The two-Josephson-junction flux qubit with large tunneling amplitude

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In this paper we discuss solid-state nanoelectronic realizations of Josephson flux qubits with large tunneling amplitude between the two macroscopic states. The latter can be controlled via the height and form of the potential barrier, which is determined by quantum-state engineering of the flux qubit circuit. The simplest circuit of the flux qubit is a superconducting loop interrupted by a Josephson nanoscale tunnel junction. The tunneling amplitude between two macroscopically different states can be increased substantially by engineering of the qubit circuit if the tunnel junction is replaced by a ScS contact. However, only Josephson tunnel junctions are particularly suitable for large-scale integration circuits and quantum detectors with present-day technology. To overcome this difficulty we consider here a flux qubit with high energy-level separation between the "ground" and "excited" states, consisting of a superconducting loop with two low-capacitance Josephson tunnel junctions in series. We demonstrate that for real parameters of resonant superposition between the two macroscopic states the tunneling amplitude can reach values greater than 1 K. Analytical results for the tunneling amplitude obtained within the semiclassical approximation by the instanton technique show good correlation with a numerical solution. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2967504]

I. INTRODUCTION

Since the successful demonstration of Rabi oscillations and Landau–Zener coherence effects,[1–5] the superconducting qubits (quantum bits) based on mesoscopic Josephson junctions have become the subject of consideration as possible candidates to be the basic elements of quantum computer hardware,[6,7] including detectors for measuring the state of an individual qubit.[8–12] The Josephson junction (JJ) qubits have two energy scales, which are the Josephson coupling energy E_J and the charging energy E_C of the JJ, and they are subdivided into flux qubits, charge qubits, and charge-phase qubits. In principle, all circuits of a quantum computer can be fabricated by modern techniques using these superconducting qubits. However, it is but poor quality[6,13] of the experimentally tested elements that is the limiting factor on the way of implementation of quantum registers. For example, an important but still unsolved problem in the physics of a qubit working in the charge regime with $E_C/E_J \gg 1$ is a substantial decrease of the high spectral density of the noise associated with the motion of charge in traps.

In turn, the phase qubit ($E_J/E_C \gg 1$), which utilizes the phase of the superconducting order parameter as a dynamical variable, is much less sensitive to the charge fluctuations but is subject to the influence of noise in the critical current of the JJ, spin fluctuations, and Nyquist noise currents generated by excess ambient temperature.

The tunnel splitting of the energy levels arising from the coherent superposition of the macroscopic states is usually small, $\Delta E_{01} \sim 150-250$ mK. Taking into account the effective noise temperature, which can reach $T_{\text{eff}} \sim 50-100$ mK in experimental studies of the qubit dynamics, leads to a dramatic fall of the decoherence times τ_φ and relaxation times τ_ε.[14–16] This means that, in order to enhance considerably the qubit quality[6] (the number of one-bit operations during the coherence time), a system with large ($\Delta E_{01} \gtrsim 1$ K) tunnel splitting of the energy levels should be created.

Undoubtedly, the problem of creation of a quantum register based on Josephson qubits brings up many issues, but presently the invention of a high-quality qubit is the most important one among them. It is easy to show that the rate of energy exchange between two macroscopic states in a flux qubit is bounded by the "cosine" shape of the potential barrier and cannot be increased owing to decreasing the barrier height, since the latter determines the characteristic rate of thermal decay of the current-flow states. A similar limitation associated with the lowering of the effective barrier height can appear also when greatly increasing the pre-exponential factor. It is absolutely obvious that the ideal case for a flux qubit is when the tunnel barrier in the phase space looks like Π-shaped function having sufficiently large height and small action. It was this issue that motivated the authors of Ref. 13 to analyze the phase-slip qubit, whose creation required de-

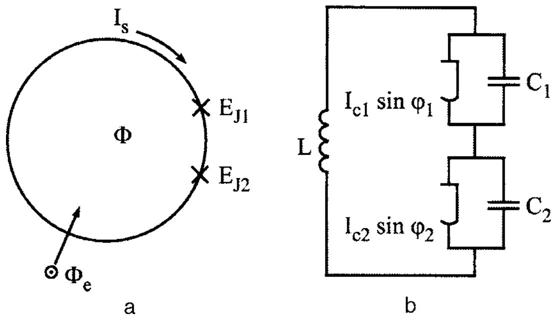

FIG. 1. Schematic picture of the proposed 2JJ flux qubit with a SQUID configuration (a) and its circuit diagram (b). The loop carrying supercurrent I_s is pierced by an externally applied magnetic flux Φ_e (towards the viewer). The individual SIS Josephson junctions are characterized by coupling energies E_{J1}, E_{J2}, critical currents I_{c1}, I_{c2}, and capacitances C_1, C_2 which do not differ significantly. The loop inductance L is small enough that the 2JJ SQUID has only two metastable flux states. The parameter $g_0^{\min}=E_J(\pi)/E_C \gg 1$ (see below).

veloping a new non-Josephson technology. In this paper we search for an improved barrier design for the JJ flux qubit.

Recently in Ref. 11 it was demonstrated how the level splitting can be increased at low temperatures ($T \to 0$) by an order of magnitude with the potential barrier height kept unchanged by modifying the qubit's potential barrier shape through the use of a clean-limit ScS junction in the superconducting ring. However, the fabrication difficulties of obtaining pure and reproducible ScS junctions are a serious hindrance in the way of designing large-scale integrated qubit circuits.

