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Phase diagram of a current-carrying superconducting film in absence of the magnetic field

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It is present the phase diagram for the current states of superconducting films, based on the experimental investigation of the resistive transitions induced by the transport current. It is found that a comparatively narrow film with the width $w < 5\lambda_\perp(T)$ ($\lambda_\perp$ is the penetration depth of the magnetic field) never enters the vortex state, but experiences direct transition from the purely superconducting state to the resistive state with phase-slip centers as soon as the current exceeds the critical Ginzburg–Landau current I_c^{GL}. The Meissner current state of the films of intermediate width, $5\lambda_\perp(T) < w < 10\lambda_\perp(T)$, transforms at $I > 0.8I_c^{GL}$ to the vortex resistive state which exists within the current interval $0.8I_c^{GL} < I < I_m$, where the value of the upper critical current is in a good agreement with the heory. The vortex state of wide films, $w > 10\lambda_\perp(T)$, is realized within the current region $I_c^{AL} < I < I_m$, where I_c^{AL} is the transition point to the vortex state calculated for the limiting case $w \gg \lambda_\perp$. At $I > I_m$, the films with the width $w > 5\lambda_\perp(T)$ enter a vortex-free resistive state with phase-slip lines.

PACS **74.40.+k** Fluctuations (noise, chaos, nonequilibrium superconductivity, localization, etc.);
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According to the Ginzburg–Landau (GL) theory [1], if the transport current through a narrow superconducting film (superconducting channel) exceeds the depairing current,

$$I_c^{GL}(T) = \frac{c\Phi_0 w}{6\sqrt{3}\pi^2\xi(0)\lambda_\perp(0)}(1 - T/T_c)^{3/2}, \qquad (1)$$

the superconducting state of the channel is destroyed and transforms to the normal state, as shown in Fig. 1. In (1), Φ_0 is the magnetic flux quantum, w is the film width, $\lambda_\perp(0) = 2\lambda^2(0)/d$ is the penetration depth of the magnetic field into the superconducting film, $\xi(0)$ and $\lambda(0)$ are the coherence length and London length, respectively, at zero temperature, and d is the film thickness. Later it was found that the real scenario of the resistive phase transition of the superconducting channel is more complex. Namely, as the transport current exceeds $I_c^{GL}(T)$, an inhomogeneous resistive state appears in the channel, consisting of alternating superconducting and quasi-normal regions [2]; the latter are the specific dynamic formations known as phase-slip centers (PSCs). The number of PSCs increases with the transport current, and at $I = I_{cn} \gg I_c^{GL}$ the resistive state turns to the completely normal state. The basic feature of the current-voltage characteristics (IVCs) of superconducting channels in the resistive state are regular voltage steps (Fig. 2), which were first observed in tin whiskers [3,4] and in narrow tin films [5]. We note the following important peculiarities of the step-like IVCs: the

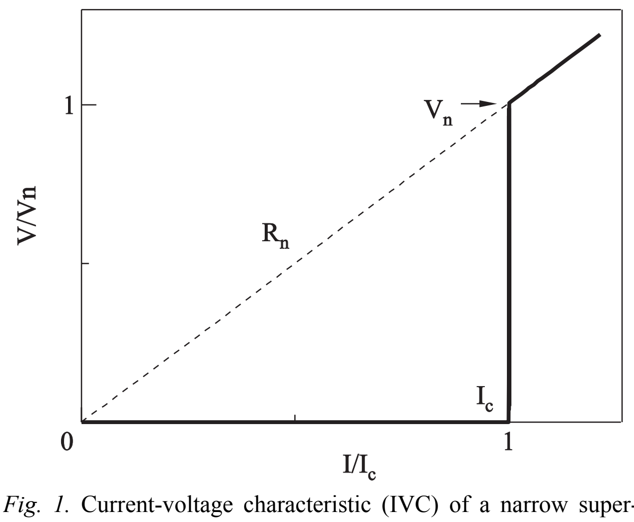

Fig. 1. Current-voltage characteristic (IVC) of a narrow superconducting channel according to the GL theory. R_n is the channel resistance in the normal state, and V_n is the voltage jump at the point of the resistive transition.

