Consistency of Ground State and Spectroscopic Measurements on Flux Qubits

A. Izmalkov,[1,*] S. H. W. van der Ploeg,[1] S. N. Shevchenko,[1,2] M. Grajcar,[1,3] E. Il'ichev,[1,†] U. Hübner,[1] A. N. Omelyanchouk,[2] and H.-G. Meyer[1]

[1]Institute of Photonic Technology, P.O. Box 100239, D-07702 Jena, Germany
[2]B. Verkin Institute for Low Temperature Physics and Engineering, 47 Lenin Avenue, 61103, Kharkov, Ukraine
[3]Department of Solid State Physics, Comenius University, SK-84248 Bratislava, Slovakia

(Received 28 March 2008; published 2 July 2008)

We compare the results of ground state and spectroscopic measurements carried out on superconducting flux qubits which are effective two-level quantum systems. For a single qubit and for two coupled qubits we show excellent agreement between the parameters of the pseudospin Hamiltonian found using both methods. We argue that by making use of the ground state measurements the Hamiltonian of N coupled flux qubits can be reconstructed as well at temperatures smaller than the energy level separation. Such a reconstruction of a many-qubit Hamiltonian can be useful for future quantum information processing devices.

DOI: 10.1103/PhysRevLett.101.017003 PACS numbers: 85.25.Cp, 03.67.Lx, 84.37.+q, 85.25.Dq

Quantum systems are generally characterized by spectroscopic measurements; the system is excited by electromagnetic radiation, with a frequency which matches the level spacing, and the response of this excitation is detected. On the other hand, quantum theory predicts that the Hamiltonian of some quantum-mechanical systems can be completely reconstructed from their ground-state properties. For instance, quantum-mechanical treatment of the ammonia molecule in a two-level approximation shows that its ground state contains information about time-independent Hamiltonian parameters [1]. Superconducting qubits are also described by a similar Hamiltonian [2]. They are micrometer-size quantum systems [3] which can be easily accessed by a macroscopic measuring device. For example, the Hamiltonian parameters of a superconducting flux qubit [4] can be determined from the measurement of its magnetic susceptibility in the ground state [5]. In this Letter we will demonstrate that for a single and two coupled flux qubits the ground-state and the spectroscopic measurements give the same results.

The persistent current, or flux, qubit is a small superconducting loop with three submicron Josephson junctions [4]. Because of the flux quantization only two phases are independent. Thus, the circuit is characterized by a two-dimensional potential $U(\phi_1, \phi_2)$ which, for suitable qubit parameters, exhibits two minima. In the classical case these minima correspond to clockwise and anticlockwise supercurrents in the loop. If the applied magnetic flux equals half a flux quantum, $\Phi_x = \Phi_0/2$ ($\Phi_0 = h/2e$), both minima have the same potential, leading to a degenerate ground state. According to the quantum mechanics the degeneracy is lifted close to this point and the flux qubit can be described by the Hamiltonian [2]:

$$H(t) = -\Delta\sigma_x - \varepsilon\sigma_z + A\cos(\omega t)\sigma_z, \tag{1}$$

where σ_x, σ_z are the Pauli matrices for the spin basis and Δ is the tunneling amplitude. The qubit bias is given by $\varepsilon = I_p(\Phi_x - \Phi_0/2)$, where I_p is the magnitude of the qubit persistent current. The last term describes the microwave irradiation necessary for the spectroscopy which aims to probe the stationary energy levels represented by the first two, time-independent, terms of this Hamiltonian. The eigenvalues of the Hamiltonian (1) depend on the flux bias $\Phi = \Phi_x - \Phi_0/2$:

$$E_\pm(\Phi) = \pm\sqrt{(I_p\Phi)^2 + \Delta^2}. \tag{2}$$

Spectroscopic measurements detect the excitation of a qubit close to the point where the microwave irradiation matches the level spacing: $\hbar\omega = \Delta E(\Phi) \equiv E_+(\Phi) - E_-(\Phi)$. By measuring ΔE as a function of Φ the qubit parameters Δ and I_p can be obtained as has been shown by van der Wal *et al.* [6].