To solve this problem, in the present paper an analysis is made of the two-Josephson-junction flux qubit (2JJ flux qubit), which can be considered as a superconducting ring of inductance L interrupted by two almost equivalent tunnel SIS junctions with Josephson energies E_{J1} and E_{J2}, critical currents I_{c1} and I_{c2}, and capacitances C_1 and C_2, respectively (see Fig. 1a and 1b). The difference between the two SIS mesoscopic junctions will be characterized by the asymmetry parameter $\lambda = I_{c2}/I_{c1} = E_{J2}/E_{J1} = C_2/C_1 \leqslant 1$, so that "junction 1" would have greater or equal values of the Josephson energy, critical current, and capacitance as compared to "junction 2." The external magnetic flux Φ_e can be coupled to the qubit by a separate coil located in close proximity to the qubit's loop. It is well known that in the classical limit the circulating current I_s as a function of external magnetic flux for dc SQUIDs with $I_{c1}=I_{c2}$ has singularities at the points $\Phi_e = \Phi_0(n+1/2)$ (Φ_0 is the flux quantum), so that the two-junction interferometer can be considered as a "single-junction" one with a modified potential energy shape in phase space. Below we indicate conditions for the proposed 2JJ flux qubit under which the classical Josephson relationship between the phase differences on the JJ contacts is retained in the quantum regime, phase (flux) is a good quantum variable, and the charging effect on the island between JJ contacts is negligible. The problem lies in determining and analyzing the tunnel splitting $\Delta E_{01} = E_1 - E_0$ of the degenerate zero energy level in the double-well symmetric potential of a 2JJ flux qubit (at corresponding external conditions) resulting from the coherent quantum tunneling of the magnetic flux between the wells. In the proposed mesoscopic system in the quantum regime, the two lower energy levels E_0 and E_1 arising from coherent superposition of the macroscopically distinct flux or persistent-current states form a qubit. It turns out that, because of the change in the form of the potential energy of the 2JJ flux qubit as compared to the 1JJ qubit, the tunnel splitting ΔE_{01} can rise manyfold, reaching values $\gtrsim 1$ K (in temperature units) and substantially enhance the properties of the qubit as a basic element for quantum computations. The sensitivity of the ΔE_{01} magnitude to λ as well as to the junction parameters can limit applications based on the 2JJ flux qubit both for quantum computation and quantum detectors.

II. THEORETICAL MODEL AND RESULTS

We will discuss the 2JJ flux qubit in the approximation of the Hamiltonian of an isolated system in the zero temperature limit. All the dissipative processes associated with the internal and the external (with respect to the system) degrees of freedom—the quasiparticles, the magnetic flux fluctuations in the qubit and in the outer measuring circuit, etc.—are neglected in this approximation. In the framework of this approximation, only the supercurrent component flows in the qubit ring, and in the classical regime, according to the Josephson relation, it is equal to

$$I_s = I_{c1}\sin\varphi_1 = I_{c2}\sin\varphi_2, \tag{1}$$

where φ_1 and φ_2 are the order parameter phase differences at the corresponding tunnel junctions. It is convenient to measure the values of the supercurrent I_s and the phase differences at the junctions clockwise, the applied magnetic flux Φ_e, the total magnetic flux in the ring Φ, and the supercurrent I_s being tied by the relation $\Phi = \Phi_e - LI_s(\Phi)$. The classical Hamiltonian of the 2JJ flux qubit in the approximation of an isolated system contains the contributions of the electrostatic energy of the charges in the junction capacitances, the junction Josephson energies, and the magnetic energy of the supercurrent in the ring, and has the form:

$$H = \frac{(2eN_0)^2}{2}\left(\frac{1}{C_1}+\frac{1}{C_2}\right) - (E_{J1}\cos\varphi_1 + E_{J2}\cos\varphi_2) + \frac{(\Phi-\Phi_e)^2}{2L} + E_0, \tag{2}$$

where N_0 is the number of the excess (deficient) Cooper pairs in the banks of the SIS Josephson junctions, and E_0 is a constant fixing the reference level for the potential energy. Using relation (1), we will reduce the expression for the Josephson energy in the classical Hamiltonian (2) to the form

$$U_J^0(\phi) = -E_J(\phi) = -(E_{J1}\cos\varphi_1 + E_{J2}\cos\varphi_2) = -E_{J1}\sqrt{(1-\lambda)^2 + 4\lambda\cos^2(\phi/2)},$$

where a new variable of the overall phase $\phi = \varphi_1 + \varphi_2$ is introduced.

