Temperature dependences of microwave-enhanced critical current in wide tin films

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(Submitted April 18, 2006)
Fiz. Nizk. Temp. **33**, 406–415 (April 2007)

The temperature dependences of the critical current of wide and narrow tin films irradiated with microwaves are investigated experimentally and analyzed. It is found that a high-frequency electromagnetic field stabilizes the current state of a wide film with respect to the entry of Abrikosov vortices into the film. The stabilizing effect of the irradiation increases with the radiation frequency. The fact that the enhancement of superconductivity is a general effect and is observed for uniform (narrow films) and nonuniform (wide films) distributions of the superconducting current over the film width made it possible to extend, in part, Éliashberg's theory to wide films. [DOI: 10.1063/1.2720075]

I. INTRODUCTION

The temperature dependence of the equilibrium critical current $I_c(T)$ of wide superconducting films in the absence of external magnetic and microwave fields has now been studied in detail. The main property of wide films that distinguishes them from narrow channels is that the transport current is distributed nonuniformly over the width of the film. This distribution is characterized by an increase of the current density near the edges of the film as a result of Meissner screening of the current-induced magnetic field. In this situation the mechanism of the suppression of superconductivity by the transport current is different from the typical mechanism for narrow channels (uniform Ginzburg–Landau depairing) and is associated with the vanishing of the edge barrier for entry of vortices into the film when the current density at the edges of a film reaches a value of the order of the Ginzburg–Landau depairing current density j_c^{GL}.[1–4]

It should be noted that even though the current state of a wide film outwardly appears to be similar to the Meissner state of a current-carrying bulk superconductor the two states are qualitatively different from one another. While the transport current in a bulk superconductor is concentrated in thin surface layer which thickness is of the order of the penetration London depth $\lambda(T)$, in a wide film the current is distributed over the entire width w of the film approximately as $[x(w-x)]^{-1/2}$, where x is the transverse coordinate.[3,4] Thus the characteristic length $\lambda_\perp(T)=2\lambda^2(T)/d$ (d is the thickness of the film), which in the theory of the current state of wide films is ordinarily called the penetration depth of the perpendicular magnetic field, actually determines not the spatial scale of the current decay away from the edges but rather the edge density of the current, playing the role of a "cutoff" factor in the above-indicated current distribution over distances x, $w-x\sim\lambda_\perp$ away from the edges of the film. Larkin and Ovchinnikov established that in films whose width is much greater than $\lambda_\perp(T)$ and the coherence length $\xi(T)$ the edge current density j_0 with total current I below the uniform depairing current I_c^{GL} reaches the value $j_0=I/d\sqrt{(\pi w\lambda_\perp)}$.[3] Using $j_0\approx j_c^{GL}$ as a qualitative estimate for the current density which suppresses the edge barrier, this result yields the relation

$$I_c(T)\approx j_c^{GL}(T)d\sqrt{\pi w\lambda_\perp(T)}$$

for the critical current of a wide film. This relation is often used for analyzing the experimental data (see, for example, Ref. 5) and yields a linear temperature dependence of the critical current near $T_c: I_c(T)\propto 1-T/T_c$. The quantitative theory proposed by Aslamazov and Lempitskiĭ in Ref. 4 for the resistive state of wide films also predicts a linear temperature dependence of I_c but it gives a critical current that is 1.5 times greater than the estimate of I_c given above. Recent experimental measurements of critical currents in wide films confirm this results.[6]

Since the parameters $\xi(T)$ and $\lambda_\perp(T)$ grow without bound as the temperature approaches the superconducting transition temperature, any film near T_c is in a narrow-channel regime and the critical current of the film exhibits the typical temperature dependence for the Ginzburg–Landau depairing current: $I_c^{GL}(T)\propto(1-T/T_c)^{3/2}$. A quantitative criterion for a transition between the wide- and narrow-film regimes can be found in Ref. 6, where it is shown that for $w<4\lambda_\perp(T)$ a superconducting film is a narrow channel and for $w>4\lambda_\perp(T)$ it becomes a wide film with a strongly nonuniform superconducting current distribution and vortices which appear when a subsequent transition into the resistive state occurs. However, as noted in Ref. 6, when a transition into the wide-film regime occurs the critical current remains proportional to $(1-T/T_c)^{3/2}$ in a quite wide temperature range though less in absolute value than the depairing current $I_c^{GL}(T)$ characteristic for a narrow channel, and the critical current becomes a linear function[4] only at sufficiently low temperatures where the film width is 10–20 times greater than $\lambda_\perp(T)$.