Alternatively, the same information can be obtained from ground-state measurements. Indeed, let us consider a flux qubit weakly coupled to a classical oscillator consisting of an inductor L_T and a capacitor C_T forming a tank circuit [7]. Because of the mutual inductance M the tank biases the qubit resulting in $\Phi = \Phi_{\rm dc} + \Phi_{\rm rf}$. Provided that the resonant frequency of the tank is small, $\omega_T = 1/\sqrt{L_T C_T} \ll \Delta/\hbar$, and the temperature is low enough, $k_B T \ll 2\Delta$ (k_B is Boltzmann's constant), the qubit will reside in its ground state E_-. The dynamic behavior of the tank-qubit arrangement can be described by the Lagrangian

$$\mathcal{L} = \mathcal{T} - \mathcal{U} = \frac{1}{2}L_T\dot{q}^2 + E_-(\Phi_{\rm dc} + M\dot{q}) - \frac{1}{2}\frac{q^2}{C_T}, \tag{3}$$

where q is the charge on the tank capacitor and $\dot{q}$ is the circulating current in the tank. Such a Lagrangian would lead to the nonlinear equation of motion:

$$0 = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = \left(L_T + M^2 \frac{\partial^2 E_-(\Phi_{\rm dc} + M\dot{q})}{\partial \Phi^2}\right)\ddot{q} + \frac{q}{C_T}; \tag{4}$$

however, for small amplitude of $\dot{q}$ the Lagrangian can be linearized [8] by replacing $E_-(\dot{q})$ by its second order Taylor expansion around $\Phi_{\rm dc}$. Consequently, we obtain the simple Lagrangian of a particle in a parabolic potential well:

$$\mathcal{L} = \frac{1}{2}m^*\dot{q}^2 - \frac{1}{2}k^*q^2. \tag{5}$$

The equation of motion which can be obtained from this Lagrangian is just the simple equation for a particle in a parabolic potential, $m^*\ddot{q} = k^*q$, where

$$m^* = \left(L_T + M^2 \frac{d^2E_-(\Phi_{\rm dc})}{d\Phi_{\rm dc}^2}\right) \tag{6}$$

is the effective mass and $k^* = 1/C_T$ is the curvature of the parabolic potential well. Thus, the resonant frequency of the tank-qubit arrangement

$$\omega_0 = \sqrt{\frac{k^*}{m^*}} \approx \omega_T\left(1 - \frac{M^2}{2L_T}\frac{d^2E_-(\Phi_{\rm dc})}{d\Phi_{\rm dc}^2}\right), \tag{7}$$

contains information on the curvature of the ground state of the qubit. Differentiating Eq. (2) results in

$$\frac{d^2E_-(\Phi_{\rm dc})}{d\Phi_{\rm dc}^2} = -\frac{(I_p\Delta)^2}{(\varepsilon^2(\Phi_{\rm dc}) + \Delta^2)^{3/2}}, \tag{8}$$

showing that Δ and I_p can be determined from the dependence of the resonance frequency of the tank circuit on the applied flux.

In order to compare both methods we fabricated a two-qubit sample like the one shown in Fig. 1. As either one of the qubits can be biased far away from degeneracy, the single qubit properties can be studied as well. This can be understood if we consider the Hamiltonian of two coupled flux qubits:

$$H_{2qbs} = -\Delta_a\sigma_x^{(a)} - \Delta_b\sigma_x^{(b)} - \varepsilon_a\sigma_z^{(a)} - \varepsilon_b\sigma_z^{(b)} + J\sigma_z^{(a)}\sigma_z^{(b)}, \tag{9}$$