The proposed 2JJ qubit system is topologically analogous to the charge-phase qubit,[8,17] representing a single-Cooper-pair tunneling (SCPT) transistor (which consists of two Josephson junction contacts with the voltage gate next to the island between them) inserted in a superconducting ring. Therefore the structure of the Josephson energy in the Hamiltonian (2) of the 2JJ qubit is similar to that of the

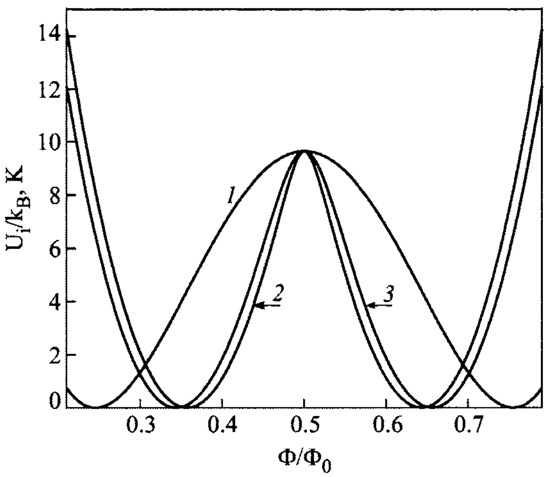

FIG. 2. Potential $U(\Phi/\Phi_0)/k_B$ in temperature units for the 1JJ qubit with $\beta_L=1.602$ (*1*) and for the 2JJ qubit with the parameter pairs $(\lambda,\beta_L)=(0.9,1.058)$ (*2*) and (0.8, 1.276) (*3*) at external magnetic flux $\Phi_e=\Phi_0/2$. The geometric ring inductance is $L=3.0\times10^{-10}$ H for both qubits; the potential barrier heights U_0 in curves *1–3* are equal, $U_0/k_B=9.64$ K.

charge-phase qubit. The main difference, affecting the form of the Josephson energy, lies in that: (i) in the 2JJ qubit there is no charge gate, and no polarization charge Q_0 is induced through it on the island; (ii) the charge-phase qubit is designed to work in the charge mode, whereas the 2JJ qubit is designed to work in the flux mode, that is, in the extreme opposite dynamic regime. In an earlier paper[9] devoted to a quantum detector based on the SCPT transistor, a study was made of its working regimes that depend on the form of the Josephson energy of the system [formulas (1) and (2) of Ref. 9]: $U_J(\phi,\varphi)=-(E_{J1}\cos\varphi_1+E_{J2}\cos\varphi_2)=-E_J(\phi)\cos(\varphi+\gamma(\phi))=-E_J(\phi)\cos\chi$, $\varphi=(\varphi_1-\varphi_2)/2$, $\tan\gamma(\phi)=(\lambda-1)/(\lambda+1)\tan(\phi/2)$), and can be characterized by the parameter $g_0=E_J(\phi)/E_C$, where $E_C=e^2/2C$ is the characteristic charging energy of the island between JJ contacts, $C=C_1+C_2+C_g$ being the total capacitance of the island with respect to the rest of the system ($C=C_1+C_2$ for the 2JJ qubit, as $C_g=0$). The parameter g_0 crucially determines the mutually conditioned quantum-averaged supercurrent (current-phase relation) $I_s(\phi)$ and effective Josephson energy $U_J(\phi)$ of the SCPT transistor and of the charge-phase qubit based on it, respectively.[8] In solving the Schrödinger equation, the supercurrent $I_s(\phi)=I_s^0(\phi)\langle\cos\chi\rangle$ is represented as an appropriate supercurrent in the classical limit $I_s^0(\phi)$, multiplied by the function $\langle\cos\chi\rangle$ that describes an effective influence of charge fluctuations on the island between JJ contacts [formulas (4) and (5) and Fig. 2a, with a family of $\cos\chi(Q_0)$ curves for differnt values of the parameter g_0 in Ref. 9]. This result of Ref. 9 reflects a physically clear conclusion: (i) the effect of fluctuations of Cooper-pair number ($\hat{n}=-id/d\varphi$) on the island, which affects $I_s(\phi)$ and $U_J(\phi)$, is well apparent in the charge mode of system dynamics ($g_0\lesssim1$), the function $\langle\cos\chi\rangle(Q_0)$ being strongly reduced and modulated; (ii) in the opposite limit $g_0\gg1$ the function $\langle\cos\chi\rangle$ becomes a constant close to unity, and because $\langle\cos\chi\rangle\equiv1$ at $g_0\to\infty$, in this limit the current-phase relation $I_s(\phi)=I_s^0(\phi)$ and the effective Josephson energy $U_J(\phi)=U_J^0(\phi)=-E_J(\phi)$ are described by classical expressions. It is interesting to note that an experimental investigation of the charge-phase qubit with the parameter $g_0\sim1$ [$E_J(\phi)\sim E_C$, i.e., $E_J(\phi)$ is rather large for a charge-phase qubit] at low temperatures (20 mK) demonstrated[18] that the influence of the gate quasicharge Q_0 is of the order of noise, and the characteristic form of the current-phase relation conforms qualitatively to the corresponding classical relation [see Eq. (5) below].

We consider the extreme case $g_0\gg1$ to make the 2JJ qubit work in the flux mode, where the effect of charge fluctuations due to the variable φ is negligible (so that φ falls out from the Hamiltonian). In this extreme case the classical expressions for the Josephson energy of the two-junction interferometer and for the supercurrent through its loop apply to the quantum regime of system dynamics, and relation (1) between the variables φ_1 and φ_2 holds.

Note that for the 1JJ flux qubit the usual condition for the phase to be a good quantum variable is that the parameter $g=E_{J1}/E_{C1}\gg1$, where $E_{C1}=e^2/2C_1$ is the characteristic electrostatic energy of the JJ contact. Then the minimum value of the parameter g_0 of the 2JJ flux qubit as a system and the parameter g characterizing a single JJ contact of the qubit are connected by the relation $g_0^{\min}=E_J(\pi)/E_C=E_{J1}(1-\lambda)/[e^2/2C_1(1+\lambda)]=(1-\lambda^2)g$. Thus the 2JJ flux qubit has to satisfy the condition $g_0^{\min}=(1-\lambda^2)g\gg1$, and the parameter λ is bounded from above by this condition.