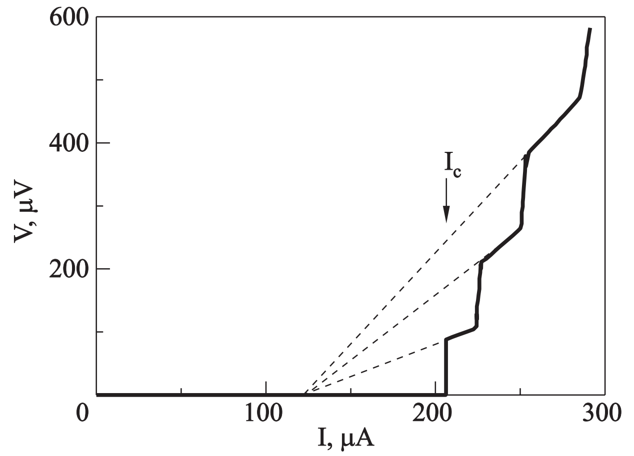

Fig. 2. Typical IVC of a superconducting film channel Sn4 at the temperature $T/T_c = 0.98$.

multiplicity of differential resistances of the sloping IVC parts, the intersection of the continued sloping parts (dashed lines) at a single point on the current axis, and the absence of hysteresis which indicates the non-heating nature of the voltage steps.

To the present time, the resistive state of narrow superconducting channels has been rather well studied experimentally, and the theory of this state has been universally recognized. In our opinion, this is not the case for the resistive state of a wide film, in spite of the fact that the study of current-carrying states in wide films started much earlier than the investigations of superconducting channels. In 1963, Tinkham [6] first involved the vortex conception for calculation of the critical fields of thin films. It was found in [7] that the motion of vortices, induced by the magnetic field of the Earth or the transport current, plays the crucial role in formation of the initial part of the IVCs. In next studies, the resistive state of wide films was associated only with the vortex motion. This looked quite natural because at that time, typical observed IVCs were similar to that shown in Fig. 3, with abrupt transition from the vortex resistive state to the normal state. We believe that in most cases, the origin of such a break-off form of the IVC is imperfection of both the experimental conditions and the samples. We note that in the experimental studies of the resistive state, the heat compliance between the film and the substrate plays an important role [8]. If the choice of the pair «film–substrate» is not optimal, or the adhesion of the film to the substrate is imperfect, the Joule overheating of the film in the resistive state leads to the break-off IVCs shown in Figs. 1 and 3.

In 1972, in study of wide tin films sputtered on the quartz substrate which provides optimal heat compliance, the authors of [9] observed not only a typical vortex region of the IVC at small transport current, but also a step-like structure at large current, which obtained no physical explanation at that time. Relying on the recommendations given in [8], the authors of [10] fabricated the films having the IVCs similar to that shown in Fig. 4. Then, using their knowledge about the phase-slip processes in narrow channels, they associated the voltage steps in the IVCs of wide films with the creation of phase-slip lines (PSLs). At the present time, it seemed that such a form of the IVC for a wide film, consisting of the vortex region and the step-like part due to the PSLs, is widely recognized; see, e.g., [9–13]. However, the break-off-type IVCs are still frequently observed [14-18].

Let us first discuss the theoretical models of the vortex resistive state. We would like to draw one's attention to the theoretical papers [20,21] which do not explain the whole form of the IVC shown in Fig. 4, but give a rather good description of the vortex part of the IVC. In wide thin superconducting films, the magnetic field of the transport current gives rise to the creation of Pearl vortices at the film edges. The motion of the vortices across the film leads to the occurrence of a voltage along the film. On the basis of such a picture of the resistive vortex state, the equation for the critical current was found by Aslamazov and Lempitskiy (AL) [20],

$$I_c^{AL}(T) = 1.5 I_c^{GL}(0)(\pi\lambda_\perp(0)/w)^{1/2}(1-T/T_c)\,, \qquad (2)$$

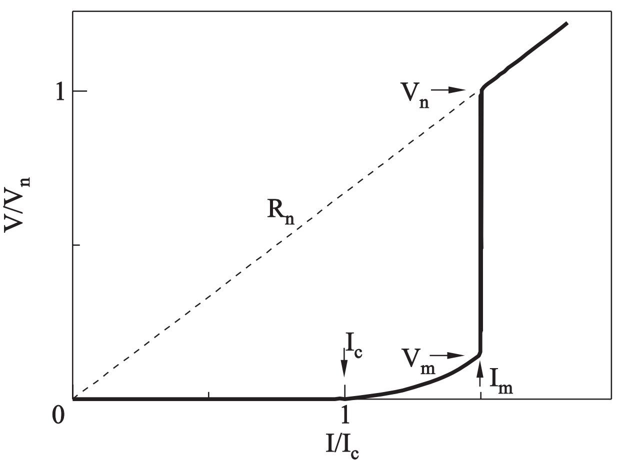

Fig. 3. Break-off-type IVC of a wide superconducting film.

