/10$. The superposition step has a periodic dependence on Φ_e, while the period is Φ_0, and is observed around the symmetrical three-well potential. The slope of this step is partly due to the noise temperature of the resonant circuit.

5–10 K due to poor heat conductivity of the heterostructures,[75] which determines the high brightness temperature T_b at the amplifier input and will lead to increased inverse effect on the qubit. At microwave frequencies the use of high-resistance loads is impossible, and the main way is to improve the transistor technologies (e.g., see ABCS-technology[76]). The lowest values of power consumption and minimum brightness temperatures are expected in specialized transistors on narrow-gap semiconductors: InAs and, particularly, InSb. For example, in Ref. 77 a decrease of more than an order of magnitude was obtained in the values of P_{10}.

It is possible to reduce the power dissipation to $P_{10} \sim 10^{-10}$–10^{-8} W when using SQUID-based amplifiers.[78] This allows to integrate the first stages of the amplifier with a qubit in almost any dilution refrigerator temperatures. For the development of experimental physics of quantum measurements of great interest are the results obtained in Refs. 79 and 80, which represent the Josephson parametric amplifier with an instantaneous bandwidth of about 1 MHz, tunable in the frequency range of 4–8 MHz. Upon cooling of the amplifier, which contains 480 DC SQUIDs, to $T \approx 15$ mK subquantum level of noise is detected. In addition, worth noting are the new bifurcation amplifiers with low power consumption[81] and SQUID-based converters,[82] built on the two-stage scheme with a chain of 100 or more SQUIDS in the second stage, capable of amplification of several thousand when the output voltage is on the order of a few microvolts. Although in SQUID-based multistage converters power dissipation rapidly increases with increasing the number of elements, it can be restricted for these problems at the level of $P_{10} \sim 10^{-8}$ W, for example, simply by reducing the number of elements.

V. CONCLUSION

Josephson effects, discovered 50 years ago, are at the basis of superconducting qubits, qutrits, new quantum detectors, and simple two-qubit elements for the realization of quantum computer registers. The behavior of Josephson qubits even in weak fields is quite unlike the behavior of atoms, since they are essentially nonlinear quantum systems strongly associated with the electromagnetic environment. Currently, the technology of manufacturing of such systems is largely based on the use of tunnel Josephson junctions Al–Al_2O_3–Al on the nanometer scale. However, a number of problems have already arisen on this path. One of them is the small value of the gap Δ_0 $(T \ll T_c)/h \approx 50$ GHz in aluminum, which requires, for example, for increasing the clock frequency, the transition to superconductors with large values of T_c. Such technological work is already underway using the new tunnel Ta–Ta_2O_5–Ta contacts.[83] One of the major problems associated with low rates of tunneling between two states, separated by a cosine potential, can be solved with the development technologies for manufacturing contacts with direct conductivity of quantum contacts[84–86] or qubits with phase-slip centers.[33] Given the recent advances of nanotechnology, in principle, one can hope for a successful solution of this question. In carrying out continuous fuzzy quantum measurements of states of individual qubits the brightness temperature of the amplifier channel can serve as a major cause of decoherence. As can be seen from the foregoing, in the near future we can expect fast-acting coolable to 10–30 mK combined "SQUID–HEMT" and "SQUBID–HEMT" signal amplification paths.

This work was supported in part by the National Academy of Sciences of Ukraine in the framework of the "Nano" project, contract number 4/11-N.

The authors thank A. N. Omelyanchik, S. N. Shechenko, and V. M. Shulga for the fruitful discussions and valuable comments.