In Refs. 7 and 8 it is established that the critical current in long ($L\gg\xi(T)$, $\lambda_\perp(T)$, where L is the length of the film) and wide superconducting films increases (enhancement of superconductivity) when the films are exposed to an external microwave field. However, up to now no purposeful experimental studies of the enhanced critical current $I_c^P(T)$ in wide films have been performed. The objective of the present work is to fill this lacuna and to give an interpretation of the results even though there is no theory of superconductivity enhancement in wide films.

II. NONEQUILIBRIUM CRITICAL CURRENT OF SUPERCONDUCTING CHANNELS IN A MICROWAVE FIELD

The theory of superconductivity enhancement for narrow channels where the equilibrium energy gap Δ and the superconducting current density j_s are distributed uniformly over the cross section of the sample has been constructed by Éliashberg.[9–11] This theory is applicable for sufficiently narrow and thin $(w,d \ll \xi(T),\lambda_\perp(T))$ films, where the spatial distribution of the microwave power and correspondingly the enhanced gap is uniform over the cross section of the film. At the same time the scattering length l_i for electrons scattered by impurities must be short compared to the coherence length. According to this theory the influence of the microwave radiation on the energy gap Δ of a superconductor through which a constant transport current with density j_s flows is described by the equation

$$\frac{T_c-T}{T_c}-\frac{7\zeta(3)\Delta^2}{8(\pi kT_c)^2}-\frac{2kT_c\hbar}{\pi e^2 D\Delta^4 N^2(0)}j_s^2+\Phi(\Delta)=0, \qquad (1)$$

where T_c is the critical temperature, $N(0)$ is the density of states at the Fermi level, $D=v_F l_i/3$ is the diffusion coefficient, v_F is the Fermi velocity, and $\Phi(\Delta)$ is the nonequilibrium increment due to the nonequilibrium nature of the electron distribution function:[9,10,12]

$$\Phi(\Delta)=-\frac{\pi\alpha}{2kT_c}\left[1+0.11\frac{(\hbar\omega)^2}{\gamma kT_c}-\frac{(\hbar\omega)^2}{2\pi\gamma\Delta}\times\left(\ln\frac{8\Delta}{\hbar\omega}-1\right)\right], \quad \hbar\omega\ll\Delta. \qquad (2)$$

Here $\alpha=Dp_s^2/\hbar$ is proportional to the power P of the external radiation, p_s is the amplitude of the superfluid momentum excited by the radiation field, and $\gamma=\hbar/\tau_\varepsilon$ where τ_ε is the energy relaxation time.

Our method of investigating the enhancement effect is based on measuring the critical current and not the energy gap. The equations (1) and (2) yield an expression for the superconducting current density j_s as a function of the energy gap, temperature, and pump power:

$$j_s=\eta\Delta^2\left(\frac{T_c-T}{T_c}-\frac{7\zeta(3)\Delta^2}{8(\pi kT_c)^2}+\Phi(\Delta)\right)^{1/2}, \qquad (3a)$$

$$\eta=eN(0)\sqrt{\frac{\pi D}{2\hbar kT_c}}. \qquad (3b)$$

The condition $\partial j_s/\partial\Delta=0$ for the superconducting current to have an extremum at fixed temperature and power gives a transcendental equation for the gap width Δ_m for which j_s reaches its maximum value, i.e. the critical current:

$$\frac{T_c-T}{T_c}-\frac{21\zeta(3)\Delta_m^2}{(4\pi kT_c)^2}-\frac{\pi\alpha}{2kT_c}\left[1+0.11\frac{(\hbar\omega)^2}{\gamma kT_c}-\frac{(\hbar\omega)^2}{4\pi\gamma\Delta_m}\left(\frac{3}{2}\ln\frac{8\Delta_m}{\hbar\omega}-1\right)\right]=0. \qquad (4)$$

