SUPERCONDUCTIVITY, INCLUDING HIGH-TEMPERATURE SUPERCONDUCTIVITY

Coherent current states in a two-band superconductor

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Homogeneous current states in thin films and Josephson current in superconducting microbridges are studied within the framework of a two-band Ginzburg–Landau theory. By solving the coupled system of equations for two order parameters the depairing current curves and Josephson current–phase relation are calculated for different values of phenomenological parameters γ and η. Coefficients γ and η describe the coupling of order parameters (proximity effect) and their gradients (drag effect), respectively. For definite parameter values the dependence of the current j on the superfluid momentum q contains local minima and corresponding bistable states. It is shown that a Josephson microbridge made from two-band superconductors can demonstrate π-junction behavior. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2737547]

I. INTRODUCTION

To the present day the overwhelming majority of works on the theory of superconductivity have been devoted to single-gap superconductors. More than 40 years ago the possibility of superconductors with two superconducting order parameters was considered by Moskalenko[1] and by Suhl, Matthias, and Walker[2] in the model of a superconductor with overlapping energy bands on the Fermi surface. Moskalenko has investigated theoretically the thermodynamic and electromagnetic properties of two-band superconductors. The real boom in investigation of multi-gap superconductivity started after the discovery[3] of two gaps in MgB_2 by the methods of scanning tunneling[4,5] and point-contact spectroscopy.[6–8] The compound MgB_2 has the highest critical temperature, $T_c=39$ K, among superconductors with a phonon mechanism of pairing, and two energy gaps $\Delta_1 \approx 7$ meV and $\Delta_2 \approx 2.5$ meV at $T=0$. At this time two-band superconductivity is also studied in other systems, e.g., in heavy-fermion compounds,[9,10] borocarbides,[11] and liquid metallic hydrogen.[12–14] Various thermodynamic and transport properties of MgB_2 have been studied in the framework of the two-band BCS model.[15–22]The Ginzburg–Landau (GL) functional for two-gap superconductors has been derived within the weak-coupling BCS theory in dirty[23] and clean[24] superconductors. The magnetic properties[25–27] and peculiar vortices[28–30] have been studied within the Ginzburg–Landau scheme.

The aim of this article is to present a Ginzburg–Landau theory of the current-carrying states in superconductors with two order parameters. In the case of several order parameters the qualitatively new features in the superconducting current state are related to mutual influence of the moduli of complex order parameters as well of the gradients of their phases. We study the manifestations of these effects in the current–momentum dependence and in the Josephson current–phase relation. In Sec. II a general phenomenological description of two-band superconductors is given within the Ginzburg–Landau theory. The Ginzburg–Landau equations for two coupled superconducting order parameters include the proximity and drag effects. In Sec. III the peculiarities of homogeneous current states in multi-gap superconductors are studied. The dependence of current on superfluid momentum is calculated for different values of the parameters. We demonstrate that for definite values of parameters it contains local minima and corresponding bistable states in the GL free energy. In Sec. IV the Josephson effect is considered in a simple model of a superconducting weak link (a generalization of the Aslamazov–Larkin theory[31] to a two-band superconductor), and the possibility of π-junction behavior is demonstrated.

II. GINZBURG—LANDAU EQUATIONS FOR TWO-BAND SUPERCONDUCTIVITY

The phenomenological Ginzburg–Landau free energy density functional for two coupled superconducting order parameters ψ_1 and ψ_2 can be written as

$$F_{GL}=F_1+F_2+F_{12}+\frac{(\mathrm{curl}\,\mathbf{A})^2}{8\pi},$$

where

$$F_1=\alpha_1|\psi_1|^2+\frac{1}{2}\beta_1|\psi_1|^4+\frac{1}{2m_1}\left|\left(-i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right)\psi_1\right|^2, \tag{1}$$

$$F_2=\alpha_2|\psi_2|^2+\frac{1}{2}\beta_2|\psi_2|^4+\frac{1}{2m_2}\left|\left(-i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right)\psi_2\right|^2, \tag{2}$$

and

$$F_{12}=-\gamma(\psi_1^*\psi_2+\psi_1\psi_2^*)+\eta\left[\left(-i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right)\psi_1\times\left(i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right)+\left(i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right)\psi_1^*\left(-i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right)\psi_2\right]. \tag{3}$$

The terms F_1 and F_2 are conventional contributions from ψ_1 and ψ_2, and the term F_{12} describes, without loss of generality, the interband coupling of the order parameters. The coefficients γ and η describe the coupling of two order parameters (proximity effect) and their gradients (drag effect),[25–27] respectively.