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[1]R. Feynman, Int. J. Theor. Phys. **21**, 467 (1982).
[2]K. A. Valiev and A. A. Kokin, *Quantum Computers: Hopes and Reality, R&C Dynamics* (Moscow-Izhevsk (2001).
[3]T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O'Brien, Nature **464**, 45 (2010).
[4]B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[5]J. Clarke and F. K. Wilhelm, Nature **453**, 1031 (2008).
[6]S. Ya. Kilin, Usp. Fiz. Nauk **169**, 507 (1999).
[7]Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature **398**, 786 (1999).
[8]J. E. Mooij, T. P. Orlando, L. Levitov, L. Tian, C. H. van der Wal, and S. Lloyd, Science **285**, 1036 (1999).
[9]J. R. Friedman, V. Patel, W. Chen, S. K. Tolpygo, and J. E. Lukens, Nature **406**, 43 (2000).
[10]D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).
[11]Y. Yu, S. Han, X. Chu, S. I. Chu, and Z. Wang, Science **296**, 889 (2002).
[12]Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[13]D. Vion, P. F. Orfila, P. Joyez, D. Esteve, and M. H. Devoret, J. Appl. Phys. **77**, 2519 (1995).
[14]M. Hashisaka, Y. Yamauchi, K. Chida, S. Nakamura, K. Kobayashi, and T. Ono, Rev. Sci. Instrum. **80**, 096105 (2009).
[15]K. W. Lehnert, K. Bladh, L. F. Spietz, D. Gunnarsson, D. I. Schuster, P. Delsing, and R. J. Schoelkopf, Phys. Rev. Lett. **90**, 027002 (2003).
[16]T. Duty, D. Gunnarsson, K. Bladh, and P. Delsing, Phys. Rev. B **69**, 140503 (2004).
[17]C. M. Wilson, T. Duty, F. Persson, M. Sandberg, G. Johansson, and P. Delsing, Phys. Rev. Lett. **98**, 257003 (2007).
[18]A. Yu. Smirnov, Phys. Rev. B **67**, 155104 (2003).
[19]J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. **89**, 117901 (2002).

