SUPERCONDUCTIVITY, INCLUDING HIGH-TEMPERATURE SUPERCONDUCTIVITY

Quantum detector based on a superposition of macroscopic states in a phase qubit

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A quantum detector whose working principle is based on magnetic-field modulation of a circulating supercurrent in the quantum ground state of a macroscopic superconducting loop with a Josephson junction. Under the influence of an external magnetic flux equal to $\Phi_0/2$ (or Φ_0), two (or three) classical states are coupled to each other by quantum tunneling through a potential barrier, and therefore the detector is a two-level (or three-level) system. In the low-temperature region and under the condition of very weak damping, the mean value of the circulating supercurrent reflects the character of the variation of the quantum superposition of macroscopic states, which is sensitive to the symmetry of the potential. The variations of the current are amplified and detected in a measurement scheme similar to the signal registration in a nonhysteretic rf SQUID. It is shown by a numerical analysis that in comparison with a qubit detector based on an SIS junction, a detector with an ScS junction is faster and has much larger amplitudes of energy splitting at the same parameters. The results presented for double- and triple-well potentials clearly indicate that a qubit with an ScS junction can act as a detector with a sensitivity determined by the quantum noise of the amplifier. © *2007 American Institute of Physics*.
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INTRODUCTION

Experimental and theoretical research on magnetometers based on rf and dc SQUIDS has led to a substantial improvement of the sensitivity in many magnetic measurements. Such sensors have naturally become used for registration of the state of quantum bits (qubits), including for solution of the fundamental problem — that of decreasing the back-action of a classical measuring system on the evolution of a two-level quantum system.[1–3] Although both types of sensors can be used to make devices with quantum limitations on the sensitivity, i.e., their energy resolution $\delta\varepsilon$ over a measurement time δt approaches the limit set by the uncertainty relation $\delta\varepsilon\,\delta t \geq \hbar/2$,[4–7] the behavior of the phase difference φ across the Josephson junctions remains classical. In other words, the structure of the formulas describing the nonlinear properties of dc and rf SQUIDs are classical, and all the variables in the equations can be measured simultaneously. Contrary to this, "topologically similar" superconducting circuits — qubits created on the basis of mesoscopic Josephson junctions with negligibly low quasiparticle current — demonstrate remarkable quantum properties at the macroscopic level: quantization of energy levels, resonant tunneling,[8] and coherent superposition of macroscopic states.[9–13]

The description of a phase qubit and a sensor based on it requires consideration of the quantum dynamics of a collective variable — the phase difference φ. The variation of this quantity is accompanied by a change of a large number of electronic states. A detailed analysis of the exact Hamiltonian of the system is complicated if one includes fluctuations and the significant nonlinearity typical for qubits based on Josephson junctions. However, the use of various approximations and phenomenological models in many cases permits comparison of the experimental data with the theory.[14–16]

Let us consider a phase qubit consisting of a superconducting loop with self-inductance L, closed by a Josephson tunnel junction with critical current I_c and capacitance C. On the one hand, for obtaining two different macroscopic states (i.e., a double-well potential) and creating a coherent superposition between them it is necessary that the characteristic parameter $\beta_L = 2\pi L I_c/\Phi_0$ be greater than unity, where $\Phi_0 = h/2e \simeq 2.07\times10^{-15}$ Wb is the magnetic flux quantum. On the other hand, the parameter β_L should not strongly exceed the value $\beta_L \approx 1$, since at high critical currents of the junction the tunneling probability becomes exponentially small because of an increase of the barrier separating these states. For example, in Ref. 9 for observation of a superposition of macroscopic states a value $\beta_L \simeq 2.3$ was chosen. At a capacitance of the SIS junction $C \approx 40$ pF only the energy levels close to the top of the barrier form a coherent superposition of qubit states characterized by different macroscopic magnetic moments $\pm10^{10}\mu_B$ (μ_B is the Bohr magneton) or currents flowing in opposite directions. For the lower energy levels the value of the matrix element of the tunneling transition is negligibly small, and the amplitude of the splitting of degenerate levels of uncoupled wells is much less than

k_BT. This necessitated making "spectroscopic" studies in which the tunneling exponent was effectively decreased by increasing the occupation probability of the upper (close to the barrier top) level was increased through the excitation of a resonant electromagnetic field.

The tunneling interaction between different macroscopic states in phase qubits is largely determined by the potential barrier which, in turn, depends on the Josephson coupling energy $E_J=I_c\Phi_0/2\pi$ and the current–phase relation $I(\varphi)=I_cf(\varphi)$ for the given type of junction. If the interaction between neighboring states, which is described with the aid of an effective action functional,[17] is sufficiently strong, the quantum nonlinearity arising in the qubit owing to superposition of states can be used to create a new sensor, which we shall refer to loosely as a qubit-SQUID or "q-SQUID."

In the present paper a comparative analysis of two q-SQUIDs, each consisting of a superconducting loop closed by an SIS or ScS contact, is carried out on the basis of a numerical solution of the Schrödinger equation. The characteristics of the qubits are analyzed over a wide range of parameters β_L and C, with particular attention to a quantum detector based on an ScS junction and to the case of the symmetric triple-well potential investigated in Refs. 1 and 18.