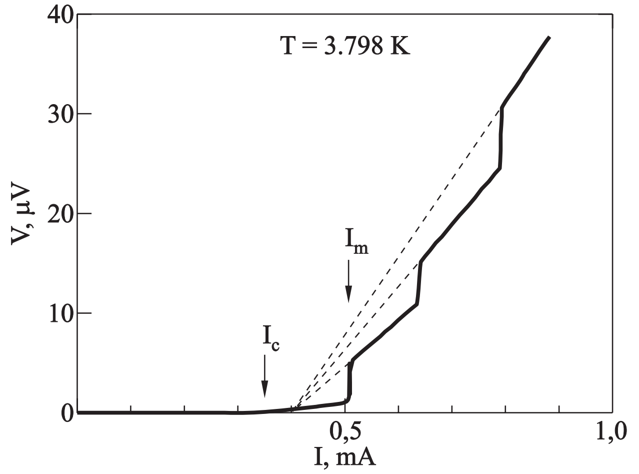

Fig. 4. Experimental IVC of a wide film SnW13 at $T = 0.99\,T_c$.

using the condition of stability of the Meissner state with respect to an infinitely small perturbation of the superconducting order parameter. Physically, at $I = I_c^{AL}$, the edge current density approaches the GL critical value, and the edge barrier for the vortex entry disappears. In order to describe the vortex part of the IVC, AL studied the viscous motion of the vortices in the film, using the hydrodynamic approximation which assumes introduction of averaged macroscopic quantities: the density of vortices and the averaged current density satisfying the macroscopic equations which connect these quantities with the averaged electric field. According to this theory, the evolution of the resistive vortex state looks as follows. As the current grows, the vortex density increases, and the current distribution across the film becomes more homogeneous. At a certain current value,

$$I_m(T) = C\, I_c^{GL}(T)\, \ln^{(-1/2)}(2w/\lambda_\perp(T)) \gg I_c^{AL}, \quad (3)$$

the current density approaches its critical value not only at the film edges, where the vortices are born, but also in the middle of the film cross-section, where the vortices and anti-vortices annihilate. At this moment the vortex state becomes unstable, although the distance between vortices is still larger than the size of the vortex core, and the authors of [20] assert that the film undergoes jump-like transition to the normal state, as shown in Fig. 3. The experimental study of wide films [12] confirmed much of the statements of the AL theory, including correctness of equations (2) and (3). Besides, these investigations also resulted in considerable refinements of several points of the theory. It turned out [12] that the vortex resistivity occurs only at large enough film width, $w > 5\lambda_\perp(T)$, and at $I > I_m$ the film undergoes transition not to the normal state, but to a vortex-free state with PSLs (Fig. 4). Such a picture of the resistive state of a wide film was later recognized in [22] by S.V. Lempitskiy, one of the authors of the resistive vortex state theory. He predicted that if the distances between PSLs are larger than the penetration depth for the electric field into the superconductor, then the IVC of a wide film will be described by known equations for a vortex-free narrow superconducting channel [2]. This result was also confirmed experimentally [23,24].