Due to the single-valuedness of the superconducting order parameter the variable ϕ satisfies the condition

$$\phi=\varphi_1+\varphi_2=2\pi\frac{\Phi}{\Phi_0}+2\pi n,\quad \Phi=n\Phi_0+\frac{\phi}{2\pi}\Phi_0,$$

$$\Phi_0=\pi\hbar/e, \tag{3}$$

where n is the integer number of flux quanta Φ_0 in the total magnetic flux Φ (below, we will consider the qubit to work in the $n=0$ mode). Owing to relations (1) and (3), there is only one independent phase variable from among φ_1,φ_2,ϕ, and in the quantum regime the physical fluctuating quantum variable is the total phase difference ϕ between the two junctions, which, to within a factor of 2π, is equal to the total magnetic flux in the ring in units of the flux quantum ($\phi/2\pi=\Phi/\Phi_0$).

The transition to the quantum description of the flux qubit consists in associating the value N_0 of the Cooper pairs tunneling through the junctions with the operator $\hat{N}_0=-i\partial/\partial\phi$, which is conjugate to the phase operator $\hat{\phi}$ ($[\hat{N}_0,\hat{\phi}]=-i$), and solving the Schrödinger equation with the resulting Hamiltonian in the ϕ representation.[19] By applying the quantization procedure to the Hamiltonian (2) and writing down the energy contributions via the variable ϕ, we will come to a canonical form of the Hamiltonian of the 2JJ flux qubit in the quantum case:

$$\hat{H}=\frac{\hat{P}^2}{2M}+\hat{U}(\phi)=-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial\phi^2}+E_{J1}\left[\varepsilon_0-\sqrt{(1-\lambda)^2+4\lambda\cos^2\frac{\phi}{2}}+\frac{(\phi-\phi_e)^2}{2\beta_L}\right], \tag{4}$$

which can be considered as the Hamiltonian of a quantum particle of mass M moving in a potential $U(\phi)$. Here $\hat{P}$

$=\hbar\hat{N}_0=-i\hbar\partial/\partial\phi$ ($[\hat{P},\hat{\phi}]=-i\hbar$) corresponds to the particle momentum operator,

$$M=\left(\frac{\Phi_0}{2\pi}\right)^2\frac{\lambda C_1}{(\lambda+1)}$$

is its mass,

$$\beta_L=\frac{2\pi}{\Phi_0}LI_{c1}=\left(\frac{2\pi}{\Phi_0}\right)^2 LE_{J1}$$

is the potential parameter, $\phi_e=2\pi\Phi_e/\Phi_0$ is the external magnetic flux parameter (the constant ε_0 is chosen further from the condition that the symmetric potential equals zero at the minimum points). The 2JJ flux qubit parameter

$$g_0^{\min}=(1-\lambda)^2\frac{\beta_L}{L}\left(\frac{\Phi_0}{2\pi}\right)^2\frac{2C_1}{e^2}=(1-\lambda^2)\beta_L\frac{C_1}{L}\frac{\hbar^2}{2e^4}\gg 1.$$

The shape of the potential $U(\phi)$ depends on the parameters λ,β_L,ϕ_e. We are interested in the case of a symmetric potential, which, according to (4), is realized at $\phi_e=\pi$ ($\Phi_e=\Phi_0/2$). It should be noticed that formally in the extreme case of identical junctions, at $\lambda=1$ (though really a value $\lambda\approx 1$ must be used so as to satisfy the condition $g_0^{\min}\gg 1$), the potential $U(\phi)$ coincides with the potential of the flux qubit based on the clean ScS contact studied in Ref. 11 [Eq. (3)]:

$$U_{ScS}(\phi)=E_J\left[-2|\cos(\phi/2)|+\frac{(\phi-\phi_e)^2}{2\beta_L}\right].$$

At the same time, owing to renormalization of the mass M for the 2JJ flux qubit by the factor $\lambda/(\lambda+1)$ with respect to the corresponding mass for the ScS flux qubit (provided that the capacitances of the SIS and ScS junctions are equal, $C_1=C$), for $\lambda\approx 1$, the relation for the masses is $M_{2JJ}\approx M_{ScS}/2$. Hence, at $\lambda\approx 1$ the splitting ΔE_{01} in the 2JJ qubit is expected to be more than in the ScS qubit. The parameter β_L determines the height of the potential barrier of the double-well potential, so that the barrier height goes down with decreasing β_L. Like in the case of ScS qubit, the 2JJ qubit potential has two local minima even at $\beta<1$ (unlike the SIS qubit, where the double-well potential exists at $\beta>1$ only), which gives the possibility of considerably scaling down the geometric dimension (inductance) of the system with the mesoscopic junctions.