THEORETICAL MODEL

The microscopic theory of microbridges[19] predicts a substantial difference between the properties of ScS and SIS junctions in the low-temperature region. For the analysis below it is important to note that the current–phase relation $I(\varphi)/I_c=f(\varphi)$ and the critical current amplitude I_c of the microbridge is different from the resistive model often used to describe tunnel junctions. In the clean limit, when the characteristic neck size of the microbridge becomes much less than the electron mean free path and the coherence length in the superconductor, the current of an ScS contact with a normal resistance R_N and a temperature-dependent superconducting gap $\Delta_0(T)$ can be written in the form[19]

$$I(\varphi)=\frac{\pi\Delta_0(T)}{eR_N}\sin(\varphi/2)\tanh\frac{\Delta_0(T)\cos(\varphi/2)}{2k_BT}. \quad (1)$$

It follows from this relation that, first, at low temperatures (formally at $T=0$) the supercurrent as a function of phase undergoes jumps at the points $\varphi=2\pi(n+1/2)$, where n is an integer. As the temperature is raised, the $I(\varphi)$ curve gradually approaches the sinusoidal dependence typical of SIS junctions. Second, in a clean ScS junction the critical current amplitude and the Josephson coupling energy are twice as high as in a tunnel junction with the same normal resistance. At a fixed value of the critical current the change of the shape of the tunnel barrier separating the two states in the qubit based on the clean ScS junction (1) will lead to a sharp increase of the tunneling probability[17,18] and, as a consequence, to a significant increase of the level splitting upon the formation of a coherent superposition of states.

Using relation (1) in the limit $T=0$ and the fluxoid quantization condition for a superconducting loop closed by an ScS junction, one can obtain a stationary equation describing the classical relation between the external magnetic flux Φ_e and the magnetic flux in the loop, Φ, in the interval $\varphi\in(-\pi,\pi)$:

$$\varphi+\beta_L\,\mathrm{sgn}(\cos(\varphi/2))\sin(\varphi/2)=\varphi_e. \quad (2)$$

Here $\varphi=2\pi\Phi/\Phi_0$ and $\varphi_e=2\pi\Phi_e/\Phi_0$ are dimensionless internal Φ and external Φ_e magnetic fluxes. The condition of stationarity means that all the time derivatives are negligible and the current through the contact is determined by the superconducting component (1). The total potential energy of the qubit with the ScS (or SIS) junction is made up of the energy of the Josephson junction and the magnetic energy stored in the loop. In the limit $T=0$ these energies depend on the type of junction and are substantially different:[1,17–19]

$$U_{ScS}(\Phi,\Phi_e)=\frac{(\Phi-\Phi_e)^2}{2L}-\frac{I_c(0)\Phi_0}{2\pi}\left|\cos\frac{\pi\Phi}{\Phi_0}\right|, \quad (3)$$

$$U_{SIS}(\Phi,\Phi_e)=\frac{(\Phi-\Phi_e)^2}{2L}-\frac{I_c^t(0)\Phi_0}{2\pi}\cos\frac{\pi\Phi}{\Phi_0}. \quad (4)$$

Here $I_c(0)=\pi\Delta_0/eR_N$ is the critical current of the ScS junction, and $I_c^t(0)=\pi\Delta_0/2eR_N$ is the critical current of an SIS junction at low temperatures $(T\ll T_c)$. Analysis of the classical dynamics of a superconducting loop closed by a Josephson junction reduces to the study of the motion of a particle having a mass proportional to the junction capacitance, in a one-dimensional potential (3) or (4). It follows from expressions (2) and (3) that a q-SQUID based on a pure ScS junction has two local minima even in the region $\beta_L<1$, since the condition for the onset of the hysteretic regime has the form $\beta_L^*=\beta_L[\partial f(\varphi)/\partial\varphi]_{\varphi=\pi}\geq 1$. As a consequence of this difference from model (4), in constructing qubits based on potential (3) one can substantially reduce the geometric size (inductance) of the quantum system. We note that an analogous assumption, though based on a different physical phenomenon, was discussed recently in Ref. 15. Currently, for decreasing the inductance of the phase qubit, one connects three tunnel junctions in the loop;[20] although this causes twice as much capacitance (mass) to be involved in the tunneling, it preserves the shape of the barrier in the tunnel direction.

In the classical limit the decay of metastable current states depends exponentially on the height of the barrier separating the two states. Since the shape of the potential barrier (3) in a qubit with an ScS junction leads to a substantial decrease of the tunneling exponent at the same barrier height, a potential of the type (3) is preferable from the standpoint of constructing quantum chains. We note that in the present paper we consider a rather general case, since it is easy to show that such a potential can be constructed (quantum-state engineering) as well in the case of a series connection of two tunnel Josephson junctions in the loop.

The quantum behavior of a macroscopic variable qubit Φ can be studied in an approximation using the Hamiltonian of an isolated system. Such an approximation describes the case when the qubit does not interact with a large number of degrees of freedom of the detector (an LC circuit). However, to obtain any, even a very small amount of information about the state of a qubit in the method of weak continuous measurements leads to an increase in the rate of its decoherence.