By minimization of the free energy $F=\int\left(F_1+F_2+F_{12}+\frac{H^2}{8\pi}\right)d^3r$ with respect to ψ_1, ψ_2, and $\mathbf{A}$ we obtain the differential GL equations for a two-band superconductor:

$$\begin{cases}\dfrac{1}{2m_1}\left(-i\hbar\nabla-\dfrac{2e}{c}\mathbf{A}\right)^2\psi_1+\alpha_1\psi_1+\beta_1|\psi_1|^2\psi_1-\gamma\psi_2+\eta\left(-i\hbar\nabla-\dfrac{2e}{c}\mathbf{A}\right)^2\psi_2=0,\\ \dfrac{1}{2m_2}\left(-i\hbar\nabla-\dfrac{2e}{c}\mathbf{A}\right)^2\psi_2+\alpha_2\psi_2+\beta_2|\psi_2|^2\psi_2-\gamma\psi_1+\eta\left(-i\hbar\nabla-\dfrac{2e}{c}\mathbf{A}\right)^2\psi_1=0,\end{cases} \tag{4}$$

and an expression for the supercurrent:

$$\mathbf{j}=-\frac{ie\hbar}{m_1}(\psi_1^*\nabla\psi_1-\psi_1\nabla\psi_1^*)-\frac{ie\hbar}{m_2}(\psi_2^*\nabla\psi_2-\psi_2\nabla\psi_2^*)-2ie\hbar\eta(\psi_1^*\nabla\psi_2-\psi_2\nabla\psi_1^*-\psi_1\nabla\psi_2^*+\psi_2^*\nabla\psi_1)-\left(\frac{4e^2}{m_1c}|\psi_1|^2+\frac{4e^2}{m_2c}|\psi_2|^2+\frac{8\eta e^2}{c}(\psi_1^*\psi_2+\psi_2^*\psi_1)\right)\mathbf{A}. \tag{5}$$

In the absence of currents and gradients of the order parameter moduli, the equilibrium values of order parameters $\psi_{1,2}=\psi_{1,2}^{(0)}e^{i\chi_{1,2}}$ are determined by the set of coupled equations

$$\alpha_1\psi_1^{(0)}+\beta_1\psi_1^{(0)3}-\gamma\psi_2^{(0)}e^{i(\chi_2-\chi_1)}=0,$$

$$\alpha_2\psi_2^{(0)}+\beta_2\psi_2^{(0)3}-\gamma\psi_1^{(0)}e^{i(\chi_1-\chi_2)}=0. \tag{6}$$

For the case of two order parameters the question arises as to the phase difference $\varphi=\chi_1-\chi_2$ between ψ_1 and ψ_2. In a homogeneous no-current state, by analyzing the free energy term F_{12} (3) one can show that in the case $\gamma>0$ the phase shift $\varphi=0$, while for $\gamma<0$ one has $\varphi=\pi$. The statement that φ can have only values 0 or π also applies in the current-carrying state, but for a coefficient $\eta\neq 0$ the criterion for whether ε equals 0 or π now depends on the value of the current (see below).

If the interband interaction is ignored, Eqs. (4) are decoupled into two ordinary GL equations with two different critical temperatures, T_{c1} and T_{c2}. In general, independently of the sign of γ, the superconducting phase transition occurs at a well-defined temperature exceeding both T_{c1} and T_{c2}, which is determined from the equation

$$\alpha_1(T_c)\alpha_2(T_c)=\gamma^2. \tag{7}$$

Let the first order parameter be stronger than the second, i.e., $T_{c1}>T_{c2}$. Following Ref. 24 we represent the temperature-dependent coefficients as

$$\alpha_1(T)=-a_1(1-T/T_{c1}),$$

$$\alpha_2(T)=a_{20}-a_2(1-T/T_{c1}). \tag{8}$$

The phenomenological constants $a_{1,2}$, a_{20}, and $\beta_{1,2}$ can be related to the microscopic parameters in the two-band BCS model. From (7) and (8) we obtain for the critical temperature T_c:

$$T_c=T_{c1}\left(1+\sqrt{\left(\frac{a_{20}}{2a_2}\right)^2+\frac{\gamma^2}{a_1a_2}}-\frac{a_{20}}{2a_2}\right). \tag{9}$$