Figure 2 shows the potential $U(\Phi/\Phi_0)/k_B$ of 2JJ flux qubit for two parameters couples (λ,β_L) and also, for comparison, the well-known potential $U_{SIS}(\Phi/\Phi_0)/k_B$ of 1JJ flux qubit at external magnetic flux $\Phi_e=\Phi_0/2$. The inductances L for both types of the qubits can be supposed equal (to specify the magnetic flux fluctuation level) while the parameter β_L (i.e., the critical currents I_{c1},I_c of the corresponding SIS junctions) in all the dependences is chosen such that the potential barriers in all the potentials are of the same height U_0. The latter requirement implies roughly equal decay rates for the metastable states due to thermal fluctuations; taking them into account is beyond the scope of this paper. Apparently, to realize the quantum regime in a physical experiment, the value U_0/k_B must greatly exceed the system temperature. As seen from Fig. 2, the potentials $U(\Phi/\Phi_0)/k_B$ for a 2JJ qubit have lesser width (between the potential minimum points) as compared to the corresponding potentials for a 1JJ qubit,

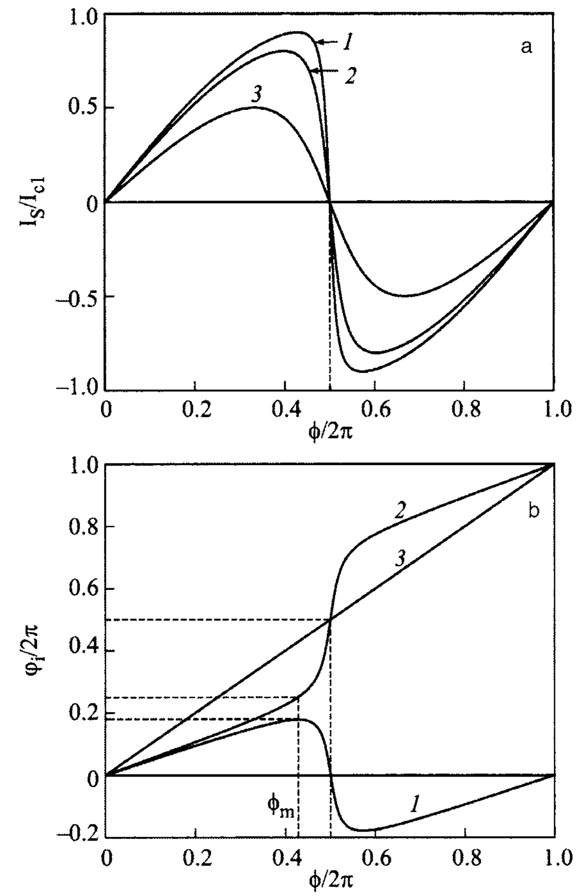

FIG. 3. Integral phase-current relation $I_s(\phi/2\pi)/I_{c1}$ for 2JJ qubit at various λ: 0.9 (*1*), 0.8 (*2*), 0.5 (*3*) (a); the functions $(\varphi_1/2\pi)[\phi/2\pi]$ (*1*), $(\varphi_2/2\pi)\times[\phi/2\pi]$ (*2*) for the 2JJ qubit at $\lambda=0.9$. The straight line $\varphi_1(\phi)+\varphi_2(\phi)=\phi$ (*3*) corresponds to the definition of ϕ. The values of $\varphi_1/2\pi=\arcsin(\lambda)/2\pi\approx 0.18$ and $\varphi_2/2\pi=0.25$ (the latter being λ-independent) correspond to $\phi_m/2\pi=\arccos(-0.9)/2\pi\approx 0.43$ (b).

while the area under the potential curve between the points of its minima for the 2JJ qubit shrinks greatly against the corresponding area for the 1JJ qubit. Additionally, if the corresponding capacitances of the SIS junctions in both 2JJ and 1JJ qubits are equal $(C_1=C)$ then the ratio of the effective masses for these qubits is $\lambda/(\lambda+1)$. As will be shown below, it is the change in the potential shape and the decrease of the effective mass in the 2JJ qubit that lead to the manyfold rise in the amplitude of its tunnel splitting.

The current-phase relation for the 2JJ qubit, directly related to the Josephson potential energy $U_J(\phi)$, is derived from Eqs. (1) and (3):

$$\frac{I_s(\phi)}{I_{c1}}=\sin\varphi_1=\lambda\sin\varphi_2=\frac{\lambda\sin\phi}{\sqrt{(1-\lambda)^2+4\lambda\cos^2(\phi/2)}}. \tag{5}$$