Therefore, the results of the numerical analysis, presented below, describe only the general regularities, while for numerical comparison with the characteristics obtained experimentally it is necessary to take into account the influence (even though weak) of the measurements on the state of the qubit.[2] Nevertheless, it is shown in the present paper that when a superposition of three states appears, fundamentally new effects arise which are described well by the model of an isolated system and can explain the experimental results obtained in Ref. 1.

It follows from the theory of clean ScS junctions that in the limit $T=0$ and under adiabatic conditions the Ohmic dissipation vanishes, and to first approximation the superposition of macroscopic states of the qubit can be analyzed without taking it into account.[17] Adiabatic conditions for a tunneling process are satisfied under the conditions $\hbar(LC)^{-1/2} \ll E_{10} \ll \Delta_0$ for the characteristic energy of the resonance (tank) circuit and the minimum value of the level splitting E_{10}. Furthermore, it was shown in Ref. 17 that at a current close to the critical value, $I \sim I_c(0) = \pi\Delta_0/eR_N$, the junction capacitance is renormalized as follows: $C \approx C^* + \pi\hbar/(4R_N\Delta_0)$, where C^* is the geometric capacitance of the junction. For ScS junctions with resistance $R_N \approx R_Q = \hbar/e^2 \approx 4.1\ \mathrm{k}\Omega$ the electrodynamic correction to the geometric capacitance is rather small (~ 0.1 pF). However, taking into account the geometric capacitance of the toroidal structure used in Ref. 1, one obtains an estimate $C \approx 4.0-8.0$ pF. For an ScS junction based onniobium with a normal resistance $R_N \approx 4.0\ \mathrm{k}\Omega$ the critical current is equal to $I_c(0) \sim 1.0\ \mu\mathrm{A}$,[19] which, for a loop with inductance $L = 3\times 10^{-10}$ H, gives a value $\beta_L \sim 1$.

In the adiabatic limit and without dissipation, the Hamiltonians of q-SQUIDs with potentials (3), (4) can be represented in the simple form

$$H(\Phi,\Phi_e) = \frac{Q_2}{2C^*} + U(\Phi,\Phi_e). \tag{5}$$

Here $Q = -i\hbar\partial/\partial\Phi$ is the charge operator, and $U(\Phi,\Phi_e)$ is the nonlinear term of Eq. (3) or (4), depending on the type of junction. The eigenenergy levels and the squares of the wave functions of the states are calculated by numerical solution of the stationary Schrödinger equations with potential energies (3), (4). In the dimensionless variables these equations have the form

$$\frac{1}{2m}\frac{\partial^2}{\partial\varphi^2}\Psi = \left[\frac{(\varphi-\varphi_e)^2}{2} - 2\beta_l|\cos(\varphi/2)|\right]\Psi; \tag{6}$$

$$\frac{1}{2m}\frac{\partial^2}{\partial\varphi^2}\Psi = \left[\frac{(\varphi-\varphi_e)^2}{2} - \beta_l\cos\varphi\right]\Psi. \tag{7}$$

For finding the solutions of the stationary equations (6), (7) a version of the finite-element method is used, with the characteristic shape of the potential energy approximated by a piecewise-constant function. The analytical solutions obtained for each finite element are joined. Boundary conditions of the zero type are specified at the boundaries of the region. The number of elements was varied in the range 10^2-10^3, depending on the required accuracy of the calculation. For convenience of comparison of the results of the calculation, the inductances of the qubits with ScS and SIS junctions and the dimensionless masses $m=(\varepsilon/\hbar\omega_0)^2$ were set equal [$\varepsilon=(\Phi_0/2\pi)^2/L \approx k_B\cdot 24.5$ K is the characteristic energy of the loop,[1] and $\omega_0=(LC)^{-1/2}$ is its eigenfrequency].