Substituting the solution Δ_m of Eq. (4) into the relation (3a) yields an expression for the critical current in a microwave field:[13]

$$I_c^p(T)=\eta dw\Delta_m^2\left\{\frac{T_c-T}{T_c}-\frac{7\zeta(3)\Delta_m^2}{8(\pi kT_c)^2}-\frac{\pi\alpha}{2kT_c}\times\left[1+0.11\frac{(\hbar\omega)^2}{\gamma kT_c}-\frac{(\hbar\omega)^2}{2\pi\gamma\Delta_m}\left(\ln\frac{8\Delta_m}{\hbar\omega}-1\right)\right]\right\}^{1/2}. \qquad (5)$$

In the absence of an external microwave field $(\alpha=0)$ the relation (5) becomes an expression for the equilibrium depairing current:

$$I_c(T)=\eta dw\Delta_m^2\left[\frac{T_c-T}{T_c}-\frac{7\zeta(3)\Delta_m^2}{8(\pi kT_c)^2}\right]^{1/2}. \qquad (6)$$

In this case $\Delta_m=\Delta_0\sqrt{(2/3)}$, where

$$\Delta_0(T)=\pi kT_c\sqrt{\frac{8(T_c-T)}{7\zeta(3)T_c}}=3.062kT_c\sqrt{1-\frac{T}{T_c}} \qquad (7)$$

is the equilibrium gap width in the absence of a transport current.

We note that when the expression (3b) with the density of states $N(0)=m^2v_F/\pi^2\hbar^3$, calculated in the free-electron model, is used in the relation (6) a large discrepancy obtains between the theoretical and experimentally measured values of the equilibrium critical current. This indicates that such an estimate is comparatively rough for the metal (tin) used for preparing the samples. At the same time, expressing the density of states in terms of an experimentally measured quantity – the square resistance of the film $R^{\square}=R_{4.2}w/L$, where $R_{4.2}$ is the total resistance of the film at $T=4.2$ K, we arrive at an expression for the quantity

$$\eta=(edR^{\square})^{-1}\sqrt{3\pi/2kT_c v_F l_i\hbar},$$

substituting which into the relation (6) gives good agreement between $I_c(T)$ given by Eq. (6) and both the experimental values of the equilibrium depairing current as well as the values calculated from the Ginzburg–Landau theory $I_c^{GL}(T)$ (see Eq. (8)). This expression for η will also be used in the relation (5) for the enhanced critical current when the latter is compared with the experimental results.

It is curious that up to now, as far as we know, no one has made a direct comparison of the temperature dependences of the enhanced critical current given by Eq. (5) and the experimental data for $I_c^P(T)$. However, we note that qualitative attempts at comparing the experimental dependences $I_c^P(T)$ with Éliashberg's theory were still made. For example, in Ref. 12 the Ginzburg–Landau depairing current was represented, using Eq. (7) for the equilibrium gap width, in the form

TABLE I. Parameters of film samples.

Sample	L, μm	w, μm	d, nm	$R_{4,2}$, Ω	$R^{\square}$, Ω	T_c, K	l_i, nm	R_{300}, Ω
SnW8	84	25	136	0.206	0.061	3.824	148	3.425
SnW10	88	7	181	0.487	0.040	3.809	169	9.156
SnW13	90	18	332	0.038	0.008	3.836	466	1.880

$$I_c^{GL}(T) = \frac{c\Phi_0 w}{6\sqrt{3}\pi^2\xi(0)\lambda_\perp(0)}(1 - T/T_c)^{3/2} = K_1\Delta_0^3(T), \quad (8)$$

where $\Phi_0=hc/2e$ is the flux quantum. Since the temperature dependence of the enhanced critical current in a narrow channel has been found to be close in form to the equilibrium dependence $I_c^P(T) \propto (1-T/T_c^P)^{3/2}$, where T_c^P is the temperature of the superconducting transition in a microwave field, the current $I_c^P(T)$ was approximated by an expression similar to Eq. (8):

$$I_c^P(T) = K_2\Delta_P^3(T), \quad (9)$$

where the enhanced energy gap width $\Delta_P(T)$ was calculated using Éliashberg's theory with a zero superconducting current (Eq. (1) with $j_s=0$). Then, assuming $K_1=K_2$ and using the microwave power as an adjustable parameter, the authors of Ref. 12 were able to attain agreement, to a definite degree of accuracy, between the computed and experimentally measured values of $I_c^P(T)$.