[20]F. K. Wilhelm, U. Hartmann, M. J. Storcz, and M. R. Geller, in *Manipulating Quantum Coherence in Solid State Systems*, edited by M. E. Flatté and I. Tifrea (Springer, Dordrecht, 2007), p. 195.
[21]M. A. Sillanpää, J. I. Park, and R. W. Simmonds, Nature **449**, 438 (2007).
[22]T. Yamamoto, Yu. A. Pashkin, O. Astafiev, Y. Nakamura, and J. S. Tsai, Nature **425**, 941 (2003).
[23]A. B. Zorin, Phys. Rev. Lett. **86**, 3388 (2001).
[24]D. Born, V. I. Shnyrkov, W. Krech, Th. Wagner, E. Il'ichev, M. Grajcar, U. Hübner, and H.-G. Meyer, Phys. Rev. B **70**, 180501(R) (2004).
[25]V. I. Shnyrkov, Th. Wagner, D. Born, S. N. Shevchenko, W. Krech, A. N. Omelyanchouk, E. Il'ichev, and H.-G. Meyer, Phys. Rev. B **73**, 024506 (2006).
[26]J. Koch, T. M. Yu, J. M. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **76**, 042319 (2007).
[27]J. Q. You, X. Hu, S. Ashhab, and F. Nori, Phys. Rev. B **75**, 140515(R) (2007).
[28]J. B. Majer, F. G. Paauw, A. C. J. ter Haar, C. J. P. M. Harmans, and J. E. Mooij, Phys. Rev. Lett. **94**, 090501 (2005).
[29]M. Grajcar, A. Izmalkov, E. Il'ichev, Th. Wagner, N. Oukhanski, U. Hübner, T. May, I. Zhilyaev, H. E. Hoenig, Ya. S. Greenberg, V. I. Shnyrkov, D. Born, W. Krech, H.-G. Meyer, A. M. van den Brink, and M. H. S. Amin, Phys. Rev. B **69**, 060501 (2004).
[30]M. Steffen, S. Kumar, D. P. DiVincenzo, J. R. Rozen, G. A. Keefe, M. B. Rothwell, and M. B. Ketchen, Phys. Rev. Lett. **105**, 100502 (2010).
[31]R. J. Schoelkopf, P. Wahlgren, A. A. Kozhevnikov, P. Delsing, and D. E. Prober, Science **280**, 1238 (1998).
[32]M. H. Devoret and R. Schoelkopf, Nature **406**, 1039 (2000).
[33]J. E. Mooij and C. J. P. M. Harmans, New J. Phys. **7**, 219 (2005).
[34]V. I. Shnyrkov, G. M. Tsoi, D. A. Konotop, and I. M. Dmitrenko in *Single-Electron Tunneling and Mesoscopic Devices (SQUID'91)*, edited by H. Koch and H. Lübbig (Springer-Verlag, Berlin, 1991), p. 208.
[35]I. O. Kulik and A. N. Omelyanchouk, Fiz. Nizk. Temp. **3**, 945 (1977) [Sov. J. Low Temp. Phys. **3**, 459 (1977)].
[36]A. Gumann, T. Dahm, and N. Schopohl, Phys. Rev. B **76**, 064529 (2007).
[37]M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996).
[38]A. Barone and G. Paterno, *The Josephson Effect* (Mir, Moscow, 1984).
[39]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[40]K. K. Likharev, Usp. Fiz. Nauk **139**(1), 169 (1983).
[41]Yu. M. Ivanchenko and L. A. Zilberman, Zh. Exp. Teor. Fiz. **55**, 2395 (1968).
[42]D. J. Scalapino, in *Tunneling Phenomena in Solids*, edited by E. Burstein and S. Lundqvist (Plenum, New York, 1969), p. 477.
[43]A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981).
[44]L. D. Jackel, J. P. Gordon, E. L. Hu, R. E. Howard, L. A. Fetter, D. M. Tennant, R. W. Epworth, and J. Kurkijarvi, Phys. Rev. Lett. **47**, 697 (1981).
[45]R. F. Voss and R. A. Webb, Phys. Rev. Lett. **47**, 265 (1981).
[46]I. M. Dmitrenko, G. M. Tsoi, and V. I. Shnyrkov, Fiz. Nizk. Temp. **8**, 660 (1982) [Sov. J. Low Temp. Phys. **8**, 330 (1982)].
[47]V. Ambegaokar, U. Eckern, and G. Schön, Phys. Rev. Lett. **48**, 1745 (1982).
[48]A. I. Larkin and Yu. N. Ovchinnikov, Phys. Rev. B **28**, 6281 (1983).
[49]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Exp. Teor. Fiz. **85**, 1510 (1983).
[50]V. A. Khlus, Fiz. Nizk. Temp. **12**, 25 (1986) [Sov. J. Low Temp. Phys. **12**, 14 (1986)].
[51]M. H. Devoret, J. M. Martinis, and J. Clarke, Phys. Rev. Lett. **55**, 1908 (1985).
[52]I. M. Dmitrenko, V. A. Khlus, G. M. Tsoi, and V. I. Shnyrkov, Fiz. Nizk. Temp. **11**, 146 (1985) [Sov. J. Low Temp. Phys. **11**, 77 (1985)].
[53]J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987).
[54]I. M. Dmitrenko, V. A. Khlus, G. M. Tsoi, and V. I. Shnyrkov, Il Nuovo Cimento D **9**, 1057 (1987); I. M. Dmitrenko, G. M. Khlus, and V. I. Shnyrkov, Fiz. Nizk. Temp. **10**, 211 (1984) [Sov. J. Low Temp. Phys. **10**, 111 (1984)].
[55]M. C. Koops, G. V. van Duyneveldt, and R. de Bruyn Ouboter, Phys. Rev. Lett. **77**, 2542 (1996).
[56]P. Silvestrini, V. G. Palmieri, B. Ruggiero, and M. Russo, Phys. Rev. Lett. **79**, 3046 (1997); D. V. Averin, J. R. Friedman, and J. E. Lukens, Phys. Rev. B **62**, 11802 (2000).