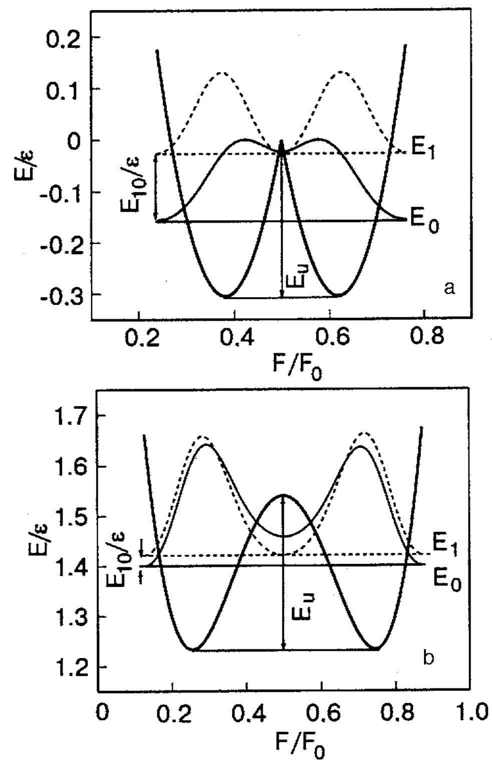


FIG. 1. Superposition of two macroscopic states in q-SQUIDs with junctions of the ScS type (a) and of the SIS type (b) with the following values of the parameters: geometric inductance of the loop $L=3\times10^{-10}$ H, the total capacitance $C\approx 2.7\times10^{-15}$ F ($m=8$), external magnetic flux $\Phi_e=\Phi_0/2$, potential barrier height $E_U=0.32\varepsilon$, $\beta_L=0.8$ (a), $\beta_L=1.54$ (b). All of the energies are given in units of $\varepsilon=\Phi_0^2/4\pi^2L$ ($\varepsilon/k_B\approx 24.5$ K).

As an example, we consider a superposition of states in qubits (6), (7) with parameters $L=3\times10^{-10}$ H, $C^*\simeq C=2.7$ pF in the case of symmetric double-well potentials, i.e., at a fixed external flux $\varphi_e=(n+1/2)2\pi$. The values of the critical currents will be chosen such that the corresponding potential barrier heights E_U are equal in magnitude and much greater than the energy of thermal fluctuations. This requirement is equivalent to the condition that the decay rates of the metastable states due to thermal fluctuations be approximately equal to each other and much less than the quantum ones. Figure 1 shows the amplitudes of the tunneling splittings $E_{10}=E_1-E_0$ and the squares of the wave functions of the ground state E_0 and excited state E_1 for two types of qubits at the point $\Phi_e=\Phi_0/2$. For the chosen values of the capacitance, at equal barrier heights the amplitude of the splitting in the qubit with the ScS junction is 6.75 times larger than in the case of a tunnel junction. It is clear that with increasing capacitance the eigenvalues of the levels in potentials (3), (4) are lowered. This leads to a decrease of the critical value of the parameter β_L, at which both split levels in the qubit with the ScS junction are still below the barrier. As we have said, because of the specific shape of the current–phase relation (1), in q-SQUIDs with an ScS junction a double-well potential exists even in the region $\beta_L<1$. Therefore, by decreasing β_L, one can obtain a progressively greater difference in the level splitting. For example, at a

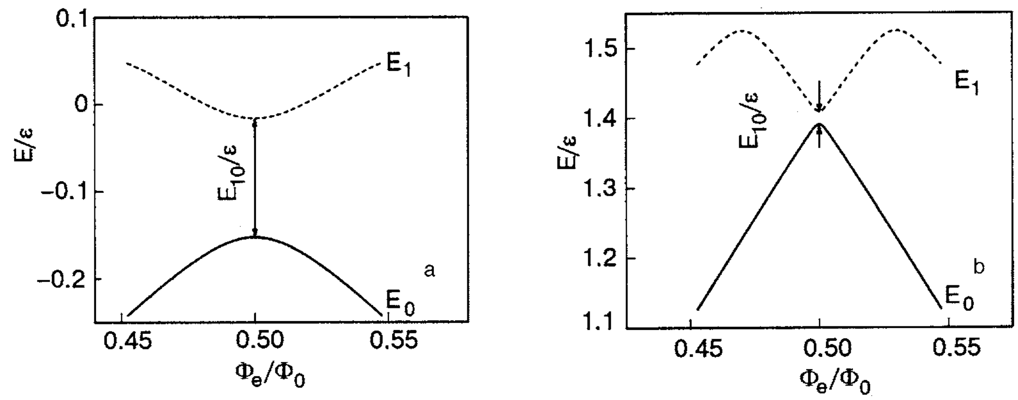


FIG. 2. Dependence of the energies of the ground state E_0 and excited state E_1 on the external magnetic flux in the presence of a superposition of double-well potentials in the region $\Phi_e=\Phi_0(1\pm0.05)$ for a q-SQUID with an ScS junction, $\beta_L=0.8$ (a) and for a q-SQUID with an SIS junction $\beta_L=1.54$ (b). All the other parameters correspond to those given in Fig. 1.

dimensionless mass $m=20$ and potential barrier heights $E_U=0.20\varepsilon$, the ratio of the splitting amplitudes E_{10} is approximately equal to 13. Moreover, a numerical analysis shows that for the case of very large masses ($m=80$) and barrier heights $\sim0.12\varepsilon$ the ratio of the splitting amplitudes increases to 22. This simple analysis shows that a superposition of two states in qubits with a specific shape of the potential barrier (3) leads to a sharp increase of the splitting amplitude E_{10}, and therefore such qubits have a clear advantage both as a basis for constructing quantum detectors and for use as quantum logic elements.

