

Josephson currents in point contacts between dirty two-band superconductors

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We develop a microscopic theory of the Josephson effect in point contacts between dirty two-band superconductors. A general expression for the Josephson current, which is valid for arbitrary temperatures, is obtained. This formula is used to calculate the current-phase relations and temperature dependence of the critical current in MgB₂ superconductors. We also examine the effect of interband scattering on the contact characteristics of dirty superconductors. The correction to the Josephson current owing to interband scattering is found to depend on the phase shift in the bulk superconductors (i.e., the *s*- or *s*[±]-wave symmetry of the order parameters).

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I. INTRODUCTION

Multiband superconductivity was first mentioned in theoretical papers by Matthis, Suhl, and Walker¹ and by Moskalenko and Palistrant² more than 50 years ago. At that time their papers were regarded as attempts to fit some characteristics of superconducting materials (refinement of the ratio of an energy gap to critical temperature, heat capacity jump, London penetration depth etc.) into the BSC theory. True multiband superconductivity became a hot topic in condensed matter physics in 2001, when Nagamatsu discovered two-band superconductivity in MgB₂ with an anomalously high $T_c=39$ K.³ It was noteworthy that the pairing mechanism had an electron–phonon origin in magnesium diboride, while the order parameters, as attributes of the superconductor energy gaps, had *s*-wave symmetry.

The recently discovered iron-based superconductors⁴ are most likely multiband systems. For example, ARPES implies the existence of two full gaps in Ba_{0.6}K_{0.4}Fe₂As₂.⁵ Moreover there is an assumption that in this iron-based superconductor, a π -shift between phases of the order parameters and, thus, with sign-reversal of the order parameter or *s*[±]-wave symmetry is realized.⁶ A phase-shift of this sort can lead to new, fundamental phenomena and effects in these superconducting systems. Unfortunately, most modern experimental methods (measuring magnetic penetration length, calorimetric method, Knight shift and spin relaxation velocity in NMR, and the above mentioned ARPES) cannot provide a unique answer regarding gap pairing symmetry. However, a new phase-sensitive technique based on the proximity effect between niobium wire and a massive NdFeAsO_{0.88}F_{0.12} plate for probing unconventional pairing symmetry in the iron pnictides has recently been reported.⁷

Thus, there is some interest in studying phase coherent effects in two-band superconductors which are sensitive to phase shifts. Josephson effects are one of these phase sensitive phenomena. Many theoretical papers deal with this problem.^{8–12} These discuss the Josephson effect in tunnel junctions between one-band and two-band superconductors.

In this paper we study the stationary Josephson effect in superconductor–constriction–superconductor (S–C–S) con-

tacts, whose behavior differs qualitatively from superconductor–insulator–superconductor (S–I–S) tunnel junctions, even in the case of one-band superconductors, as revealed in the papers of Kulik and Omelyanchouk^{13,14} (KO theory). We construct a microscopic theory of “dirty” S–C–S contacts for two-band superconductors, as a generalization of the KO theory to this case. We show that interband scattering effects in dirty superconductors cause mixing of Josephson currents from different bands.

II. MODEL AND BASIC EQUATIONS

We assume a weak superconducting link in the form of a thin filament of length L and diameter d , connecting two bulk superconductors (“banks”) (Fig. 1). This model describes S–C–S contacts with direct conductivity (point contacts, microbridges), which qualitatively differ from the tunnel S–I–S junctions. On condition that $d \ll L$ and $d \ll \min[\xi_1(0), \xi_2(0)]$ (the $\xi_i(T)$ are the coherence lengths) we can solve a one-dimensional problem with “rigid” boundary conditions inside the filament ($0 \leq x \leq L$). At $x=0$ and L all functions are assumed equal to the values in the homogeneous zero-current state of corresponding bank.