For arbitrary values of the interband coupling, Eq. (6) can be solved numerically. For $\gamma=0$, $T_c=T_{c1}$, and for temperature close to T_c (hence for $T_{c2}<T\leq T_c$), the equilibrium values of the order parameters are $\psi_2^{(0)}(T)=0$ and $\psi_1^{(0)}(T)=\sqrt{a_1(1-T/T_c)/\beta_1}$. Henceforth assuming weak interband coupling, we have from Eqs. (6)–(9) corrections $\sim\gamma^2$ to these values:

$$\psi_1^{(0)}(T)^2=\frac{a_1}{\beta_1}\left(1-\frac{T}{T_c}\right)+\frac{\gamma^2}{\beta_1}\left(\frac{1}{a_{20}-a_2\left(1-\frac{T}{T_c}\right)}-\frac{T}{T_c}\frac{1}{a_{20}}\right),$$

$$\psi_2^{(0)}(T)^2=\frac{a_1}{\beta_1}\left(1-\frac{T}{T_c}\right)\frac{\gamma^2}{\left(a_{20}-a_2\left(1-\frac{T}{T_c}\right)\right)^2}. \tag{10}$$

Expanding Eq. (9) over $\left(1-\frac{T}{T_c}\right)\ll 1$, we have conventional temperature dependence of the equilibrium order parameters in the weak interband coupling limit:

$$\Psi_1^{(0)}(T) \approx \sqrt{\frac{a_1}{\beta_1}}\left(1+\frac{1}{2}\frac{a_{20}+a_2}{a_{20}^2 a_1}\gamma^2\right)\sqrt{1-\frac{T}{T_c}},$$

$$\psi_2^{(0)}(T) \approx \sqrt{\frac{a_1}{\beta_1}}\frac{\gamma}{a_{20}}\sqrt{1-\frac{T}{T_c}}. \tag{11}$$

The case considered above [Eqs. (9)–(11)] corresponds to different critical temperatures $T_{c1} > T_{c2}$ in the absence of interband coupling γ. The order parameter in the second band $\psi_2^{(0)}$ arises from the "proximity effect" of stronger $\psi_1^{(0)}$ and is proportional to the value of γ [Eq. (11)]. Now consider another situation. Suppose for simplicity that the two condensates are identical in the zero-current state. In this case for arbitrary values of γ we have

$$\alpha_1(T)=\alpha_2(T)\equiv\alpha(T)=-a\left(1-\frac{T}{T_c}\right), \beta_1=\beta_2\equiv\beta, \tag{12}$$

$$\psi_1^{(0)}=\psi_2^{(0)}=\sqrt{\frac{|\gamma|-\alpha}{\beta}}. \tag{13}$$

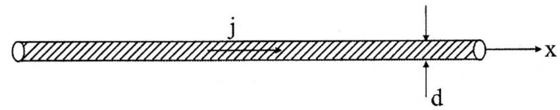

FIG. 1. Geometry of the system.

III. HOMOGENEOUS CURRENT STATES AND GINZBURG–LANDAU DEPAIRING CURRENT

In this Section we will consider the homogeneous current states in thin wires or films with transverse dimension $d \ll \xi_{1,2}(T), \lambda_{1,2}(T)$ (see Fig. 1), where $\xi_{1,2}(T)$ and $\lambda_{1,2}(T)$ are coherence lengths and London penetration depths for each order parameter, respectively, without interband interaction. This condition leads to a one-dimensional problem and permits us to neglect the self-magnetic field of the system.

The current density j and the moduli of the order parameters do not depend on the longitudinal coordinate x. Writing $\psi_{1,2}(x)$ as $\psi_{1,2}=|\psi_{1,2}|\exp(i\chi_{1,2}(x))$ and introducing the difference and weighted sum phases:

$$\begin{cases}\varphi=\chi_1-\chi_2,\\ \theta=c_1\chi_1+c_2\chi_2,\end{cases}$$

we obtain for the free energy density (1)–(3)

$$F=\alpha_1|\psi_1|^2+\alpha_2|\psi_2|^2+\frac{1}{2}\beta_1|\psi_1|^4+\frac{1}{2}\beta_2|\psi_2|^4+\hbar^2\left(\frac{|\psi_1|^2}{2m_1}+\frac{|\psi_2|^2}{2m_2}+2\eta|\psi_1||\psi_2|\cos\varphi\right)\left(\frac{d\theta}{dx}\right)^2$$
$$+\hbar^2\left(c_2^2\frac{|\psi_1|^2}{2m_1}+c_1^2\frac{|\psi_2|^2}{2m_2}-2\eta c_1c_2|\psi_1||\psi_2|\cos\varphi\right)\left(\frac{d\varphi}{dx}\right)^2-2\gamma|\psi_1||\psi_2|\cos\varphi, \tag{14}$$

where

$$c_1=\frac{\dfrac{|\psi_1|^2}{m_1}+2\eta|\psi_1||\psi_2|\cos\varphi}{\dfrac{|\psi_1|^2}{m_1}+\dfrac{|\psi_2|^2}{m_2}+4\eta|\psi_1||\psi_2|\cos\varphi},$$

$$c_2=\frac{\dfrac{|\psi_2|^2}{m_2}+2\eta|\psi_1||\psi_2|\cos\varphi}{\dfrac{|\psi_1|^2}{m_1}+\dfrac{|\psi_2|^2}{m_2}+4\eta|\psi_1||\psi_2|\cos\varphi}. \tag{15}$$