The dependence of the first two energy levels E_0 and E_1 on the external flux in the interval $\Phi_e=\Phi_0/2\pm0.05\Phi_0$ is shown in Fig. 2. The working principle of the sensor, which was implemented in Ref. 1, is based on the fact that in the ground state, i.e., in the motion along the level E_0, the Josephson quantum inductance is a periodic function of the external magnetic flux with period Φ_0:

$$L_J^{-1}(\varphi,\varphi_e)=(2\pi/\Phi_0)[\partial^2E_0(\varphi,\varphi_e)/\partial\varphi^2]. \quad (8)$$

Therefore for excitation and registration of the state of a q-sensor it can be inductively coupled to a low-frequency tank circuit having a natural frequency $f_T=1/2\pi(L_TC_T)^{1/2}$ that is much less than the characteristic frequency of the two-level system, E_{10}/h (the adiabatic excitation regime). In such a coupled system as a result of the mutual inductance M the oscillations in the circuit at frequency f_T create an external magnetic flux $\Phi_e(t)=MI_T\sin 2\pi f_T t$ applied to the qubit. If the amplitude of this flux is much less than Φ_0, then the Josephson quantum inductance (8) introduced in the circuit will be determined by the local curvature of the ground state $E_0(\varphi,\varphi_e)$, having maxima at $\Phi_e=(1+1/2)\Phi_0$. In this case the external magnetic flux Φ_e, by altering the value of the local curvature and the inductance (8), will lead to a shift of the resonance frequency of the circuit, i.e., to a signal. In such a measurement scheme the transfer coefficient is proportional to the square of the coupling coefficient $k^2=M/LL_T$, which with the use of the nonlinearity arising because of the coherent superposition of states should be small, since the back-action of the circuit on the qubit is also proportional to k^2 (Refs. 2 and 3). Therefore, in contrast to the well-known condition for the coupling coefficient in rf SQUIDs ($k^2Q\geq1$), in making weak continuous measurements of a quantum system one should choose values $k^2Q<1$. Otherwise, the dissipation introduced in the qubit by the classical tank circuit, $\sim k^2QLf_T$, will lead to an increase in the rate of decoherence and rapid collapse of the wave function of the superposition state.

As was shown above, in a qubit with an ScS junction (Fig. 2a) the level E_1 can be found rather far from the ground-level energy E_0, and to first approximation one can assume that it does not have a substantial influence on the behavior of the system in the case of a weak low-frequency excitation of the circuit. In fact, this approximation means that the value of the minimum splitting E_{10} is much greater than $k_BT/\varepsilon\approx0.015$ and the characteristic energy hf_T of the circuit.[1] Therefore, at low amplitudes of excitation, $\Phi_e(t)\ll\Phi_0$, the insertion inductance in the circuit is linked only with the mean value of the square of the ground-state wave function. Figure 3 shows the dependence of the mean value of the magnetic flux $\langle\Phi\rangle$ on the external flux Φ_e, obtained by averaging the squares of the wave functions for the ground state E_0 and excited state E_1. We note that in the adiabatic motion in the ground state the curves are of opposite character, like $\varphi(\varphi_e)$ for the nonhysteretic regime of an rf SQUID, although, of course, the analogy is incomplete. The opposite nature of $\langle\varphi\rangle(\varphi_e)$ causes the signal characteristics of the q-SQUID with a double-well potential to be qualitatively similar to the characteristics of a classical rf SQUID with parameter $\beta_L<1$. These features of the signal and amplitude–frequency characteristics have been mentioned in experimental studies of systems with ScS junctions.[1,21]

Following the results of Ref. 1, let us analyze the q-SQUIDs based on ScS junctions with parameters $2<\beta_L<4$, i.e., in the existence region of triple-well potentials. It is seen from Eqs. (3) and (4) that for a qubit inductance $L=3\times10^{-10}$ H a triple-well symmetric potential can be obtained at points $\Phi_e=n\Phi_0$, having increased the critical current of the contact to $I_c=2-4$ μA. In such a potential the energy levels in the "left" and "right" well are degenerate at any β_L, as in the previous case. In the middle (deep) well the distribution of levels depends on the critical current of the Josephson junction and the capacitance, and therefore total degeneracy (the condition of resonance of three levels for the specified mass) occurs only at certain values of β_L. Under

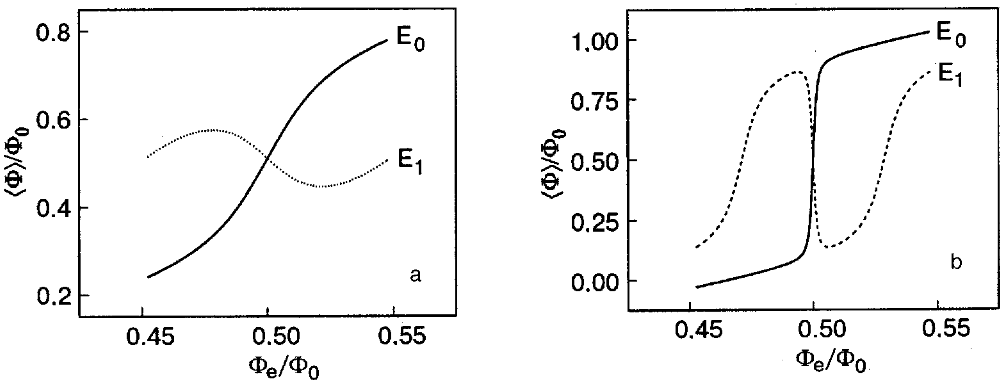


FIG. 3. Dependence of the mean values of the magnetic flux $\langle\Phi\rangle$ on the external flux Φ_e in the presence of superposition in double-well potentials for the ground state E_0 and excited state E_1, obtained by averaging the square of the wave functions in q-SQUIDs based on ScS (a) and SIS (b) Josephson junctions with parameters corresponding to Figs. 1 and 2.