Another approach to the analysis of the vortex mechanism of resistivity was used by Vodolazov and Peeters [21] which performed numerical solution of extended time-dependent Ginzburg–Landau equations [25,26] for the vortex motion in the superconducting plate, formally considering an infinitely thick superconducting slab which reduces the problem to a two-dimensional one. In this aspect, the problem becomes rather close to the problem of the vortex state in a thin film, although the structure of vortices in these cases is quite different: exponential decay of the magnetic field around the Abrikosov vortices in a massive slab and slow power-like decay around the Pearl vortices in a thin film. The results of [21] generally confirm the macroscopic analysis of the vortex state by AL [20]: when the current exceeds the critical value, at which the surface barrier for the vortex entry is suppressed, the vortices and anti-vortices enter the slab and then, being affected by the transport current, move to the middle of the sample, where they annihilate. This process leads to a maximum of the current density in the middle of the sample, in accordance with macroscopic calculations in [20]. At moderate values of the transport current, the vortex structure of the sample is close to the triangle lattice. However, when the current density in the middle of the sample approaches the depairing value, i.e., when the transport current approaches I_m, the triangle vortex lattice turns into row-like vortex structure. The authors of [21] interpret this phenomenon as the creation of quasi-PSLs due to acceleration of the vortex–anti-vortex annihilation and anomalously rapid motion of the vortices. According to this theory, the PSLs represent the rows of rapidly moving vortices which occur simultaneously along the whole length of the sample. This contradicts the experimental data shown in Fig. 4 which indicate consequent appearance of the PSLs while the transport current increases. For this reason, in order to confirm their conclusions, the authors of [21] refer only to the experiments [14–18], in which the break-off-like IVCs were observed. In our opinion, these experiments have mutual drawback: the substrates do not provide a good heat removal from the films. This is indicated by hysteresis observed, e.g., in [16], or by low quality of the films with numerous pinning centers and bad adhesion with the substrate, as in [15]. We believe that for these reasons, the PSLs occur in such films in an avalanche, which is accompanied by a break-off in the IVC. Our point of view is supported by the results of investigation of the resistive state of wide films by the laser scanning microscope [19]. Due to specifics of this method, the film is overheated by the laser irradiation, which gives rise to the break-off behavior of the IVC (Fig. 3). However, the visualization of the resistive state by this microscope shows that at $I > I_m$ the number of PSLs increases smoothly with the transport current, starting from the single PSL at $I = I_m$.

Now we proceed to the results of our experimental studies. The samples whose characteristics are given in the Table were fabricated by using an original technique which provides minimization of bulk defects and results in perfect, almost specular, film edges. All such samples show full GL critical current in a near vicinity of T_c, where the condition of narrow channel regime $w < \lambda_\perp(T)$ is satisfied; actually, this criterion was used for the selection of samples for next experiments in the resistive state. Special attention was devoted to the quality of the substrate — optically polished crystalline quartz, which seems to be the better material for the heat removal from tin films.

Table. Parameters of the film samples

Sample	L, μm	w, μm	d, nm	$R_{4.2}$, Ω	$R^{\square}$, Ω	T_c, K	l_i, nm	R_{300}, Ω
Sn4	30	1	199	1.45	0.048	3.783	131	21.50
SnW9	95	17	159	0.319	0.057	3.825	138	4.900
SnW10	88	7	181	0.487	0.040	3.809	169	9.156
SnW13	90	18	332	0.038	0.008	3.836	466	1.880

C o m m e n t s: Here L, w and d are the length, the width and the thickness of a sample; l_i is the electron mean free path.

As follows from the AL theory [20] (see (2) and (3)), the reduced critical currents $I_c^{AL}/I_c^{GL}(T)$ and $I_m/I_c^{GL}(T)$ for wide films, $w/\lambda_\perp \gg 1$, must be universal functions of the basic parameter of the theory, $w/\lambda_\perp(T)$. In other words, the dependencies of the reduced critical currents on this parameter should be not affected either by the geometry of the films, or by their material properties. Our experimental results completely confirm such universality and, moreover, extend it over the small values of $w/\lambda_\perp$. Examples of the experimental dependencies of the reduced critical currents are presented in Fig. 5. To avoid overloading of the figure, we plot the results obtained on a single pair of essentially different films; other samples demonstrate quite similar behavior. The triangles (direct and inverted) correspond to the upper boundary of the purely superconducting state. In the regime of narrow film (superconducting channel), $w/\lambda_\perp < 5$, the sample is completely superconducting until the transport current reaches the GL depairing current; at $I \geq I_c^{GL}$ the film undergoes transition to the resistive state with one-dimensional PSCs. Correspondingly, this part of the dependence of I_c/I_c^{GL} can be approximated by horizontal straight line. When the ratio $w/\lambda_\perp$ exceeds 5, an unusual phenomenon is observed: the value of the critical current sharply falls to $0.8I_c^{GL}$ (vertical approximating line) and then holds this value until $w/\lambda_\perp$ reaches 10 (second horizontal line). As soon as the transport current exceeds $0.8I_c^{GL}$, the film enters the vortex resistive state, in accordance with general conclusions of the theory. However, such behavior of I_c is inconsistent with the AL theory, even with its generalized version valid for the case of arbitrary value of $w/\lambda_\perp$ and developed by us. Formal consideration of this problem leads to the conclusion that at $5 < w/\lambda_\perp < 10$ the vortices can overcome the edge barrier when the edge current density approaches the value $j_c \sim (1-T/T_c)^2$ much smaller than the GL critical density $j_c^{GL} \sim (1-T/T_c)^{3/2}$, but the physical mechanism of such anomalous penetration of vortices into the film is unclear. For very wide films with the transversal size $w > 10\lambda_\perp$, the experimental data excellently agree with the AL theory (curve *1* in Fig. 5). The experimental values of the upper boundary of the stability of the vortex state, $I_m/I_c^{GL}(T)$, are plotted in Fig. 5 for two different films by circles and squares. As it is obvious from Fig. 5, these data well correlate with the result (3) of the AL theory with the fitting parameter $C = 1.6$ (curve *2*).