where J is the Josephson coupling energy provided by the large connecting Josephson junction. Suppose qubit a is the one biased far from its degeneracy point in such a way that ε_a is large in comparison with the other energy variables. Then, qubit a has a well-defined ground state with averaged spin variables $\langle\sigma_z^{(a)}\rangle = 1$ and $\langle\sigma_x^{(a)}\rangle = 0$, which can be averaged out of the two-qubit Hamiltonian (9) reducing it to $H_{2qbs,\rm red} = -\Delta_b\sigma_x^{(b)} - (\varepsilon_b - J)\sigma_z^{(b)}$. Apart from the offset in the bias term due to the coupling this is identical to the single qubit Hamiltonian (1). This offset can be easily compensated and measured allowing the determination of the coupling energy J [9]. The qubit parameters, Δ_b and $I_p^{(b)}$, are determined from the ground state measurement, as it is described above. Analogously, biasing qubit b far from the degeneracy point the parameters for qubit a, Δ_a, and $I_p^{(a)}$ can be determined. In a similar way the parameters of a N-qubit Hamiltonian can be completely reconstructed from the ground state measurements as has already been demonstrated, for instance, for four qubits circuits [10].

The qubit parameters can be probed by either the ground state (adiabatic) measurements or by making use of spectroscopy. However, it naturally raises the question of whether the ground state and the spectroscopy measurements are consistent? While addressing this problem in this Letter we study both approaches *in situ* for one and two coupled flux qubits.

Experimentally, the shift of the resonance frequency can be obtained by driving the tank circuit with a rf current $I_{\rm rf}$ at a frequency close to the resonant frequency ω_T and measure the phase shift Θ between the rf voltage and driving current. For a small qubit inductance L, the phase

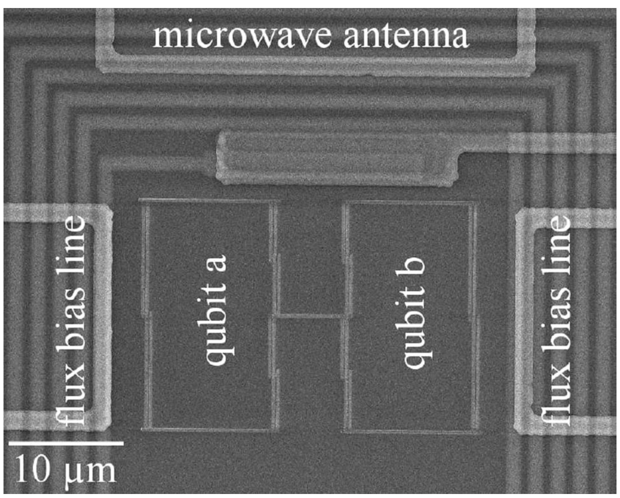

FIG. 1. Scanning electron micrograph showing two coupled aluminum flux qubits, placed inside a niobium tank coil and dc and microwave lines. The qubits are fabricated on an oxidized silicon substrate by making use of electron-beam lithography and shadow evaporation. The loops share a large Josephson junction visible in the center. This junction provides a coupling between the qubits with a coupling energy $J/h = 1.9$ GHz. The qubit loops are interrupted by three junctions each. Two of them, located in the inner sides, have nominal areas of 200×700 nm while the outer junctions are 70% smaller. The qubit system is placed inside a 30 turn superconducting niobium pancake coil which forms a resonant tank circuit, with a resonance frequency of 20.8 MHz, with a capacitor mounted on the sample holder. The resonator typically has a quality $Q = 300$. Individual control of the dc-flux bias of the individual qubits (Φ_a, Φ_b), is provided by the niobium bias lines visible on top of the coil windings. The antenna used for supplying the MW excitation is visible at the top of the picture.

shift Θ is defined by [5]

$$\tan\Theta = \frac{M^2 Q}{L_T} \frac{d^2 E_-}{d\Phi_{dc}^2}. \quad (10)$$

The mutual inductance M, tank inductance L_T and quality factor Q can be measured independently giving a value of 23.4 pH for this prefactor. The results of such measurements are shown in Fig. 2(a). Note that the sample was thermally anchored to the mixing chamber of a dilution refrigerator at a temperature $T_{mix} \approx 10$ mK. The effective temperature of the sample T is higher and we estimated from the best theoretical fits that $T \approx 70$ mK [11].