The current density j in terms of phases θ and φ has the form

$$j=2e\hbar\left(\frac{|\psi_1|^2}{m_1}+\frac{|\psi_2|^2}{m_2}+4\eta|\psi_1||\psi_2|\cos\varphi\right)\frac{d\theta}{dx} \tag{16}$$

and includes the partial inputs $j_{1,2}$ and the drag current j_{12} proportional to η.

In contrast to the case of a single order parameter,[32] the condition div $\mathbf{j}=0$ does not fix the constancy of the superfluid velocity. In the Appendix we present the Euler–Lagrange equations for $\theta(x)$ and $\varphi(x)$. They are complicated coupled nonlinear equations, which generally permit soliton-like solutions (in the case $\eta=0$ they were considered in Ref. 33). The possibility of states with inhomogeneous phase $\varphi(x)$ requires separate investigation.

Here we restrict consideration to a homogeneous phase difference between the order parameters, $\varphi=$const. For $\varphi=$const it follows from Eqs. (A4) (see Appendix) that $\theta(x)=qx$ (q is the total superfluid momentum) and $\sin\varphi=0$, i.e., φ equals 0 or π. Minimization of the free energy gives the following equation for φ:

$$\cos\varphi=\mathrm{sign}(\gamma-\eta\hbar^2q^2). \tag{17}$$

Note that now the value of φ, in principle, depends on q and thus on the current density j.

Finally, Eqs. (14) and (16) take the form:

$$F=\alpha_1|\psi_1|^2+\frac{1}{2}\beta_1|\psi_1|^4+\frac{\hbar^2}{2m_1}|\psi_1|^2q^2+\alpha_2|\psi_2|^2++\frac{1}{2}\beta_2|\psi_2|^4+\frac{\hbar^2}{2m_2}|\psi_2|^2q^2-2(\gamma-\eta\hbar^2q^2)|\psi_1||\psi_2|\mathrm{sign}(\gamma-\eta\hbar^2q^2), \tag{18}$$

$$j=2e\hbar\left(\frac{|\psi_1|^2}{m_1}+\frac{|\psi_2|^2}{m_2}+4\eta|\psi_1||\psi_2|\mathrm{sign}(\gamma-\eta\hbar^2q^2)\right)q. \tag{19}$$

We will parameterize the current states by the value of the superfluid momentum q, which, for a given value of j, is determined by Eq. (19). The dependence of the order parameter moduli on q is determined by the GL equations:

$$\alpha_1|\psi_1|+\beta_1|\psi_1|^3+\frac{\hbar^2}{2m_1}|\psi_1|q^2-|\psi_2|(\gamma-\eta\hbar^2q^2)\times\mathrm{sign}(\gamma-\eta\hbar^2q^2)=0, \tag{20}$$

$$\alpha_2|\psi_2|+\beta_2|\psi_2|^3+\frac{\hbar^2}{2m_2}|\psi_2|q^2-|\psi_1|(\gamma-\eta\hbar^2q^2)\times\mathrm{sign}(\gamma-\eta\hbar^2q^2)=0. \tag{21}$$

At first we consider the case of small values of the interband coupling γ and dragging coefficient η. In the same manner as for $q=0$ (Sec. II) we obtain for $|\psi_1|(q)$ and $|\psi_2|(q)$, instead of expression (11),

$$\psi_1^2(q)=\frac{a_1\left(1-\frac{T}{T_c}\right)}{\beta_1}-\frac{\hbar^2}{2m_1\beta_1}q^2-\frac{\gamma^2}{a_{20}\beta_1}\frac{T}{T_c}+\frac{(\gamma-\eta\hbar^2q^2)^2}{\beta_1\left(a_{20}-a_2\left(1-\frac{T}{T_c}\right)+\frac{\hbar^2}{2m_2}q^2\right)}, \tag{22}$$

$$\psi_1^2(q)=\left(\frac{a_1\left(1-\frac{T}{T_c}\right)}{\beta_1}-\frac{\hbar^2}{2m_1\beta_1}q^2\right)\times\frac{(\gamma-\eta\hbar^2q^2)^2}{\beta_1\left(a_{20}-a_2\left(1-\frac{T}{T_c}\right)+\frac{\hbar^2}{2m_2}q^2\right)}. \tag{23}$$

The system of Eqs. (19), (22), and (23) describes the depairing curve $j(q,T)$ and the dependences $|\psi_1|$ and $|\psi_2|$ on the current j and temperature T. It can be solved numerically for a given superconductor with concrete values of the phenomenological parameters.