It is clear that such a comparison between the experimental results and Éliashberg's theory is only a qualitative approximation and cannot be used to obtain quantitative results. In the first place, the gap width in the zero-current regime ($j_s=0$), which is different from the gap width when a current is flowing, enters into the relations (8) and (9). In the second place, the depairing curves $j_s(\Delta)$ in the equilibrium state ($P=0$) and in the presence of a microwave field differ strongly from one another.[13] Finally, as shown in Refs. 11–13, as $T\to T_c^P$ the enhanced order parameter $\Delta_P(T)$ goes to a finite (though small) value $\Delta_P(T_c^P)=(l/2)\hbar\omega$ and vanishes abruptly when $T>T_c^P$, while the critical current vanishes continuously (not abruptly) as $T\to T_c^P$ and, therefore, cannot in general be satisfactorily described by a relation of the type (9) (the appreciable deviation of the relation (9) from the experimental points close to T_c also attests to this). In the present work the experimental data will be analyzed on the basis of the exact relation (5) using a numerical solution of Eq. (4).

III. EXPERIMENTAL RESULTS

We investigated superconducting tin thin films prepared using an innovative technology[6] which made it possible to reduce to a minimum the number of defects at the edge and in the interior volume of the film. The critical current of such samples is determined by the suppression of the barrier for vortex entry when the current density at the edge of the film reaches a value of the order of j_c^{GL} and reaches the maximum possible theoretical value,[4] which attests to the absence of edge defects which lower the barrier locally and thereby lower the current I_c. To measure the current–voltage characteristics (IVCs) the samples were placed in a double screen made of annealed permalloy. The parameters of some of the experimental films are presented in Table I; the generally accepted values of v_F and the inelastic relaxation time in tin at T=4.2 K were used: $v_F=6.5\cdot 10^7$ cm/s and $\tau_\varepsilon=4\cdot 10^{-10}$ s.

Figure 1 shows the IVC of one of the experimental samples. The film resistivity, due to the motion of the Abrikosov-vortex lattice, exists in the current range $I_c<I<I_m$ (vortex section of the IVC), where I_m is the maximum current at which a vortex state exists.[4,6] Voltage steps due to the appearance of phase-slip lines appear on the IVC above I_m.

Figure 2 displays the experimentally measured temperature dependences of the critical current for sample SnW10. We shall examine first the behavior of $I_c(T)$ in the absence of an external electromagnetic field (Fig. 2(■)). The width of the SnW10 film is comparatively small (w=7.3 μm), so that in the temperature range $T_{\text{cros1}}<T<T_c$ sufficiently close to T_c the sample behaves like a narrow channel and the critical current equals the Ginsburg–Landau depairing current $I_c^{GL} \propto (1-T/T_c)^{3/2}$. The cross-over temperature T_{cros1}=3.769 K corresponds to a transition into the wide-film regime: a vortex section is observed on the IVC at $T<T_{\text{cros1}}$. The temperature dependence $I_c(T)$ for $T<T_{\text{cros1}}$ at first remains of the form $(1-T/T_c)^{3/2}$, though the value of $I_c(T)$ is found to be less than the depairing current $I_c^{GL}(T)$ because a nonuniform distribution of the current density appears and decreases away from the edge of the film. Finally, for $T<T_{\text{cros2}}$ =3.717 K the temperature dependence of the critical current becomes linear $I_c(T)=I_c^{AL}(T)=9.12\cdot 10^1(1-T/T_c)$ mA, which

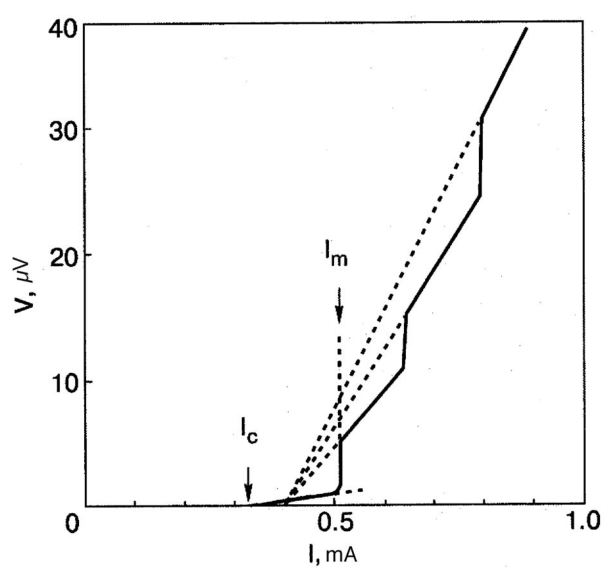

FIG. 1. Current-voltage characteristic of a wide superconducting film SnW13 at temperature 3.798 K.