conditions close to degeneracy, a system of split levels is formed which, by analogy with the case of double-well potentials, we denote as follows: E_0 is the lowest, E_1 the middle, and E_2 the highest level. The minimum value of the splitting between them at the point $\Phi_e=\Phi_0$ will be equal to $E_1-E_0=E_{10}$ and $E_2-E_1=E_{21}$, respectively.

Figure 4a a shows the dependence of the splitting on the dimensionless parameter β_L for the first two energy levels, obtained by solution of the time-independent Schrödinger equation for four values of the capacitance of the q-SQUID. For values $m=10$ upon a resonance superposition of three states the magnitude of the splitting reaches values $E_{10}/k_B=E_{21}/k_B\approx 3.5$ K, where E_{21} is equal to the minimum (i.e., at the point $\Phi_e=\Phi_0$) value of the splitting between the corresponding levels. We note that at such points of resonance superposition the splitting amplitude can reach values greater than 1 K, even for a very large mass $m=40$, which corresponds to a total capacitance of the q-SQUID $C\approx 1.33\times 10^{-14}$ F.

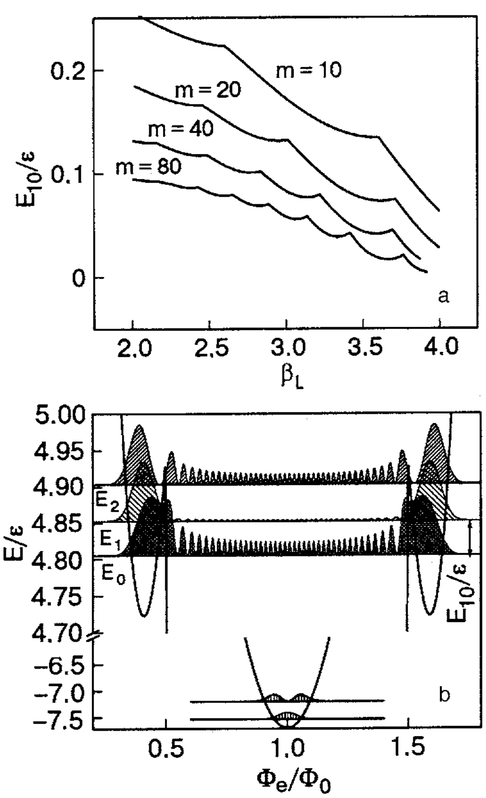
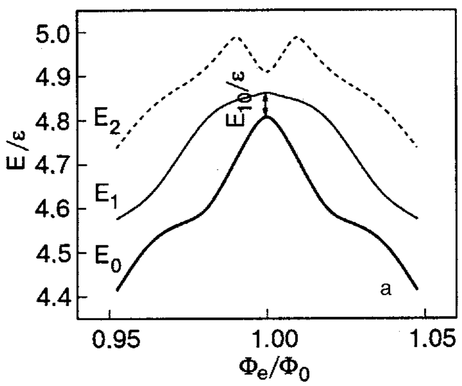


FIG. 4. Superposition of macroscopic states in a triple-well symmetric potential in the presence of an external magnetic flux $\Phi_e=\Phi_0$. The family of curves of the level splitting $E_{10}=E_1-E_0$ versus the values of the dimensionless parameter β_L for a q-SQUID with an ScS junction. The parameter of the family is the dimensionless mass $m=(\varepsilon/\hbar\omega_0)^2$. The extrema on the $E_{10}(\beta_L)$ curve appear at exact degeneracy of the three energy levels in the triple-well potential (a). The shape of the potential energy and square of the wave functions for split levels in a q-SQUID with an ScS junction at $m=26$, $\beta_L=3.83$ at a point of degeneracy of three levels, $\Phi_e=\Phi_0$. The position of the two lower levels in the central well is shown with a break of the energy scale (b).

As to the choice of the values of β_L, we note that in Ref. 1, starting from a resistive model of the junction (4), this parameter was estimated to be around 5.8, but going to a model of a clean ScS junction decreases this value by $\pi/2$. In the present paper we therefore have carried out our numerical analysis for values in the range $2<\beta_L\leq 4$.