We investigate the case of dirty two-band superconductor with strong impurity intraband scattering rates (dirty limit) and weak interband scattering. In the dirty limit, the

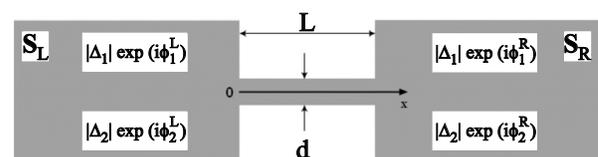


FIG. 1. Model of an S–C–S contact. The right and left banks are bulk two-band superconductors connected by a thin filament of length L and diameter d .

superconductor is described by the Usadel equations for the normal and anomalous Green functions g and f , which for a two-band superconductor are given by¹⁵

$$\omega f_1 - D_1(g_1 \nabla^2 f_1 - f_1 \nabla^2 g_1) = \Delta_1 g_1 + \gamma_{12}(g_1 f_2 - g_2 f_1), \tag{1}$$

$$\omega f_2 - D_2(g_2 \nabla^2 f_2 - f_2 \nabla^2 g_2) = \Delta_2 g_2 + \gamma_{21}(g_2 f_1 - g_1 f_2). \tag{2}$$

The Usadel equations are supplemented by self-consistency equations for the order parameters Δ_i ,

$$\Delta_i = 2\pi T \sum_j \sum_{\omega > 0}^{\omega_D} \lambda_{ij} f_j, \tag{3}$$

and a formula for the current density

$$j = -ie\pi T \sum_i \sum_{\omega} N_i D_i (f_i^* \nabla f_i - f_i \nabla f_i^*). \tag{4}$$

The subscript $i=1, 2$ enumerates the first and second bands. The normal and anomalous Green functions g_i and f_i , which

are related by the normalization condition $g_i^2 + |f_i|^2 = 1$, are functions of x and the Matsubara frequency $\omega = (2n+1)\pi T$. The D_i are the intraband diffusivities owing to nonmagnetic impurity scattering, the N_i are the densities of states at the Fermi surface of the i -th band, the electron-phonon constants λ_{ij} take into account the Coulomb pseudopotentials, and the γ_{ij} are the interband scattering rates. The symmetry relations $\lambda_{12}N_1 = \lambda_{21}N_2$ and $\gamma_{12}N_1 = \gamma_{21}N_2$ also hold.

In the case of short weak links examined here, we can neglect all terms in Eqs. (1) and (2) except the gradient term. Using the normalization condition we obtain equations for $f_{1,2}$,

$$\sqrt{1 - |f_1|^2} \frac{d^2}{dx^2} f_1 - f_1 \frac{d^2}{dx^2} \sqrt{1 - |f_1|^2} = 0, \tag{5}$$

$$\sqrt{1 - |f_2|^2} \frac{d^2}{dx^2} f_2 - f_2 \frac{d^2}{dx^2} \sqrt{1 - |f_2|^2} = 0. \tag{6}$$

The boundary conditions for Eqs. (5) and (6) are found by solving the equations for the Green functions in the left (right) banks:

$$\begin{cases} \omega f_1^{L(R)} = \Delta_1^{L(R)} \sqrt{1 - |f_1^{L(R)}|^2} + \gamma_{12} (\sqrt{1 - |f_1^{L(R)}|^2} f_2^{L(R)} - \sqrt{1 - |f_2^{L(R)}|^2} f_1^{L(R)}), \\ \omega f_2^{L(R)} = \Delta_2^{L(R)} \sqrt{1 - |f_2^{L(R)}|^2} + \gamma_{21} (\sqrt{1 - |f_2^{L(R)}|^2} f_1^{L(R)} - \sqrt{1 - |f_1^{L(R)}|^2} f_2^{L(R)}). \end{cases} \tag{7}$$

Note, that the solutions f_1 and f_2 of Eqs. (5) and (6) are coupled because of interband scattering in the banks through the boundary conditions (7).

Introducing the phases of the order parameters in the banks,

$$\Delta_1^{L(R)} = |\Delta_1| \exp(i\phi_1^{L(R)}), \quad \Delta_2^{L(R)} = |\Delta_2| \exp(i\phi_2^{L(R)}), \tag{8}$$

and writing $f_i(x)$ in Eqs. (5) and (6) as $f_i(x) = |f_i(x)| \times \exp(i\chi_i(x))$, we have

$$|f_i(0)| = |f_i(L)| = |f_i|, \quad \chi_i(0) = \chi_i^L, \quad \chi_i(L) = \chi_i^R, \tag{9}$$

where $|f_i|$ and $\chi_i^{L(R)}$ are connected with $|\Delta_i|$ and $\phi_i^{L(R)}$ through Eq. (7).