The extrema of the current-phase relation $I_s(\phi)$ (which are equal in absolute value) are located at the points $\phi_m=\arccos(-\lambda)$ (maximum; $\pi/2\leqslant\phi_m\leqslant\pi$) and $\phi_{m1}=2\pi-\arccos(-\lambda)$ (minimum) symmetrically around the point $\phi=\pi$, where the supercurrent vanishes to zero ($I_s=0$) alternating its direction. Thus, at λ near unity, in the interval (ϕ_m,ϕ_{m1}) the supercurrent I_s changes dramatically from its maximum to minimum value when the current direction changes at the point $\phi=\pi$. Figure 3a displays the integral current-phase relation $I_s(\phi/2\pi)/I_{c1}$ for the 2JJ qubit for several parameters λ. The interval (ϕ_m, ϕ_{m1}) shrinks as the parameter λ increases, and the maximum-to-minimum transition in current becomes sharper (the extreme case $\lambda=1$, which is valid in classical SQUID dynamics, corresponds to $\phi_m=\phi_{m1}=\pi$, with an infinite derivative of the current-phase relation at the point π). Let us also consider the order-parameter phase differences $\varphi_1(\phi)$, $\varphi_2(\phi)$ derived directly from (5). Analysis of formula (5) shows that the function $\varphi_1(\phi)$ for a junction with high critical current has extrema at the points ϕ_m, ϕ_{m1}. The transition from the maximum positive value $\varphi_1(\phi_m)=\arcsin\lambda$ $(0\leqslant\varphi_1(\phi_m)\leqslant\pi)$ to the minimum negative value $\varphi_1(\phi_m)=-\arcsin\lambda$, with the phase difference changing sign at the point π $(\varphi_1(\pi)=0)$, takes place in the interval (ϕ_m, ϕ_{m1}), and $\varphi_1(0)=\varphi_1(2\pi)=0$. The function $\varphi_2(\phi)$ for a junction with lower critical current is a monotonically increasing one from $\varphi_2(0)=0$ to $\varphi_2(2\pi)=2\pi$, which is symmetrical with respect to the line $y=\phi$; $\varphi_2(\pi)=\pi$, and $\varphi_2(\phi_m)\equiv\pi/2$, $\varphi_2(\phi_{m1})=3\pi/2$. For the classical 2JJ SQUID, in the extreme case of $\lambda=1$ the functions $\varphi_1(\phi)$, $\varphi_2(\phi)$ behave as follows: $\varphi_1=\varphi_2=\phi/2$ at $0\leqslant\phi<\pi$; at the point π a jump appears in the function $\varphi_1(\phi)$ between the values $\pi/2$ and $-\pi/2$, with a further linear rise up to $\varphi_1(2\pi)=0$, while the function $\varphi_2(\phi)$ demonstrates a jump between the values $\pi/2$ and $3\pi/2$, with a further linear increase up to $\varphi_2(2\pi)=2\pi$. Figure 3b displays the dependences $(\varphi_1/2\pi)[\phi/2\pi]$ and $(\varphi_2/2\pi)[\phi/2\pi]$ for a certain λ, with their distinctive appearance. The straight line $\varphi_1(\phi)+\varphi_2(\phi)=\phi$ corresponds to the definition of ϕ, showing how the total phase difference over the two junctions is decomposed into the component phase differences of the order parameter over each of them.

We will find the tunnel splitting ΔE_{01} of the degenerate zero level in the symmetrical (at $\phi_e=\pi$) double-well potential $U(\phi)$ in the 2JJ flux qubit by numerical solution of the Schrödinger equation and analytically by using the instanton technique in the semiclassical approximation. To find a numerical solution of the stationary Schrödinger equation

$$\hat{H}\Psi(\phi)=E\Psi(\phi) \tag{6}$$

with Hamiltonian (4), a kind of finite-element method is used, with the potential $U(\phi)$ approximated by a piecewise-constant function. Zero boundary conditions are used for the wave function $\Psi(\phi)$, and the domain width and number of elements are set so as to provide good accuracy of the calculation.

In the semiclassical approximation the problem of a tunneling quantum particle can be solved using the instanton technique.[20,21] For a particle of mass M moving at zero temperature in a symmetric double-well potential $V(x)$, referenced from its minimum level $[V(\pm a)=0$, where $\pm a$ are the minimum points], the expressions for the energy levels $E_{1,0}$ and the tunnel splitting ΔE_{01} read like

$$E_{1,0}=E_0\pm\frac{\Delta E_{01}}{2}=\frac{\hbar\omega_0}{2}\pm\hbar K\exp\left(-\frac{S_0}{\hbar}\right),$$

$$S_0=\int_{-a}^{a}dx\sqrt{2MV(x)},$$

$$K=A\omega_0\sqrt{\frac{M\omega_0a^2}{\pi\hbar}},\qquad \omega_0^2=\frac{V''(\pm a)}{M},$$

$$\Delta E_{01}=2A\hbar\omega_0\sqrt{\frac{M\omega_0a^2}{\pi\hbar}}\exp\left(-\frac{S_0}{\hbar}\right). \tag{7}$$

Here ω_0 is the frequency of the zero-point oscillations of the particle in each of the wells, S_0 is the particle action on the instanton trajectory, and the dimensionless constant A is found from the equation for the instanton's function $t(x)$ in the asymptotic limit:

$$t(x)\big|_{x\to a}=\int_0^{x\to a}\frac{dx}{\sqrt{2V(x)/M}}=-\frac{1}{\omega_0}\ln\frac{a-x}{Aa},$$

$$A=\lim_{x\to a}\frac{(a-x)}{a}\exp\left(\int_0^x dx\sqrt{\frac{M\omega_0^2}{2V(x)}}\right). \tag{8}$$