It is important to note that thermal excitations can modify the measured signal, which would result in erroneous qubit parameters. In practice, thermal excitations are not negligible when $k_B T \gtrsim 2\Delta$. Nevertheless, if $k_B T < 2\Delta$ the dispersive measurement provides a correct value of qubit parameters [11]. This statement is also confirmed by a good agreement between both ground state and spectroscopic measurements [see Fig. 2(c)].

In fact the tank circuit can be used as detector for the spectroscopy measurements as well, since the variation in the population of the qubits' energy levels results in the change of the effective impedance of the tank circuit [12]. The tank circuit is insensitive to the microwave signal itself since $\omega_T \ll \Delta/\hbar$ and $Q \gg 1$. However, if the microwave frequency is close to the qubit level separation, the system damps or amplifies the voltage on the tank, mimicking the Sisyphus mechanism of damping (and heating) of the tank known from quantum optics [13]. This effect generates the peak-dip structure in the $V_T(\Phi_{dc})$ dependence around the resonance [see Fig. 2(b)] [12,14]. The position of the resonances is the point where a peak changes to a dip. From the positions of the mid points of the peak-dip structures one can determine the energy gap ΔE between the energy levels. The obtained agreement between the adiabatic and spectroscopic measurement for weak driving regime is excellent [see Fig. 2(c)].

With increasing the microwave power, the Landau-Zener interference pattern of the qubit is clearly visible. The qubit's response in the strong driving regime is demonstrated in Fig. 3 where the tank voltage phase shift is presented as a function of the microwave amplitude and the dc-flux bias. The position of the multiphoton resonances is approximately given by the relation $\Delta E(\Phi_{dc}) \approx n\hbar\omega$, where the energy gap ΔE is calculated using the parameters I_p and Δ obtained from the ground state measurement.

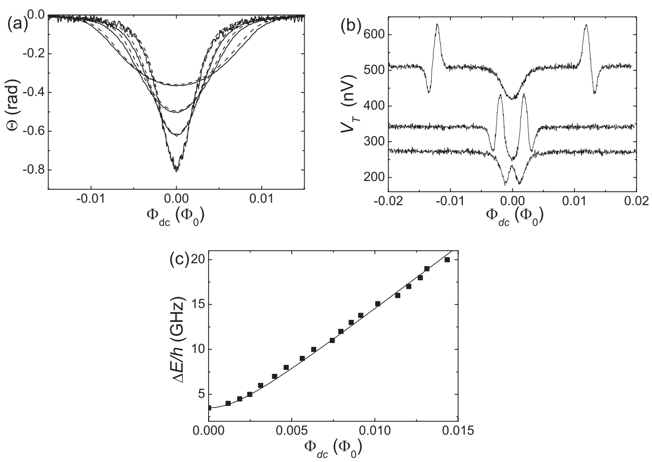

FIG. 2. Comparison of the ground state and spectroscopic measurements for qubit b. (a) Ground state measurements. Presented is the dependence of the phase shift between the tank circuit voltage and bias current on the flux bias. The solid lines are experimental data fitted by the theoretical curves (dashed curves) for qubit parameters $I_p = 225$ nA and $\Delta/h = 1.75$ GHz. The curves correspond to various values of the rf-bias current on the tank circuit resulting in rf voltage amplitudes, from top to bottom, $V_T = 4.3$, 2.9, 0.5, and 0.3 μV. (b) Amplitude of the tank voltage as a function of the normalized magnetic flux in the qubit at the driving frequencies, from top to bottom, $\omega/2\pi = 18$, 5, and 3.5 GHz. The curves have been shifted for clarity. The resonant excitation in the flux qubit results in the peak-and-dip at the positions defined by the condition $\Delta E(\Phi_{dc}) = \hbar\omega$. (c) Energy gap ΔE between the qubit energy levels determined from the positions of the mid points of the peak-and-dip structures (solid squares). The solid line is the theoretical curve calculated from Eq. (2) using the parameters $I_p = 225$ nA and $\Delta/h = 1.75$ GHz obtained from the ground state measurements. The effective temperature $T \approx 70$ mK $\approx$ 1.4 GHzh/k_B is smaller than the minimal energy level separation 2Δ.