The solution of Eqs. (5)–(9) determines the Josephson current in the system. It depends on the phase difference at the contact, which we define as $\phi \equiv \phi_1^R - \phi_1^L = \phi_2^R - \phi_2^L$, and on the possible phase shift in each bank $\delta = \phi_2^L - \phi_1^L = \phi_2^R - \phi_1^R$. The phase shift δ between the phases of the two order parameters in the two-band superconductor can be 0 or π , depending on the sign of the interband coupling constants, the interband scattering rates, and the temperature of the system (see Appendix).

Equations (5) and (6) with the boundary conditions (9) admit an analytical solution, and for the current density (4) we obtain:

$$\begin{aligned} I = & \frac{2\pi T}{eR_{N1}} \sum_{\omega} \frac{|f_1| \cos \frac{\chi_1^L - \chi_1^R}{2}}{\sqrt{(1 - |f_1|^2) \sin^2 \frac{\chi_1^L - \chi_1^R}{2} + \cos^2 \frac{\chi_1^L - \chi_1^R}{2}}} \\ & \times \arctan \frac{|f_1| \sin \frac{\chi_1^L - \chi_1^R}{2}}{\sqrt{(1 - |f_1|^2) \sin^2 \frac{\chi_1^L - \chi_1^R}{2} + \cos^2 \frac{\chi_1^L - \chi_1^R}{2}}} \\ & + \frac{2\pi T}{eR_{N2}} \sum_{\omega} \frac{|f_2| \cos \frac{\chi_2^L - \chi_2^R}{2}}{\sqrt{(1 - |f_2|^2) \sin^2 \frac{\chi_2^L - \chi_2^R}{2} + \cos^2 \frac{\chi_2^L - \chi_2^R}{2}}} \\ & \times \arctan \frac{|f_2| \sin \frac{\chi_2^L - \chi_2^R}{2}}{\sqrt{(1 - |f_2|^2) \sin^2 \frac{\chi_2^L - \chi_2^R}{2} + \cos^2 \frac{\chi_2^L - \chi_2^R}{2}}}. \end{aligned} \tag{10}$$

Here R_{N1} and R_{N2} are the contributions of the normal resistance for each band, where $R_{Ni}^{-1} = (2Se^2/L)N_i D_i$ ($S = \pi d^2/4$ is the cross sectional area).

The general expression (10), together with Eqs. (7)–(9), describes the Josephson current as a function of the gaps Δ_i in the banks and phase difference φ at the contact.

III. JOSEPHSON CURRENTS

A. Josephson current without interband scattering

When there is no interband scattering, the system of Eqs. (7) yields

$$\begin{cases} f_{1,2}^{L(R)} = \frac{\Delta_{1,2}^{L(R)}}{\sqrt{|\Delta_{1,2}^{L(R)}|^2 + \omega^2}} \\ g_{1,2}^{L(R)} = \frac{\omega}{\sqrt{|\Delta_{1,2}^{L(R)}|^2 + \omega^2}} \end{cases} \quad (11)$$

Using Eq. (11), we rewrite the current (10) in terms of the order parameters as

$$\begin{aligned} I = & \frac{2\pi T}{eR_{N1}} \sum_{\omega} \frac{|\Delta_1| \cos \frac{\phi}{2}}{\sqrt{\omega^2 + |\Delta_1|^2 \cos^2 \frac{\phi}{2}}} \arctan \frac{|\Delta_1| \sin \frac{\phi}{2}}{\sqrt{\omega^2 + |\Delta_1|^2 \cos^2 \frac{\phi}{2}}} \\ & + \frac{2\pi T}{eR_{N2}} \sum_{\omega} \frac{|\Delta_2| \cos \frac{\phi}{2}}{\sqrt{\omega^2 + |\Delta_2|^2 \cos^2 \frac{\phi}{2}}} \arctan \frac{|\Delta_2| \sin \frac{\phi}{2}}{\sqrt{\omega^2 + |\Delta_2|^2 \cos^2 \frac{\phi}{2}}}. \end{aligned} \quad (12)$$

Thus, if the interband scattering rates γ_{ik} are neglected, the Josephson current (10) decomposes into two parts: current flows independently from the first band to the first and from the second band to the second. At zero temperature, $T=0$, Eq. (12) takes the form

$$\begin{aligned} I = & \frac{\pi|\Delta_1|}{2eR_{N1}} \cos \frac{\phi}{2} \text{Arctanh} \sin \frac{\phi}{2} \\ & + \frac{\pi|\Delta_2|}{2eR_{N2}} \cos \frac{\phi}{2} \text{Arctanh} \sin \frac{\phi}{2}. \end{aligned} \quad (13)$$

Equation (12) is a straightforward generalization of the Josephson current for a one-band superconductor.¹³ The partial inputs from the band currents $I(1 \rightarrow 1)$ and $I(2 \rightarrow 2)$ are determined by the ratio of the normal resistances $r = R_{N1}/R_{N2}$. Introducing the total resistance $R_N = R_{N1}R_{N2}/(R_{N1}+R_{N2})$ and normalizing the current to $I_0 = (2\pi/eR_N)T_c$, we plot the current–phase relation (12) for different values of r and T in Fig. 2.