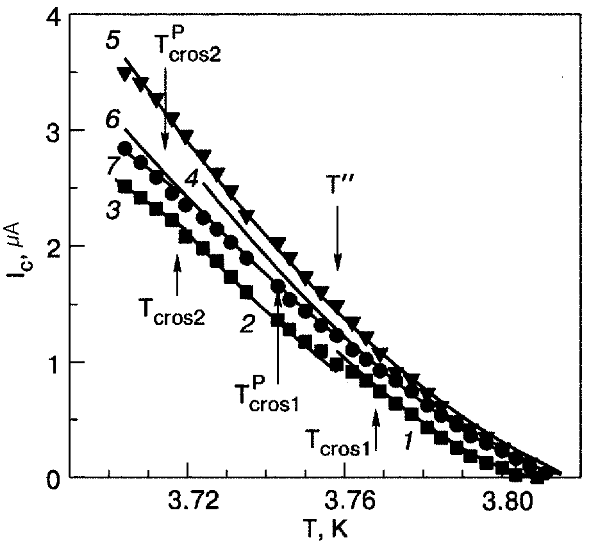

FIG. 2. Experimental temperature dependences of critical currents $I_c(P=0)$(■), $I_c(f=9.2\ \mathrm{GHz})$(•), and $I_c(f=12.9\ \mathrm{GHz})$(▼) of the sample SnW10. The theoretical dependence $I_c^{GL}(T)=7.07\cdot 10^2(1-T/T_c)^{3/2}$ mA (see Eq. (8)) (curve 1); computed dependence $I_c(T)=5.9\cdot 10^2(1-T/T_c)^{3/2}$ mA (curve 2); theoretical dependence $I_c^{AL}(T)=9.12\cdot 10^1(1-T/T_c)$ mA (Ref. 4) (straight line 3); theoretical dependence $I_c(f=9.2\ \mathrm{GHz})$ calculated using the relation (5) and the fitting function $I_c(T)=6.5\cdot 10^2(1-T/3.818)^{3/2}$ mA (curve 4); theoretical dependence $I_c(f=12.9\ \mathrm{GHz})$ calculated using the relation (5) and the fitting function $I_c(T)=6.7\cdot 10^2(1-T/3.822)^{3/2}$ mA (curve 5); the theoretical dependence $I_c(f=9.2\ \mathrm{GHz})$ calculated using the relation (5), normalized to the curve 2, and the fitting function $I_c(T)=5.9\cdot 10^2(1-T/3.818)^{3/2}$ mA (curve 6); computed dependence $I_c(T)=9.4\cdot 10^1(1-T/3.818)$ mA (straight line 7).

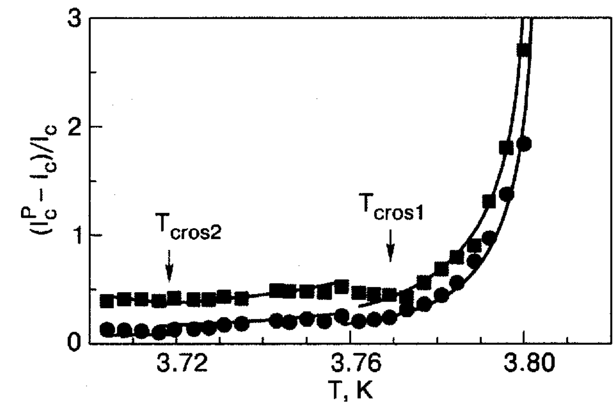

FIG. 3. Temperature dependences of the largest relative increment to the critical current of the sample SnW10 at irradiation frequencies 9.2 GHz (•) and 12.9 GHz (■).

corresponds to the Aslamazov–Lempitskiĭ theory.[4]