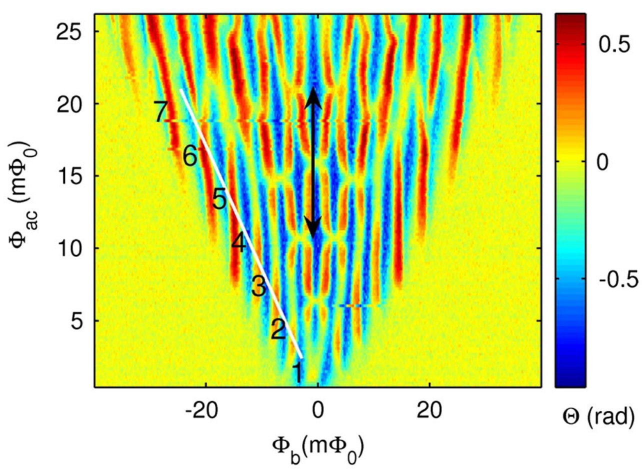

FIG. 3 (color online). Landau-Zener interferometry for qubit b. Dependence of the tank voltage phase shift Θ on the dc-flux bias Φ_{dc} and the ac flux amplitude Φ_{ac} (the microwave amplitude). The spots along the Φ_{dc} axis correspond to the multiphoton resonances at the positions defined by the relation $\Delta E(\Phi_{dc}) \approx n\hbar\omega$; the numbers from 1 to 7 show the position of the n-photon resonances. The changes along the Φ_{ac} axis are due to Stückelberg oscillations in the qubit. The calibration of the driving power of the ac flux can be done either with the distance between these oscillations (black arrow) or from the slope of the interference fringes (white line).

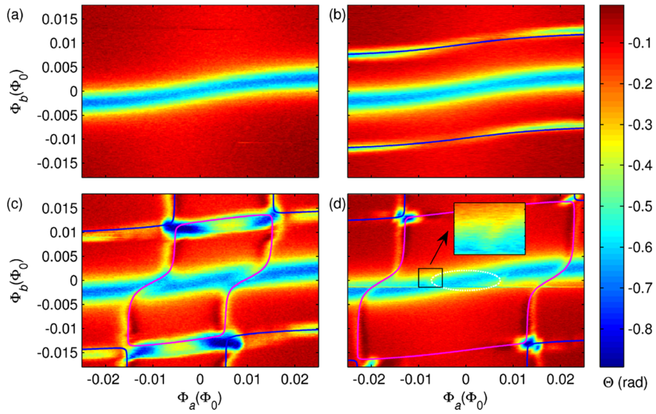

FIG. 4 (color online). Spectroscopy of the system of two coupled flux qubits. The dependence of the tank voltage phase shift Θ on the flux biases in qubits a and b is presented for measurements without microwave excitation in (a) and for microwave excitation with $\omega/2\pi = 14.125$, 17.625 and 20.75 GHz in (b) till (d), respectively. Inset shows the transition to the third excited level. The blue, magenta and white-dotted lines in the pictures with microwave excitation show the expected positions of the resonant excitations of the qubits to the first, second and third excited levels, respectively, calculated from the energy eigenvalues of Hamiltonian (10) with parameters: $\Delta_a/h = 7.9$ GHz, $\Delta_b/h = 1.75$ GHz, $I_p^{(a)} = 120$ nA, $I_p^{(b)} = 225$ nA, and $J/h = 1.9$ GHz. The trough around $\Phi_b = 0$ is due to the ground state curvature of qubit a and corresponds to the ground state measurements of Fig. 2. The shallow trough around $\Phi_a = 0$ visible in figure (a), is due to qubit a.