Figure 4b shows the form of the potential energy and the value of the splitting for the case of a resonance superposition of three macroscopic states in a q-SQUID with an ScS junction. The parameters chosen for the calculation, $m=26$, $\beta_L=3.83$ are rather close to the values obtained in Ref. 1: $m=20\pm 10$, $\beta_L\approx 4.2$. In the figure we give the squares of the wave functions for the levels E_0, E_1, and E_2. We note that, unlike the case of the double-well potential, in the triple-well potential there always exist levels lying below E_0. The number of these levels in the central well is large and corresponds to the number of zeros of the square of the wave function. This fundamental difference means that over the energy relaxation time τ_ε an irreversible collapse of the states can occur due to energy relaxation, and the system can end up in the main energy minimum. In other words, the state of superposition in the triple-well potential is always metastable in the sense of energy relaxation. Therefore, to make a sensor based on a q-SQUID with such a potential, it is necessary that the energy relaxation time τ_ε of the sensor be much greater than the characteristic time of the measurements. Interestingly, energy relaxation was not observed in Ref. 1, where the signal characteristics were recorded with rather long measurement times (5 min). This means that the system being measured was very well isolated from the environment (e.g., because of the toroidal "self-shielding" con-



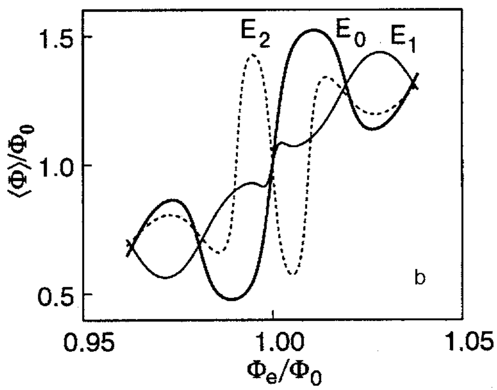


FIG. 5. Superposition of three macroscopic states in a q-SQUID with an ScS junction at $m=26$, $\beta_L=3.83$. The dependence of the three split energy levels on the external magnetic flux in the interval $\Phi_e=(1\pm0.05)\Phi_0$ (a). The mean values of the magnetic flux $\langle\Phi\rangle$ in a q-SQUID, obtained by averaging the square of the wave functions for the three split energy levels E_0, E_1, and E_2, versus the magnetic flux in the same interval (b).

struction of the resonator) and that the tank circuit had a rather weak influence on the state of the qubit.[2,3,22] The long energy relaxation times obtained experimentally allow us to keep the term "ground state" for the E_0 level in the sense that the main nonlinearity of the q-SQUID is due to the motion along this level only.

The dependence of the level energies E_0, E_1, and E_2 on the external magnetic flux Φ_e in the interval $\Phi_e=(1\pm0.05)\Phi_0$ and the corresponding dependence of the mean values of the magnetic flux $\langle\Phi\rangle$ are presented in Fig. 5. The amplitude of the variation of $\langle\Phi\rangle$ around $\Phi_e=\Phi_0$ (Fig. 5b) demonstrates that, first, the coherent superposition of macroscopic states in the triple-well potential arises between states separated by an interval $\sim2\Phi_0$, which differs sharply from the classical dependence typical of an rf SQUID in the nonhysteretic regime. Furthermore, in this case the nonlinearity is maximum at a value $\Phi_e=\Phi_0$, whereas for double-well potentials and for classical rf SQUIDs the maximum is reached at the point $\Phi_e=\Phi_0/2$. The character of the nonlinear dependence of $\langle\Phi\rangle$ on the flux Φ_e that arises for motion along the ground state is shown in Fig. 5b by the solid curve. Since precisely these features of the signal and current–voltage characteristics were noted as "anomalous" in Ref. 1, this can be considered to be the first implementation of a q-SQUID with a niobium ScS junction. It is seen in Fig. 5a that for the parameter $\beta_L=3.83$, even for large masses $m=26$ ($C\approx8\times10^{-15}$ F) the resonance superposition of three states leads to a value of the splitting between the first two levels $E_{10}=0.06\varepsilon$, which corresponds to 1.5 K.

Turning to a discussion of the sensitivity of q-SQUIDs, we note that a linear superposition of two (three) macroscopic states leads to nonlinear dependence of $\langle\Phi\rangle$ on Φ_e (see Fig. 3b and 5b). If the rate of diffusion of the wave packet is much higher than the rate of change of the potential energy, then these relations are single-valued. This fact becomes very important when one considers that in such a regime the SQUID can be treated as an ideal parametric frequency up-converter, and therefore its contribution to the total noise of an amplifying device can be made negligibly small.[23,24] At low temperatures ($T<0.1$ K) — and it is at such temperatures that measurements of the states of superconducting qubits are proposed — the total sensitivity of a q-SQUID as a narrow-band amplifier, ε_N, will be bounded from below by the Heisenberg uncertainty principle:

$$(\varepsilon_N)_{\min}=\varepsilon_N(T_T,T_A)(\omega_S/\omega_T)\geq\hbar\,\omega_S/2. \tag{9}$$

Here T_T and T_A denote the noise temperatures of the tank circuit and amplifier, and ω_S is the signal frequency. The sensitivity of an rf SQUID at 4.2 K in the nonhysteretic regime, with the fluctuation back-effect taken into account, is more than an order of magnitude greater than the limit (9) and is usually determined by the noise temperature T_T of the tank circuit and the transistor noise. However, an analysis of results obtained on cooled amplifiers[25] in recent years shows that in the temperature region $T<0.1$ K both of these factors can be substantially reduced, making it possible to approach values close to Planck's constant $h/2\pi$.