For measurements in a microwave field the power of the radiation was chosen from the condition that the critical current reaches is maximum value $I_c^P(T)$. We shall examine the behavior of $I_c^P(T)$ for the sample SnW10 in a field with frequency $f=9.2$ GHz (Fig. 2(•)). In the temperature range $T_{\mathrm{cros1}}^P(9.2\ \mathrm{GHz})<T<T_c^P(9.2\ \mathrm{GHz})$ $(T_{\mathrm{cros1}}^P(9.2\ \mathrm{GHz})=3.744$ K and $T_c^P(9.2\ \mathrm{GHz})=3.818$ K) the IVC has no vortex section, i.e. the sample behaves as a narrow channel. We note that $T_{\mathrm{cros1}}^P(9.2\ \mathrm{GHz})<T_{\mathrm{cros1}}(P=0)$, while $T_c<T_c^P$, i.e., under the optimal enhancement conditions the narrow-channel regime remains in a wider temperature range. In the range $T_{\mathrm{cros1}}<T<T_c$, where the sample in the absence of a microwave field is a narrow channel (and even at somewhat lower temperatures $T>3.760$ K) the experimental values of $I_c^P(T)$ (Fig. 2(•)) agree quite well with the values calculated using Eq. (5) (Fig. 2, curve 4), where α serves an adjustable parameter.[1)] However, for $T<3.760$ K the experimental points fall below the computed curve 4 and, finally, for $T<T_{\mathrm{cros2}}^P(9.2\ \mathrm{GHz})=3.717$ K the temperature dependence of the critical current becomes linear (Fig. 2, straight line 7).

Figure 2 also shows the temperature dependence of the highest critical current of the sample SnW10 at irradiation frequency 12.9 GHz (Fig. 2 (▼)). It is evident that, just as in a narrow channel, as the irradiation frequency increases, the highest critical current also increases. We note that at this frequency of the electromagnetic field there is no vortex section on the IVC in our entire experimental temperature interval (down to $T=3.700$ K and even somewhat lower). In other words, in the temperature range $T_{\mathrm{cros1}}^P(12.9\ \mathrm{GHz})<T<T_c^P(12.9\ \mathrm{GHz})(T_{\mathrm{cros1}}^P(12.9\ \mathrm{GHz})<3.700$ K and is not shown in Fig. 2, $T_c^P(12.9\ \mathrm{GHz})=3.822$ K) the sample behaves as a narrow channel. We note that $T_{\mathrm{cros1}}^P(12.9\ \mathrm{GHz})<T_{\mathrm{cros1}}^P(9.2\ \mathrm{GHz})<T_{\mathrm{cros1}}(P=0)$ while $T<T_c^P(9.2\ \mathrm{GHz})<T_c^P(12.9\ \mathrm{GHz})$. Thus, under optimal enhancement conditions the temperature interval of the narrow-channel regime increases with the irradiation frequency.

It is also important to note that the experimental dependence $I_c^P(T)$ for $f=12.9$ GHz agrees well with the critical current calculated using Eq. (5) for a narrow channel (Fig. 2, curve 5), in our entire experimental temperature range and is fit well by the function $I_c(T)=6.7\cdot 10^2(1-T/3.822)^{3/2}$ mA. Hence it follows that the temperature of the transition into the wide-film regime T_{cros1}^P, where a vortex section appears on the IVC and the temperature T^{**} at which the experimental curve $I_c^P(T)$ deviates from the curve calculated using Eq. (5) decrease with increasing irradiation frequency.

Figure 3 displays the temperature dependences of the reduced excess of the highest critical current $[I_c^P(T)-I_c(T)]/I_c(T)=\Delta I_c^P(T)/I_c(T)$ of the sample Sn10 for two microwave irradiation frequencies. It is evident that as the irradiation frequency increases, the ratio $\Delta I_c^P(T)/I_c(T)$ increases in our entire experimental temperature range. Near T_c the functions $\Delta I_c^P(T)/I_c(T)$ are very similar to the analogous functions which are characteristic for narrow channels. This is unsurprising, since in the temperature range $T_{\mathrm{cros1}}<T<T_c$ this sample is indeed a narrow channel. However, at lower temperatures the function $\Delta I_c^P(T)/I_c(T)$ becomes nonmonotonic, which has not been observed in narrow channels for these irradiation frequencies.[14]