[57]A. O. Caldeira and A. J. Leggett, Ann. Phys. **149**, 374 (1983).
[58]A. J. Leggett, in *Proceedings of the International Symposium on the Foundations of Quantum Mechanics*, edited by S. Kamefuchi (Tokio, 1983), p. 74.
[59]V. I. Shnyrkov, A. A. Soroka, and O. G. Turutanov, *Qubits, Quitrit, Couplers, and Detectors for Quantum Information Science, A Joint Russian-Ukranian Seminar "Physics of Superconducting Heterostructures*," September 14–16 (Institute of Solid State Physics, Chernogolovka, Moscow Region, 2011).
[60]A. N. Korotkov and D. V. Averin, Phys. Rev. B **64**, 165310 (2001).
[61]M. J. Everitt, P. Stiffell, T. D. Clark, A. Vourdas, J. F. Ralph, H. Prance, and R. J. Prance, Phys. Rev. B **63**, 144530 (2001).
[62]A. Yu. Smirnov, Phys. Rev. B **68**, 134514 (2003).
[63]A. B. Zorin, Zh. Exp. Teor. Fiz. **125**, 1423 (2004).
[64]E. M. Lifshitz and L. D. Landau, *Statistical Physics: Course of Theoretical Physics*, 3rd ed. (Butterworth, London, 1984) Vol. 5, Chap. 12, p. 359.
[65]S. N. Shevchenko, S. H. W. van der Ploeg, M. Grajcar, E. Il'ichev, A. N. Omelyanchouk, and H.-G. Meyer, Phys. Rev. B **78**, 174527 (2008).
[66]A. M. Korolev, V. I. Shnyrkov, and V. M. Shulga, Rev. Sci. Instrum. **82**, 016101 (2011).
[67]I. M. Dmitrenko, G. M. Tsoi, and V. I. Shnyrkov, in *II Proceeding Meeting on Quantum Metrology and Fundamental Physical Constants* (NGO VNIIM, Leningrad, 1985), p. 81.
[68]V. I. Shnyrkov, G. M. Tsoi, D. A. Konotop, and I. M. Dmitrenko, *5-th Czchoslovak Symposium on Weak Superconductivity, Smolenice* (Nova Science, New York, 1989), p. 223.
[69]V. I. Shnyrkov, G. M. Tsoi, D. A. Konotop, and I. M. Dmitrenko, *Proceedings of 4-th International Conference SQUID'91 (Sessions on SET and Mesoscopic Devices)* (Springer-Verlag, Berlin, Germany 1991), p. 211.
[70]K. A. Valiev, Usp. Fiz. Nauk **175**(1), 3 (2005).
[71]V. I. Shnyrkov, A. A. Soroka, and S. I. Melnik, Fiz. Nizk. Temp. **34**, 773 (2008) [Low Temp. Phys. **34**, 610 (2008)].
[72]N. Oukhanski and E. Hoenig, Appl. Phys. Lett. **85**, 2956 (2004).
[73]E. Gremion, A. Cavanna, Y.-X. Liang, U. Gennser, M.-C. Cheng, M. Fesquet, G. Chardin, A. Benoit, and Y. Jin, J. Low Temp. Phys. **151**, 971 (2008).
[74]N. Wadefalk, A. Mellberg, I. Angelov, M. Barsky, S. Bui, E. Choumas, R. W. Grundbacher, E. L. Kollberg, R. Lai, N. Rorsman, P. Starski, J. Stenarson, D. C. Streit, and H. Zirath, IEEE Trans. Microwave Theory Tech. **52**, 1705 (2003).
[75]J. J. Bautista and E. M. Long, The Interplanetary Network Progress Report, IPN PR, 42–170, August 15, 2007, p. 1.
[76]C. Liu, Y. Li, and Y. Zeng, Engineering **2**, 617 (2010).
[77]A. Noudeviwa, Y. Roelens, F. Danneville, A. Olivier, N. Wichmann, N. Waldhoff, S. Lepilliet, G. Dambrine, L. Desplanque, X. Wallart, G. Moschetti, J. Grahn, and S. Bollaert, IEEE Trans. Electron Devices **57**, 1903 (2010).
[78]M. Mück, and J. Clarke, J. Appl. Phys. **88**, 6910 (2000).
[79]M. A. Castellanos-Beltran, K. D. Irwin, G. C. Hilton, L. R. Vale, and K. W. Lehnert, Nat. Phys. **4**, 929 (2008).
[80]M. A. Castellanos-Beltran, K. D. Irwin, L. R. Vale, G. C. Hilton, and K. W. Lehnert, IEEE Trans. Appl. Supercond. **19**, 944 (2009).
[81]R. Vijay, M. H. Devoret, and I. Siddiqi, Rev. Sci. Instrum. **80**, 111101 (2009).
[82]V. K. Kornev, I. I. Soloviev, N. V. Klenov, and O. A. Mukhanov, Supercond. Sci. Technol. **20**, 362 (2007); I. I. Soloviev, V. K. Kornev, N. V. Klenov, and O. A. Mukhanov, Fiz. Tverd. Tela **52**, 2110 (2010).
[83]V. I. Gurtovoi and V. A. Tulin, in The 26th International Conference on Low Temperature Physics (LT26), Beijing, China (to be published in Journal of Physics: Conference Series).
[84]C. W. J. Beenakker and H. van Houten, Phys. Rev. Lett. **66**, 3056 (1991).
[85]J. M. Krans, C. J. Muller, I. K. Yanson, T. C. M. Govaert, R. Hesper, and J. M. van Ruitenbeek, Phys. Rev. B **48**, 14721 (1993).
[86]E. Scheer, P. Joyez, M. H. Devoret, D. Esteve, and C. Urbina, Superlattices Microstruct. **23**, 747 (1998).

Translated by D. K. Maraoulaite