In order to study the specific effects produced by interband coupling and dragging, let us now consider the model case when the order parameters coincide at $j=0$ [Eqs. (12) and (13)] but the gradient terms in Eq. (4) are different. In this case Eqs. (19)–(21) take the form

$$f_1(1-(1+\tilde{\gamma})f_1^2)-f_1q^2+f_2(\tilde{\gamma}-\tilde{\eta}q^2)\mathrm{sign}(\tilde{\gamma}-\tilde{\eta}q^2)=0, \tag{24}$$

$$f_2(1-(1+\tilde{\gamma})f_2^2)-kf_2q^2+f_1(\tilde{\gamma}-\tilde{\eta}q^2)\mathrm{sign}(\tilde{\gamma}-\tilde{\eta}q^2)=0, \tag{25}$$

$$j=f_1^2q+kf_2^2q+2\tilde{\eta}f_1f_2q\;\mathrm{sign}(\tilde{\gamma}-\tilde{\eta}q^2). \tag{26}$$

Here we normalize $\psi_{1,2}$ to the value of the order parameters at $j=0$ (13), j is measured in units of $2\sqrt{2}e\frac{|\gamma|+|\alpha|}{\beta}\sqrt{\frac{|\alpha|}{m_1}}$, q is measured in units of $\sqrt{\frac{\hbar^2}{2m_1|\alpha|}}$, $\tilde{\gamma}=\frac{\gamma}{|\alpha|}$, $\tilde{\eta}=2\eta m_1$, $k=\frac{m_1}{m_2}$. If $k=1$ the order parameters also coincide in the current-carrying state, $f_1=f_2=f$, and from Eqs. (24)–(26) we have the expressions

$$f^2(q)=\frac{1-q^2-|\tilde{\gamma}-\tilde{\eta}q^2|}{1+|\tilde{\gamma}|}, \tag{27}$$

$$j(q)=2f^2(1-\tilde{\eta}\;\mathrm{sign}(\tilde{\gamma}-\tilde{\eta}q^2))q, \tag{28}$$

which for $\tilde{\gamma}=\tilde{\eta}=0$ are the conventional dependences for a one-band superconductor[32] (see Fig. 2).

For $k\neq 1$ the depairing curve $j(q)$ can contain two stable, increasing with q, branches, which corresponds to the possi-

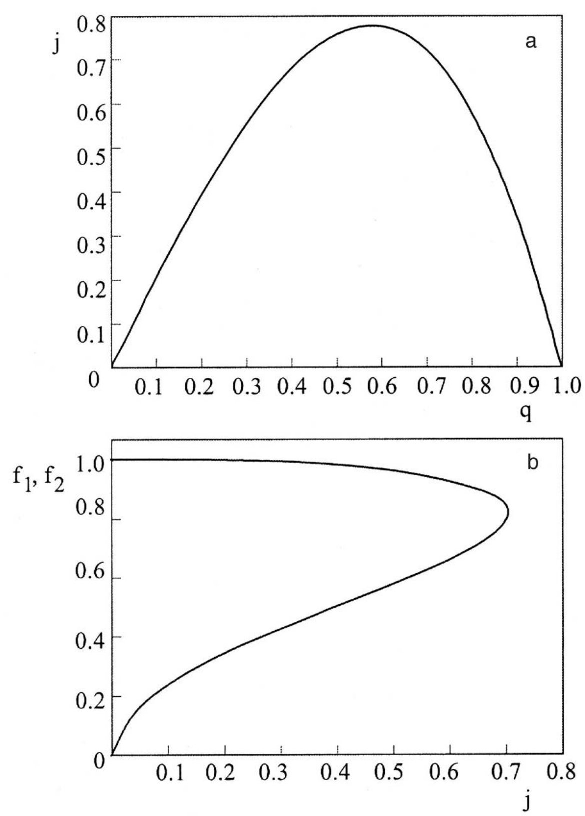

FIG. 2. Depairing current curve (a) and the graph of the order parameter moduli versus current (b) for coincident order parameters.