The current–phase relation (12) determines the temperature dependence of the critical current I_c , which is plotted in Fig. 3.

In calculating $I(\varphi)$ and $I_c(T)$ (Figs. 2 and 3) we use the parameters of superconducting MgB_2 with $\gamma_{ij}=0$. For zero interband scattering, the Josephson current (12) is independent of any possible phase shift δ (e.g., in the case of $s \pm$ -wave symmetry).

B. Josephson current with interband scattering

We now examine the effects of interband scattering on S–C–S contact behavior. In general, the system of Eqs. (7) has no analytical solution. The case of temperatures near the

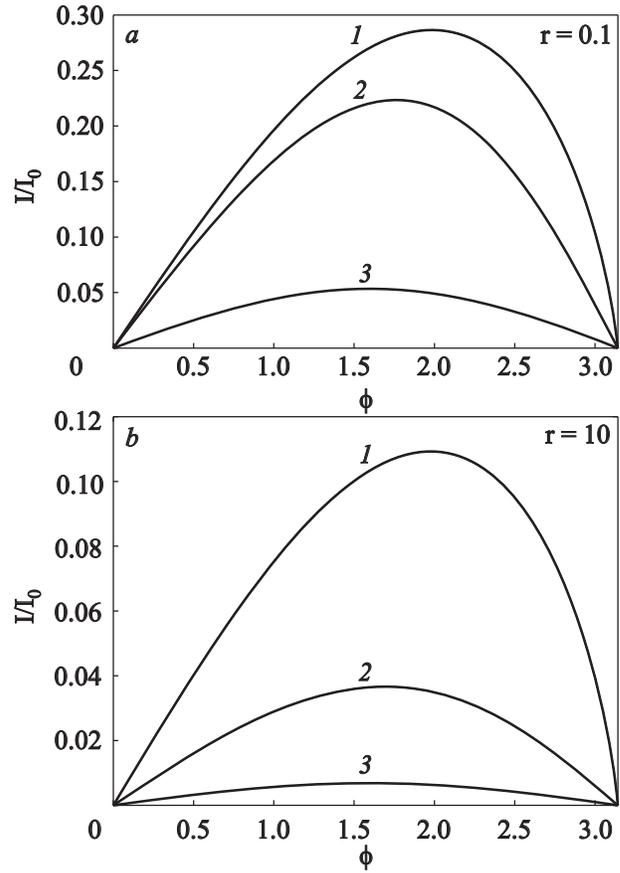


FIG. 2. Current-phase relations of S–C–S $\text{MgB}_2|\text{MgB}_2$ for different temperatures $\tau=T/T_c$: 0 (1); 0.5 (2); 0.9 (3); and a resistance ratio of $r = R_{N1}/R_{N2}$.

critical temperature T_c with an arbitrary level of interband scattering has been examined in Ref. 16. Here, we examine interband scattering γ_{ij} using perturbation theory for arbitrary temperatures $0 \leq T \leq T_c$. In the first approximation, we find the Green functions in each bank:

$$\begin{cases} f_1 = \frac{\Delta_1}{\sqrt{|\Delta_1|^2 + \omega^2}} + \gamma_{12} \frac{(2\omega^2 + |\Delta_1|^2)(\Delta_2 - \Delta_1) - \Delta_1^2(\Delta_2^* - \Delta_1^*)}{2\sqrt{(|\Delta_1|^2 + \omega^2)^3} \sqrt{|\Delta_2|^2 + \omega^2}}, \\ f_2 = \frac{\Delta_2}{\sqrt{|\Delta_2|^2 + \omega^2}} + \gamma_{21} \frac{(2\omega^2 + |\Delta_1|^2)(\Delta_1 - \Delta_2) - \Delta_2^2(\Delta_1^* - \Delta_2^*)}{2\sqrt{|\Delta_1|^2 + \omega^2} \sqrt{(|\Delta_2|^2 + \omega^2)^3}}. \end{cases} \quad (14)$$

When interband scattering is taken into account and if the phase shift $\delta \neq 0$, the phases of the Green functions f_i will not coincide with those of the order parameters, Δ_i .