Starting from the Hamiltonian (4) and using formulas (7) and (8), we obtain the tunnel splitting ΔE_{01} for the 2JJ flux qubit in the case of symmetric double-well potential:

$$\Delta E_{01}=4A\,\hbar\omega_0\sqrt{\frac{M\omega_0\alpha_0^2}{\pi\hbar}}\exp\left(-\frac{S_0}{\hbar}\right), \tag{9a}$$

$$\frac{S_0}{\hbar}=\frac{2\hbar}{e^2}\sqrt{\frac{\lambda C_1}{(\lambda+1)L}}\int_0^{\alpha_0}d\alpha\sqrt{\beta_L\left(\sqrt{(1-\lambda)^2/4+\lambda\sin^2\alpha_0}-\sqrt{(1-\lambda)^2/4+\lambda\sin^2\alpha}\right)+\alpha^2-\alpha_0^2}, \tag{9b}$$

$$\omega_0=\sqrt{\frac{(\lambda+1)}{\lambda C_1L}\left(1-\frac{\beta_L}{2}\frac{d^2}{d\alpha^2}\sqrt{(1-\lambda)^2/4+\lambda\sin^2\alpha}\bigg|_{\alpha_0}\right)},\qquad M=\left(\frac{\Phi_0}{2\pi}\right)^2\frac{\lambda C_1}{(\lambda+1)}, \tag{9c}$$

$$A=\lim_{\alpha\to\alpha_0}\frac{(\alpha_0-\alpha)}{\alpha_0}\exp\left(\sqrt{\frac{M\omega_0^2}{E_{J1}}}\int_0^{\alpha}\frac{d\alpha}{\sqrt{\sqrt{(1-\lambda)^2/4+\lambda\sin^2\alpha_0}-\sqrt{(1-\lambda)^2/4+\lambda\sin^2\alpha}+(\alpha^2-\alpha_0^2)/\beta_L}}\right), \tag{9d}$$

$$2\alpha_0\sqrt{(1-\lambda)^2/4+\lambda\sin^2\alpha_0}-\beta_L\lambda\sin\alpha_0\cos\alpha_0=0. \tag{9e}$$

A variable $\alpha=(\phi-\pi)/2$ is introduced in formulas (9) (due to the potential symmetry condition $\phi_e=\pi$), the minimum point $\alpha_0>0$ of the potential $U(\alpha)$ satisfying equation (9e). The accuracy of the semiclassical approximation is high provided that $S_0/\hbar\gg 1$; the method accuracy degrades as the dimensionless variable $S_0/\hbar$ diminishes, approaching unity. The results of a numerical analysis are of great importance in this region.

Figure 4a and 4b, presents the β_L dependence of the tunnel splitting $\Delta E_{01}(\beta_L)/k_B$ for 2JJ, ScS, and SIS flux qubits at equal capacitances of the corresponding junctions $C_1=C=2.7$ fF and at the inductance $L=0.3$ nH of the qubit loop. In both plots the curves calculated numerically are shown by hollow circles, while the ones obtained analytically using the instanton technique are plotted by solid lines. The formulas (9) were used for the 2JJ qubit, while similar formulas were taken for ScS and SIS qubits, based on the forms of their potentials. A change of the parameter β_L means a variation of the critical currents I_{c1} and I_c of the corresponding junctions at a fixed inductance L. The double-well potential height decreases with decreasing parameter β_L, the energy level E_1 being equalized to the potential barrier height U_0 at a certain β_{L0} ($E_1=U_0$) and exceeding it with further lowering of β_L. Then the wave function corresponding to the level E_1 is no longer a superposition of states localized in the left and right wells. The boundary values β_{L0} for the curves in the figure are indicated by dashed lines. In the vicinity of β_{L0}, at $(U_0-E_1)\sim k_BT$, the quantum coherence will be destroyed due to thermal fluctuations causing over-barrier transitions. One can see from Fig. 4 that the numerically and the analytically obtained curves almost coincide at large β_L and begin to diverge at lower β_L. This is because of the condition of semiclassicity $S_0/\hbar\gg 1$ starts to fail with diminishing β_L. This, in its turn, is caused by decreasing of the barrier height U_0 and therefore the action S_0. Analysis of the $S_0(\beta_L)/\hbar$ curves reveals that $S_0/\hbar\sim 1$ at $\beta\sim\beta_{L0}$, and the relative divergence between the numerical and analytical results for the ScS and 2JJ qubits is within 2 to 10 percent. For a SIS qubit a fit of the numerical and analytical results requires more stringent fulfillment of the semiclassicity condition. However, it follows even from this analysis that obtaining a tunnel splitting $\Delta E_{01}\gtrsim 1$ K in the flux qubit based on a single SIS junction is impossible under the condition of weak $(U_0-E_1\gg k_BT)$ influence of thermal fluctuations on the decay of the metastable states. The $S_0(\beta_{L0}/\hbar)$ curves are close to linear for the 2JJ, ScS, and SIS qubits, with a slope (the rate of increase of the action S_0 with β_L) that increases in the order listed. The value of the tunnel splitting in the region of its exponential smallness $S_0(\beta_L)/\hbar\gg 1$ diminishes in the same sequence.

The points on the numerical curves corresponding to equal heights of the potential barriers ($U_0=9.64$ K) are indicated by arrows in Fig. 4a. The corresponding values of the parameter pairs $(\beta_L, E_{01}(\beta_L)/k_B)$ for the 2JJ ($\lambda=0.9$), ScS, and SIS flux qubits are: (1.06, 3.45 K), (0.88, 1.79 K), (1.60, 0.16 K). It is seen that, under this condition, the tunnel splitting in the 2JJ qubit is about twice the splitting in the ScS qubit and more than 20 times higher than that of the SIS qubit. The curve of the tunnel splitting for the 2JJ qubit lies completely above the curves for the ScS and SIS qubits, and the tunnel splitting for the 2JJ qubit reaches a value of 3.45 K at $\beta_L\simeq 1>\beta_{L0}$. The advantages of a ScS qubit if compared to a SIS qubit were thoroughly analyzed in Ref. 11. Note that the still greater increase of the tunnel splitting in a 2JJ qubit in comparison with a ScS qubit with the matched parameters mentioned above results from the fact that their potentials (at $\lambda\simeq 1$) practically coincide, while the