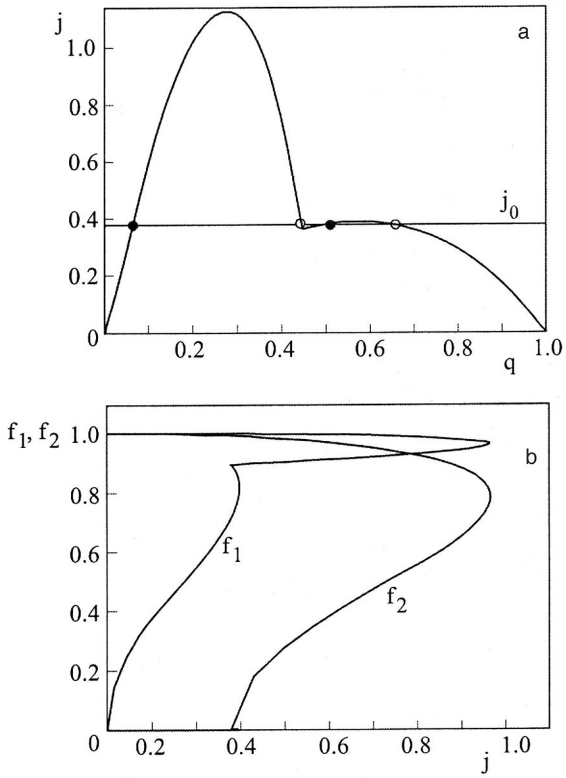

FIG. 3. Dependence of the current j on superfluid momentum q. For the value of the current $j = j_0$, stable states (●) and unstable states (○) are shown (a). Dependences of the order parameters on current j for $k=5$ and $\tilde{\gamma}=\tilde{\eta}=0$ (b).

bility of a bistable state. In Fig. 3 the $j(q)$ and $f_1(j)$ and $f_2(j)$ curves calculated numerically from Eqs. (24)–(26) are shown for $k=5$ and $\tilde{\gamma}=\tilde{\eta}=0$.

The interband scattering ($\tilde{\gamma}=0$) smears the second peak in $j(q)$; see Fig. 4.

If the dragging effect ($\tilde{\eta}=0$) is taken into account the depairing curve $j(q)$ can have a jump at a definite value of q (for $k=1$ see Eq. (28)); see Fig. 5. This jump corresponds to a switching of the relative phase difference from 0 to π.

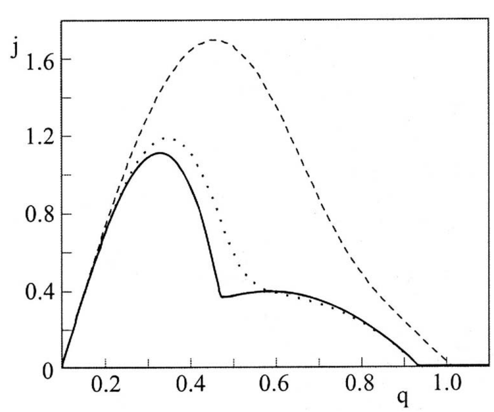

FIG. 4. Depairing current curves for different values of the interband interaction: $\tilde{\gamma}=0$ (solid line), $\tilde{\gamma}=0.1$ (dotted line) and $\tilde{\gamma}=1$ (dashed line). The ratio of effective masses $k=5$, and $\tilde{\eta}=0$.

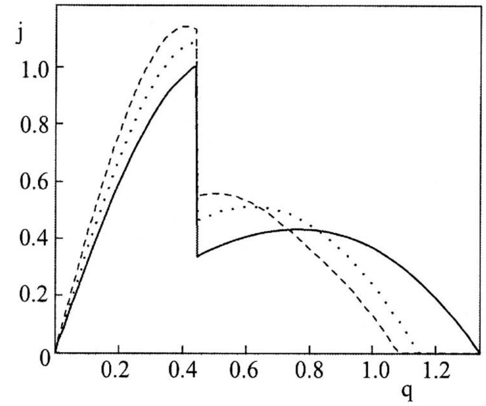

FIG. 5. Depairing current curves for different values of the effective mass ratio $k=1$ (solid line), $k=1.5$ (dotted line), and $k=2$ (dashed line). The interband interaction coefficient $\tilde{\gamma}=0.1$ and drag effect coefficient $\tilde{\eta}=0.5$.