Moreover, the Landau-Zener interferometry allows the calibration of the microwave power to the ac flux due to the periodicity of the Stückelberg oscillations on the parameter $4I_p\Phi_{ac}/\hbar\omega$ with the period 2π [15,16]. It follows that the distance between the resonances (shown by the black arrow in Fig. 3) is approximately equal to $\delta\Phi_{ac} = \frac{1}{2}\pi\hbar\omega/I_p$. Alternatively, the calibration can be made using the slope of the interference fringes (white line in Fig. 3) [17,18].

After determining the single qubit parameters far away from the degeneracy points, we investigated the two-qubit behavior. Firstly, the coupling energy J was determined from the offset of the qubit dips from the $\Phi_{a/b} = 0$ lines, visible in the pure ground state measurements presented in Fig. 4(a). Then the qubits were driven by various ac magnetic fluxes $\Phi_{ac}\sin\omega t$. In Fig. 4(b) a frequency inbetween both qubit gaps was used and therefore only the transitions to the first excited state are visible. For higher frequencies, also the second and third excited states become visible as can be seen in subfigures (c) and (d). Here also both types of the measurements (ground state and spectroscopic) result in the same set of parameters for the system. Finally, we would like to note that the theoretical calculations allow us to plot analogous to Figs. 3 and 4 graphs (to be published elsewhere [11]).

In conclusion, the equivalence of the ground state and spectroscopic approaches for the measurement of the qubit system parameters was demonstrated. We have probed the one- and two-flux qubit systems by using a dispersive measurement technique. It was shown that the ground state measurement gives the same qubit parameters as the spectroscopy in the weak (Figs. 2 and 4) as well as in the strong driving regime (Fig. 3).

This work was initiated by A. Izmalkov who untimely passed away when this work was finished. We thank Yakov Greenberg for useful discussions and gratefully acknowledge the financial support of the EU through the RSFQubit and EuroSQIP projects. M. G. was supported by Grants No. APVV-0432-07 and No. VEGA 1/0096/08. S. N. S. acknowledges the financial support of INTAS.

*Deceased.
†evgeni.ilichev@ipht-jena.de

[1] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics. Quantum Mechanics* (California Institute of Technology, Addison-Wesley, Reading, Massachusetts, 1965).
[2] Yu. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[3] J. Q. You and F. Nori, Phys. Today **58**, No. 11, 42 (2005).
[4] J. E. Mooij *et al.*, Science **285**, 1036 (1999).
[5] Ya. S. Greenberg *et al.*, Phys. Rev. B **66**, 214525 (2002).
[6] C. H. van der Wal *et al.*, Science **290**, 773 (2000).
[7] This arrangement has been already described phenomenologically—oscillator perturbed by two-level system [5]. Here we present a more general approach to this problem.
[8] For finite I_{rf} the response of the tank circuit can be calculated numerically [5].
[9] M. Grajcar *et al.*, Phys. Rev. B **72**, 020503 (2005).
[10] M. Grajcar *et al.*, Phys. Rev. Lett. **96**, 047006 (2006).
[11] S. N. Shevchenko, S. H. W. van der Ploeg, M. Grajcar, E. Il'ichev, A. N. Omelyanchouk, and H.-G. Meyer (to be published).
[12] M. Grajcar *et al.*, arXiv:0708.0665.
[13] D. Wineland, J. Dalibard, and C. Cohen-Tannouji, J. Opt. Soc. Am. B **9**, 3242 (1992).
[14] S. N. Shevchenko, Eur. Phys. J. B **61**, 187 (2008).
[15] A. V. Shytov, D. A. Ivanov, and M. V. Feigel'man, Eur. Phys. J. B **36**, 263 (2003).
[16] S. N. Shevchenko and A. N. Omelyanchouk, Low Temp. Phys. **32**, 973 (2006).
[17] W. D. Oliver *et al.*, Science **310**, 1653 (2005).
[18] M. Sillanpää, T. Lehtinen, A. Paila, Yu. Makhlin, and P. Hakonen, Phys. Rev. Lett. **96**, 187002 (2006).