CONCLUSION

A numerical analysis of the solutions of the stationary Schrödinger equation shows that a coherent superposition of two macroscopic states in a superconducting loop closed by a clean ScS junction leads to a sharp (order-of-magnitude) increase of the energy level splitting in comparison with a qubit based on an SIS junction at the same parameters. The substantial improvement of the quality of qubits is due to a change of the effective action[16] describing the quantum behavior of the phase difference φ and depending on the type of weak link or the shape of the potential barrier. Moreover, since in q-SQUIDs with a ScS junction a double-well potential exists even in the region $\beta_L<1$, nonlinearity due to superposition of macroscopic states can be obtained for lower values of L.

Superposition of three macroscopic states leads to two new effects in the ground state: the maximum nonlinearity of the function $\langle\Phi\rangle(\Phi_e)$ is observed in the neighborhood of $\Phi_e=\Phi_0$, and the characteristic amplitude of this nonlinearity increases by almost a factor of two in comparison with superposition in a double-well potential. In the adiabatic approximation, i.e., under the condition of a slow variation of the potential energy, superposition of two and three states in q-SQUIDs with an ScS junction leads to reversible (nonhysteretic) dependence $\langle\Phi\rangle(\Phi_e)$, so that a q-SQUID can be treated as an ideal parametric frequency up-converter. The rate of diffusion of the wave packet in a time-dependent potential and the restriction on the maximum frequency of parametric conversion as a result of this process requires further examination. However, since for a stationary potential the diffusion time $t_D \approx \pi\hbar/E_{10}$, one can assume that the maximum frequency of parametric conversion will be higher for larger values of E_{10}.

As is shown in Ref. 26, the use of a phase-slip center as a weak link theoretically permits one to obtain still larger values of the level splitting. However, the creation of a qubit on such a system is problematic because of the large values of the quasiparticle component of the current. Since the technology of tunnel junctions is now well developed, to improve the characteristics of phase qubits through the shape of the potential barrier (3) one can insert two niobium-based tunnel junctions with equal critical currents into the loop. If the dimensions of the junctions are made small enough, e.g., 100×100 nm, the self-capacitance of two junctions can be of the order of 10^{-15} F. It is interesting that the proposed topological phase qubit with two niobium-based tunnel junctions is similar in construction to the charge qubit implemented in Ref. 27.

The theory of signal characteristics of rf SQUIDs in the nonhysteretic regime[23,24] with superposition of three states in the q-SQUID taken into account, that the results presented in Figs. 4 and 5 describe well the new effects observed in Ref. 1 with the use of the method of weak continuous measurements.[2,22] These new effects include: a Φ_0-periodic variation of the resonance frequency of the circuit in the "formally hysteretic" regime; doubling of the amplitude of the signal characteristics of the region of small amplitudes of excitation of I_T, and unusual shape of the current–voltage characteristic $V_T(I_T)$ at $\Phi_e = \Phi_0$.

We emphasize that in the experiments discussed in Ref. 1 and in the present paper, adiabatic conditions for a coherent superposition of macroscopic states, $hf_T \ll E_{10} \ll \Delta_0$, were fulfilled, as is necessary for constructing a sensor with a minimum back-action on the measured quantum object. The question of the dynamics of the wave packet in a time-dependent potential requires further analysis. Furthermore, it must be supposed that the q-SQUID was very well isolated from the external environment, since in a triple-well potential, energy relaxation did not occur in the motion along the metastable state E_0.

It follows from the above analysis of a q-SQUID with a double-well potential that the results of Ref. 21 can be explained by a special shape of the potential energy for the ScS junction (Figs. 1–3) without invoking unrealistically low values of the capacitance of the rf SQUID.

Interestingly, the three-level structure of the qubit proposed in Ref. 28, based on a Λ SQUID, is automatically realized for q-SQUID with a triple-well potential.

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Note Added in Proof

An interesting paper devoted to an analysis of a quantum detector based on a qubit with three Josephson junctions in a superconducting loop[20] was recently published by E. El'ichev and Ya. S. Greenberg (cond-mat/0608416). In comparing this detector with the q-SQUID based on an ScS junction,[1] it should be noted that keeping the cosinusoidal barrier shape and increasing the mass $(m \rightarrow 2m)$ in a three-junction q-SQUID will lead to a substantial decrease in the rate of quantum diffusion of the wave packet because of a decrease of E_{10} at identical values of the barrier height. The increase of the local curvature of the ground state at low values of E_{10} (see Fig. 2 above) in a detector based on a three-junction qubit cannot be regarded as an automatic improvement in sensitivity, since in a time-dependent potential the adiabatic conditions for tunneling will hold at much lower excitation frequencies $(f_T \ll E_{10}/h)$. This will lead to a limitation of the sensitivity by noise in the tank circuit and amplifier, with a simultaneous decrease of the speed of the q-SQUID. In our view, the most remarkable thing about a three-junction q-SQUID is that it can be used to construct a micro-q-SQUID (with dimensions of the quantization contour of 1×1 μm, for example) to improve the spatial resolution.

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