IV. JOSEPHSON EFFECT IN A TWO-BAND SUPERCONDUCTING MICROCONSTRICTION

In the previous Section the GL theory of two-band superconductors was applied to the case of filament length $L \to \infty$. The opposite case of a strongly inhomogeneous current state is the Josephson microbridge geometry, which we model as a narrow channel connecting two massive superconductors (banks). The length L and diameter d of the channel (see Fig. 6) are assumed to be small as compared to the order-parameter coherence lengths ξ_1 and ξ_2.

For $d \ll L$ we can solve the one-dimensional GL equations (4) inside the channel with rigid boundary conditions for the order parameters at the ends of the channel.[34]

In the case $L \ll \xi_1, \xi_2$ we can neglect in Eqs. (4) all except the gradient terms and solve the equations

$$\begin{cases} \dfrac{d^2\psi_1}{dx^2} = 0, \\ \dfrac{d^2\psi_2}{dx^2} = 0, \end{cases} \tag{29}$$

with the boundary conditions:

$$\psi_1(0) = \psi_{01}\exp(i\chi_1), \quad \psi_2(0) = \psi_{02}\exp(i\chi_3),$$

$$\psi_1(L) = \psi_{01}\exp(i\chi_2), \quad \psi_2(L) = \psi_{02}\exp(i\chi_4). \tag{30}$$

Calculating the current density j in the channel, we obtain:

$$j = j_1 + j_2 + j_{12}, \tag{31}$$

$$j_1 = \frac{2e\hbar}{Lm_1}\psi_{01}^2 \sin(\chi_2 - \chi_1), \tag{32}$$

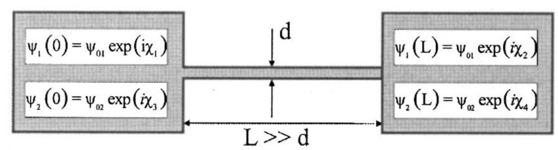

FIG. 6. Geometry of an S–C–S contact as a narrow superconducting channel in contact with bulk two-band superconductors. The values of the order parameters at the banks are indicated.

$$j_2 = \frac{2e\hbar}{Lm_2}\psi_{02}^2 \sin(\chi_4 - \chi_3), \tag{33}$$

$$j_{12} = \frac{4e\hbar}{L}\eta\psi_{01}\psi_{02}(\sin(\chi_2 - \chi_3) + \sin(\chi_4 - \chi_1)). \tag{34}$$

Let $\chi_2 - \chi_1 = \chi$. The difference between two order-parameter phases in the banks equals 0 or π, depending on the sign of the interband interaction constant γ. Therefore, if $\gamma > 0$ then $\chi_3 = \chi_1$ and $\chi_4 = \chi_2$, and if $\gamma < 0$ then $\chi_3 - \chi_1 = \pi$, $\chi_4 - \chi_2 = \pi$. Thus the current–phase relation $j(\chi)$ in the general case of arbitrary values of the phenomenological constants γ and η for two-band superconducting microbridge has the form

$$j \equiv j_0 \sin\chi = \frac{2e\hbar}{L}\left(\frac{\psi_{01}^2}{m_1} + \frac{\psi_{02}^2}{m_2} + 4\eta\psi_{01}\psi_{02}\,\mathrm{sign}(\gamma)\right)\sin\chi. \tag{35}$$

The value of j_0 in (35) can take both positive and negative values:

$$j_0 > 0 \quad \text{if } \eta\,\mathrm{sign}(\gamma) > -\left(\frac{1}{4m_1}\frac{\psi_{01}}{\psi_{02}} + \frac{1}{4m_2}\frac{\psi_{02}}{\psi_{01}}\right), \tag{36}$$

$$j_0 < 0 \quad \text{if } \eta\,\mathrm{sign}(\gamma) < -\left(\frac{1}{4m_1}\frac{\psi_{01}}{\psi_{02}} + \frac{1}{4m_2}\frac{\psi_{02}}{\psi_{01}}\right). \tag{37}$$

When condition (37) is satisfied for the set of parameters of a two-band superconductor, the microbridge behaves as a so-called π junction (see Ref. 35).