Figure 4 displays the temperature dependences of I_c for the sample SnW8. We shall examine first the behavior of $I_c(T)$ in the absence of an external electromagnetic field. The width of this film is quite large ($w=25\ \mu$m), so that such a sample is a narrow channel only very close to T_c and $T<T_{\mathrm{cros1}}=3.808$ K it behaves as a wide film. For $T_{\mathrm{cros2}}=3.740\ \mathrm{K}<T<T_{\mathrm{cros1}}$ the temperature dependence of the critical current is of the form $(1-T/T_c)^{3/2}$, even though $I_c<I_c^{GL}$. For $T<T_{\mathrm{cros2}}$ the temperature dependence of the critical current becomes linear and corresponds to the Aslamazov–Lempitskiĭ theory:[4] $I_c(T)I_c^{AL}(T)=1.47\cdot 10^2(1-T/T_c)$ mA.

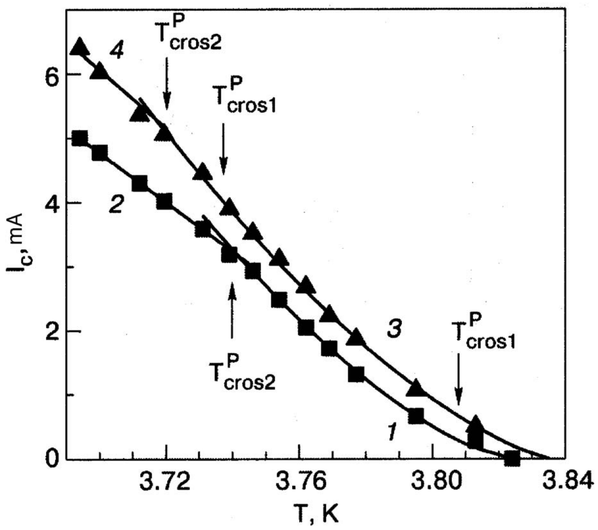

FIG. 4. Experimental temperature dependences of the critical currents $I_c(P=0)$(■), $I_c(f=15.2\text{ GHz})$(▲) of sample SnW8: computed dependence $I_c(T)=1.0\cdot 10^3(1-T/T_c)^{3/2}$ mA (curve 1); theoretical relation $I_c^{AL}(T)=1.47\cdot 10^2(1-T/T_c)$ mA (Ref. 7) (straight line 2); theoretical dependence $I_c(f=15.2\text{ GHz})$, calculated using the relation (5), normalized to the curve 1, and the fitting function $I_c(T)=1.0\cdot 10^3(1-T/3.835)^{3/2}$ mA (curve 3); computed dependence $I_c(T)=1.72\cdot 10^2(1-T/3.835)$ mA (straight line 4).

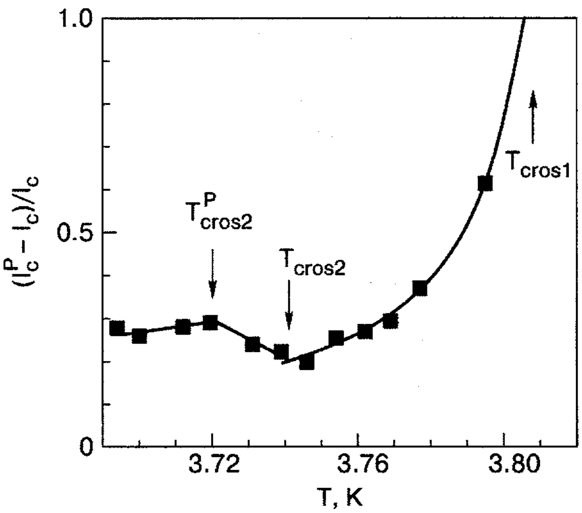

FIG. 5. Temperature dependence of the reduced increment to the highest critical current I_c^P above $I_c(P=0)$ of the sample SnW8 at irradiation frequency 15.2 GHz.