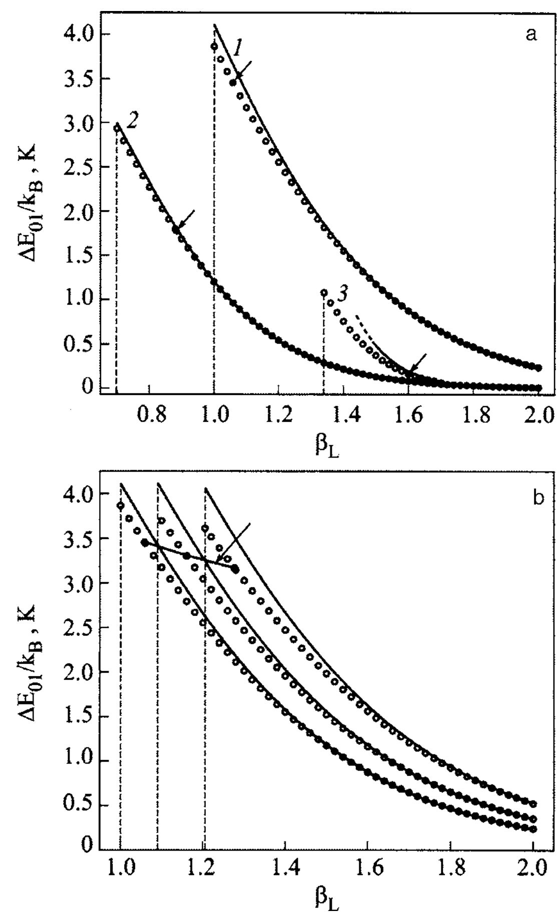

FIG. 4. The function $\Delta E_{01}(\beta_L)/k_B$ for the 2JJ qubit at $\lambda=0.9$ (*1*), ScS qubit (*2*), and 1JJ qubit (*3*); the points on the numerical curves corresponding to equal height (9.64 K) of the potential barrier for all the qubits are indicated by arrows (a); The function $\Delta E_{01}(\beta_L)/k_B$ for the 2JJ qubit and various λ: 0.9 (*1*), 0.85 (*2*), 0.8 (*3*) and the "level line" of equal heights (9.64 K) of the potential barriers at varying λ (*4*) (b). The numerically obtained results are represented by hollow circles, and the analytical results are plotted by solid lines in (a) and (b). The dashed lines show the lowest boundary β_L at which the level height E_1 becomes equal to the potential barrier height U_0. For 1JJ and ScS qubits, the capacitance of the corresponding (SIS and ScS) junctions is $C=2.7\times 10^{-15}$ F, while for the 2JJ qubit the capacitance of the larger SIS junction is $C_1=2.7\times 10^{-15}$ F. The geometric inductance of the ring is $L=30\times 10^{-10}$ H, and the parameter $g\approx 76\beta_L$ for all the qubits. For the 2JJ flux qubit the parameter $g_0^{\min}\approx 76\beta_L(1-\lambda^2)\gtrsim 10$ at $\lambda\lesssim 0.93$.

effective mass M in the 2JJ qubit is less by a factor of about two. Figure 4b shows the dependence $\Delta E_{01}(\beta_L)/k_B$ for the 2JJ qubit at several λ, and also a "level line," the line $\Delta E_{01}(\beta_L)/k_B$ corresponding to equal height (9.64 K) of the potential barriers in the 2JJ qubit with varying λ. The curve $\Delta E_{01}(\beta_L)/k_B$ shifts rightward with decreasing λ, and the smaller the value of λ, the higher the tunnel splitting at a fixed β_L. This, however, is due to the lowering of the barrier height U_0 with decreasing λ, which leads to an exponential rise of the thermal decay rate. Note that when the junctions are desymmetrized, the fit between the numerical and analytical curves gets worse because $S_0(\beta_L)/\hbar$ decreases. As is seen from the plot, the value of the tunnel splitting gradually diminishes while moving along the level line with equal height of the potential barriers towards lower values of the junction symmetry parameter λ (and higher β_L).

III. CONCLUSIONS

It should be emphasized that the principal requirements on 2JJ flux qubits, namely: $\lambda \simeq 0.9$; $C \lesssim 50$ fF/μm^2; $j_c \sim 10^3$ A/cm^2; $I_c \sim 1$ μA at the JJ area $S_J \sim 0.1$ μm^2; $L \sim 0.3$ nH, $\beta_L \sim 1$ can be met with the present-day technology based on the materials Nb, NbN, and MoRe with superconductivity gap $\Delta(0) \simeq 10$ K (see, e.g., Ref. 22). We note in conclusion that a 2JJ flux qubit with large amplitude of tunnel splitting potentially has some strong advantages: (i) weak sensitivity to the motion of charge in traps; (ii) extremely fast excitation (pumping frequency) in qubit-based readout as well as in computer circuits due to considerable increase of the quantum tunneling rate $\nu \sim \Delta E_{01}$; (iii) macroscopically large energy relaxation times τ_ε (see, e.g., Ref. 10 and references cited therein); (iv) further improvement of qubit coherence characteristics.[16]

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