V. CONCLUSIONS

We have investigated the current-carrying states in two-band superconductors within the phenomenological Ginzburg–Landau theory. Two limiting situations were considered: the homogeneous current state in a long film or channel, and the Josephson effect in a short superconducting microconstriction. We used the GL functional for two order parameters, which includes the interband coupling (proximity effect) and the effect of dragging in the current state of the two-band system. For the case of two order parameters the question arises as to the phase difference $\varphi = \chi_1 - \chi_2$ between $\psi_1 = |\psi_1| e^{i\chi_1}$ and $\psi_2 = |\psi_2| e^{i\chi_2}$. In the homogeneous no-current state the value of φ equals 0 or π, depending on the sign of the interband coupling constant γ.[36] The statement that φ can only have values 0 or π also holds in the current-carrying state, but for nonzero drag coefficient η the criterion for whether φ equals 0 or π now depends on the value of the superfluid momentum q, namely $\cos\varphi = \mathrm{sign}(\gamma - \eta\hbar^2 q^2)$. The system of coupled GL equations is analyzed for different values of the phenomenological parameters. The depairing current expression contains the term $\cos\varphi$, and, in general, depending on the parameters γ and η, with increasing momentum q the value of φ can switch from 0 to π. In the current-driven regime it leads to the existence of two ascending branches of $j(q)$, which are both stable. This bistability is an intrinsic property of a two-band superconductor. It is interesting to study the effects of relative phase switching in a magnetic-flux-driven regime in a multivalued geometry. The Josephson current–phase relation for two band superconducting weak links $j(\chi)$ also contains the phase difference of the order parameters in the banks, $j = j_0(\varphi)\sin\chi$. Here j_0 can take both positive and negative values. In the latter case we have what is called the π junction, again due to intrinsic properties of two-band superconductivity. In Sec. II we restricted consideration to a homogeneous phase difference φ between the two order parameters. The general equations (A4) admit the possibility of inhomogeneous, solitonlike distributions $\varphi(x)$, which will be the subject of a separate publication.

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APPENDIX: FREE ENERGY TRANSFORMATION

Instead of the phases χ_1 and χ_2, we introduce the new variables φ and θ:

$$\begin{cases} \chi_1 - \chi_2 = \varphi, \\ c_1\chi_1 + c_2\chi_2 = \theta, \end{cases} \tag{A1}$$

where the coefficients c_1 and c_2 are chosen as

$$c_1 = \frac{\dfrac{|\psi_1|^2}{m_1} + 2\eta|\psi_1||\psi_2|\cos\varphi}{\dfrac{|\psi_1|^2}{m_1} + \dfrac{|\psi_2|^2}{m_2} + 4\eta|\psi_1||\psi_2|\cos\varphi},$$

$$c_2 = \frac{\dfrac{|\psi_2|^2}{m_2} + 2\eta|\psi_1||\psi_2|\cos\varphi}{\dfrac{|\psi_1|^2}{m_1} + \dfrac{|\psi_2|^2}{m_2} + 4\eta|\psi_1||\psi_2|\cos\varphi}. \tag{A2}$$

The expression for the free energy density in the new variables takes a quadratic form in the derivatives of θ and φ:

$$F = A + B\left(\frac{d\theta}{dx}\right)^2 + C\left(\frac{d\varphi}{dx}\right)^2 - D\cos\varphi. \tag{A3}$$

Here A, B, C, and D are

$$A = \alpha_1|\psi_1|^2 + \frac{1}{2}\beta_1|\psi_1|^4 + \alpha_2|\psi_2|^2 + \frac{1}{2}\beta_2|\psi_2|^4, \qquad B = \left(\frac{|\psi_1|^2}{2m_1} + \frac{|\psi_2|^2}{2m_2} + 2\eta|\psi_1||\psi_2|\cos\varphi\right)\hbar^2,$$

$$C = \left(c_2^2\frac{|\psi_1|^2}{2m_1} + c_1^2\frac{|\psi_2|^2}{2m_2} - 2c_1c_2\eta|\psi_1||\psi_2|\cos\varphi\right)\hbar^2, \qquad D = 2\gamma|\psi_1||\psi_2|.$$

Doing a variation of (4) θ and φ, we obtain equations for the spatial dependence of the phases φ and θ:

$$\begin{cases} \left(\frac{|\psi_1|^2}{2m_1} + \frac{|\psi_2|^2}{2m_2} + 2\eta|\psi_1||\psi_2|\cos\varphi \right) \frac{d^2\theta}{dx^2} - 2\eta|\psi_1||\psi_2| \frac{d\varphi}{dx}\frac{d\theta}{dx}\sin\varphi = 0, \\ 2\frac{d}{dx}\left(C(\varphi)\frac{d\varphi}{dx} \right) - \frac{dB(\varphi)}{d\varphi}\left(\frac{d\theta}{dx}\right)^2 - \frac{dC(\varphi)}{d\varphi}\left(\frac{d\varphi}{dx}\right)^2 - D\sin\varphi = 0. \end{cases} \tag{A4}$$

In the particular case (5) $\eta=0$ (no drag effect) Eq. (A4) coincides with the equation obtained in Ref. 33.

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