Equation (14) yields the following corrections to the current (12):

$$\delta I = \delta I_1 + \delta I_2, \quad (15)$$

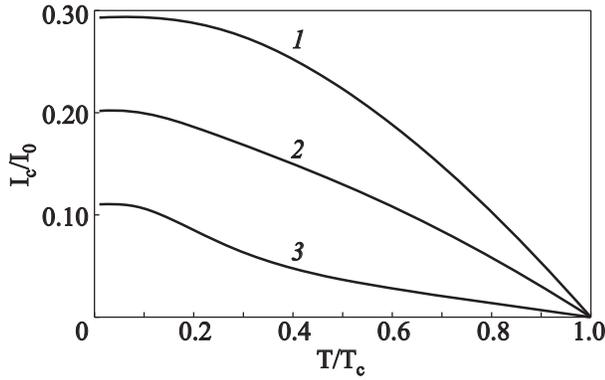


FIG. 3. Temperature variation in the critical current I_c (T) for different values of $r=R_{N1}/R_{N2}$: 0.1 (1); 1 (2); 10 (3).

$$\delta I_1 = \frac{2\pi T \gamma_{12}}{eR_{N1}} \sum_{\omega} \left(\frac{\omega^2 (|\Delta_2| e^{i\delta} - |\Delta_1|) \cos\left(\frac{\phi}{2}\right)}{\sqrt{\left(|\Delta_1|^2 \cos^2\left(\frac{\phi}{2}\right) + \omega^2\right)^3} \sqrt{|\Delta_2|^2 + \omega^2}} \right. \\ \times \arctan \frac{|\Delta_1| \sin \frac{\phi}{2}}{\sqrt{\omega^2 + |\Delta_1|^2 \cos^2 \frac{\phi}{2}}} \\ \left. + \frac{1}{2} \frac{\omega^2 |\Delta_1| (|\Delta_2| e^{i\delta} - |\Delta_1|) \sin \phi}{(|\Delta_1|^2 + \omega^2) \left(|\Delta_1|^2 \cos^2\left(\frac{\phi}{2}\right) + \omega^2\right) \sqrt{|\Delta_2|^2 + \omega^2}} \right), \quad (16)$$

$$\delta I_2 = \frac{2\pi T \gamma_{21}}{eR_{N2}} \sum_{\omega} \left(\frac{\omega^2 (|\Delta_1| - e^{i\delta} |\Delta_2|) \cos\left(\frac{\phi}{2}\right)}{\sqrt{\left(|\Delta_2|^2 \cos^2\left(\frac{\phi}{2}\right) + \omega^2\right)^3} \sqrt{|\Delta_1|^2 + \omega^2}} \right. \\ \times \arctan \frac{|\Delta_2| \sin \frac{\phi}{2}}{\sqrt{\omega^2 + |\Delta_2|^2 \cos^2 \frac{\phi}{2}}} \\ \left. + \frac{1}{2} \frac{\omega^2 |\Delta_2| (|\Delta_1| - e^{i\delta} |\Delta_2|) \sin \phi}{(|\Delta_2|^2 + \omega^2) \left(|\Delta_2|^2 \cos^2\left(\frac{\phi}{2}\right) + \omega^2\right) \sqrt{|\Delta_1|^2 + \omega^2}} \right). \quad (17)$$

The corrections to the Josephson current owing to interband scattering, Eqs. (15)–(17), depend on the phase shifts at the banks, $\delta=0$ or π . This reflects the mixing of Josephson currents between different bands.¹⁶

CONCLUSIONS

We have developed a microscopic theory of the Josephson effect at point contacts between dirty two-band superconductors. A general expression for the Josephson current,

valid for arbitrary temperatures, is obtained. We have examined dirty superconductors with interband scattering. For superconductors which are in contact, this effect causes coupling of the currents between different bands. When interband scattering is taken into account, the observable characteristics $I(\varphi)$ and $I_c(T)$ depend on the phase shift of the order parameters in the different bands. This makes it possible to distinguish between s - and s_{\pm} -wave symmetry by studying the Josephson effect in point contacts of two band superconductors.