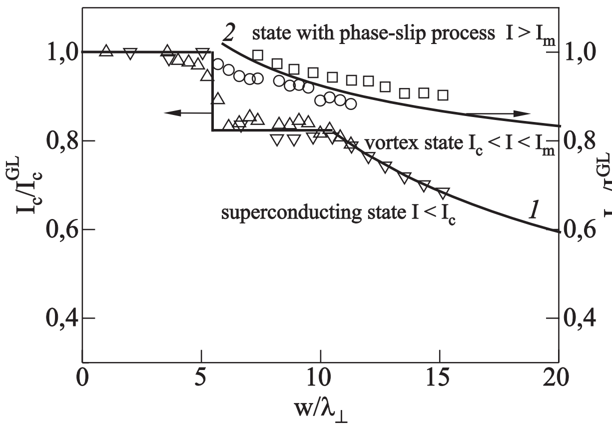

Fig. 5. Diagram of the current-carrying states of wide superconducting films in the dependence on the reduced film width and the reduced magnitude of the transport current. Different states are separated by the dependencies of the reduced critical current I_c / I_c^{GL} for the samples SnW9 (▽), SnW10 (△) and of the reduced current of the vortex instability I_m / I_c^{GL} for the samples SnW9 (□), SnW10 (○) on the quantity $w / \lambda_\perp$. The smooth part of the curve *1* represents the AL theoretical dependence of I_c^{AL} / I_c^{GL} calculated by (1) and (2); the curve *2* is the AL theoretical dependence of I_m / I_c^{GL} calculated by (1) and (3) with $C = 1.6$.

The set of lines in Fig. 5 can be considered as critical lines at the phase diagram for thin superconducting films. These lines divide the phase plane «reduced current I/I_c^{GL} – reduced film width $w/\lambda_\perp$» into three regions. The lower region, $I < I_c$, corresponds to the completely superconducting state: the homogeneous current state in narrow films ($w < 5\lambda_\perp$) or the Meissner state in wide films ($w > 5\lambda_\perp$). For the latter case, there exists the intermediate vortex resistive region, $I_c < I < I_m$, where $I_c = 0.8I_c^{GL}$ at $5\lambda_\perp < w < 10\lambda_\perp$, and $I_c = I_c^{AL}$ for $w > 10\lambda_\perp$. Then, at $I > I_m$ the wide film ($w > 5\lambda_\perp$), enters the resistive state with PSLs, while the narrow film ($w < 5\lambda_\perp$) exhibits direct transition at $I > I_c^{GL}$ to the resistive state with PSCs.

In conclusion, we develop the phase diagram for the current states of superconducting films, based on the experimental investigation of the resistive transitions induced by the transport current in the films. We found that a narrow film with the width $w < 5\lambda_\perp$ never enters the vortex state, but experiences direct transition from the purely superconducting state to the resistive state with PSCs as soon as the current exceeds the critical GL current. The Meissner current state of the films of intermediate width,

$5\lambda_{\perp}(T) < w < 10\lambda_{\perp}(T)$, transforms at $I > 0.8 I_c^{GL}$ to the vortex resistive state which exists within the current interval $0.8 I_c^{GL} < I < I_m$, where the value of the upper critical current I_m well agrees with the theory [20]. The vortex state of wide films, $w > 10\lambda_{\perp}(T)$, is realized within the current region $I_c^{AL} < I < I_m$, where I_c^{AL} is the transition point to the vortex state calculated in [20] for the limiting case $w \gg \lambda_{\perp}$. At $I > I_m$, the films with the width $w > 5\lambda_{\perp}(T)$ enter a vortex-free resistive state with phase-slip lines.

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