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APPENDIX

The free energy in microscopic terms is given by:¹⁷

$$F = \frac{1}{2} \sum_{ij} \Delta_i \Delta_j^* N_i \lambda_{ij}^{-1} + F_1 + F_2 + F_{\text{int}} + \frac{B^2}{8\pi}, \quad (A1)$$

where

$$F_i = 2\pi T \sum_{\omega>0} N_i \left[(1 - g_i) - \text{Re}(f_i^* \Delta_i) + \frac{1}{4} D_i \left(\left(\nabla + \frac{2\pi i \mathbf{A}}{\Phi_0} \right) \right. \right. \\ \left. \left. \times f_i \left(\nabla - \frac{2\pi i \mathbf{A}}{\Phi_0} \right) f_i^* + (\nabla g_i)^2 \right) \right], \quad (A2)$$

$$F_{\text{int}} = 2\pi T \text{Re} \sum_{\omega>0} (N_1 \gamma_{12} + N_2 \gamma_{21}) (g_1 g_2 + f_1^* f_2 - 1). \quad (A3)$$

Writing the order parameters as $\Delta_i^{L(R)} = |\Delta_i| \exp(i\phi_i^{L(R)})$ and the anomalous Green functions as $f_i(x) = |f_i(x)| \exp(i\chi_i(x))$ and simultaneously using the fact that $N_1 \lambda_{12} = N_2 \lambda_{21}$ and $N_1 \gamma_{12} = N_2 \gamma_{21}$, from (A1) we extract terms containing the phase difference $\delta = \phi_2^{L(R)} - \phi_1^{L(R)}$:

$$- \frac{\lambda_{12} |\Delta_1| |\Delta_2|}{\lambda_{11} \lambda_{22} - \lambda_{12} \lambda_{21}} \cos \delta + 4\pi T \gamma_{12} \sum_{\omega>0} |f_1| |f_2|. \quad (A4)$$

Substituting $|f_1|$ and $|f_2|$ into (A4) and letting $\gamma_{12} \rightarrow 0$ yields

$$\left(- \frac{\lambda_{12}}{\lambda_{11} \lambda_{22} - \lambda_{12} \lambda_{21}} + 4\pi T \gamma_{12} \right. \\ \left. \times \sum_{\omega>0} \frac{1}{\sqrt{(\omega^2 + |\Delta_1|^2)(\omega^2 + |\Delta_2|^2)}} \right) \cos \delta. \quad (A5)$$

For $\delta=0$, the free energy minimum is satisfied if the term in brackets is less than zero and, vice versa, for $\delta=\pi$, Eq. (A1) has a minimum if the term in brackets is greater than 0. This yields an existence criterion for a π -shift between phases of order parameters:

$$\text{sgn} \left(- \frac{\lambda_{12}}{\lambda_{11} \lambda_{22} - \lambda_{12} \lambda_{21}} + 4\pi T \gamma_{12} \right. \\ \left. \times \sum_{\omega>0} \frac{1}{\sqrt{(\omega^2 + |\Delta_1|^2)(\omega^2 + |\Delta_2|^2)}} \right) = \pm 1. \quad (A6)$$

Here “+1” on the right corresponds to $\delta=\pi$ and “-1” to $\delta=0$.

At $T=0$ this criterion can be simplified to

$$\operatorname{sgn}\left(-\frac{\lambda_{12}}{\lambda_{11}\lambda_{22}-\lambda_{12}\lambda_{21}}+\frac{2\gamma_{12}}{|\Delta_2(0)|}\operatorname{K}\left(\sqrt{1-\frac{|\Delta_1|^2(0)}{|\Delta_2|^2(0)}}\right)\right) = \pm 1, \quad (\text{A7})$$

where $\operatorname{K}(\sqrt{1-|\Delta_1|^2(0)/|\Delta_2|^2(0)})$ is a complete elliptic integral of the first kind, and $\Delta_1(0)$ and $\Delta_2(0)$ are the order parameters at $T=0$.

In the vicinity of T_c , condition (A6) transforms to

$$\operatorname{sgn}\left(-\frac{\lambda_{12}}{\lambda_{11}\lambda_{22}-\lambda_{12}\lambda_{21}}+\frac{\pi}{2T_c}\gamma_{12}\right) = \pm 1. \quad (\text{A8})$$

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