In a 15.2 GHz microwave field and the temperature range $T_{\text{cros2}}^P=3.720\text{ K}<T<T_c^P=3.835$ K the relation (5) describes well the temperature dependence of the highest critical current with an additional factor ensuring that for $P=0$ it agrees with the measured value of the equilibrium critical current $I_c(T)=1.0\cdot 10^3(1-T/T_c)^{3/2}$ mA (Fig. 4, curve 3). Physically, this factor is a form factor which qualitatively takes account of the nonuniformity of the current distribution over the width of the film. In this temperature range the critical current can be described by the function $I_c(T)=1.0\cdot 10^3(1-T/3.835)^{3/2}$ mA. For $T<T_{\text{cros2}}^P$ temperature dependence $I_c^P(T)$ becomes linear (Fig. 4, straight line 4). It follows from Fig. 4 that for microwave-enhanced superconductivity even a quite wide film behaves like a narrow channel down to lower temperatures than in the absence of irradiation ($T_{\text{cros1}}^P<T_{\text{cros1}}$); there is no vortex section on the IVC in this temperature range.

It is also important to note that the quantity $\Delta I_c^P(T)$ in the temperature range $T_{\text{cros2}}<T<T_c$ is much smaller than for $T<T_{\text{cros2}}^P$. Figure 5 shows the temperature dependence of the ratio $\Delta I_c^P(T)/I_c(T)$ for the sample SnW8. It is evident that as the temperature decreases, this ratio initially decreases, just as happened for a narrow channel. However, at still lower temperatures in the range $T_{\text{cros2}}^P<T<T_{\text{cros2}}$ the ratio $\Delta I_c^P(T)/I_c(T)$ increases, which was not observed, as a matter of principle, in narrow channels at such irradiation frequencies. As the temperature drops below T_{cros2}^P the ratio $\Delta I_c^P(T)/I_c(T)$ is once again observed to decrease. But this effect is much stronger in the wide SnW8 film than in the narrower SnW10 film.

IV. DISCUSSION

Our experimental investigations of the temperature dependences of the critical current in films in a microwave field revealed that with the enhancement of superconductivity the temperature range near T_c where the film behaves as a narrow channel becomes wider ($T_{\text{cros1}}^P<T_{\text{cros1}}, T_c^P>T_c^P$), and this effect becomes stronger as the irradiation frequency increases. One possible reason for this observation is deep modulation of the constant current by the high-frequency current, since estimates show that they could be comparable in magnitude.

It seems to us that such behavior of films in a microwave field is associated with the stabilizing effect of the radiation on the vortices which are located at the edge of a film. Since our experimental samples were small compared with the wavelength of the electromagnetic field (the sample length $\sim 10^{-4}$ m and the shortest wavelength $\sim 10^{-2}$ m), we are actually dealing with a variable high-frequency current flowing through the sample $I_f\propto\sqrt{P}$ and this current keeps vortices from leaving the edge of the film. It is important to note that the relative power P/P_c at which the highest enhanced current $I_c^P(T)$ is observed is quite high ($P/P_c\sim 0.1-0.2$), and this ratio increases with the irradiation frequency.[8] As far as the value of the critical power is concerned, it has been shown[15,16] that as the irradiation frequency increases right up to the frequency $f_\Delta\approx(1-T/T_c)^{1/2}/2.4\pi\tau_\varepsilon$, which is the reciprocal of the relaxation time of the gap width, P_c is observed to increase and remain virtually constant as the frequency increases to higher values. For our samples $f_\Delta<10^8$ Hz in the experimental temperature range. In the regime where superconductivity is enahnced the microwave irradiation frequency $f>f_\Delta$.

Consequently, as the irradiation frequency increases $P_c=\text{const}$; the ratio P/P_c increases because the absolute power at which the enhanced current $I_c^P(T)$ reaches its maximum value increases. Thus, as the irradiation frequency increases, the absolute value of the power at which optimal enhancement of superconductivity and, therefore, a stabilizing effect of the field on the vortices occur increases.

In summary, turning to Fig. 2, the following can be asserted. At irradiation frequency $f=12.9$ GHz the highest value of $I_c^P(T)$ for sample SnW10 obtains at a high power of the external microwave field, which prevents the motion of vortices, so that this sample behaves like a narrow channel at temperatures from the critical value to $T<3.700$ K. The curve 5 in Fig. 2, constructed using Eq. (5) of Éliashberg's theory for a narrow channel and giving the Ginzburg–Landau depairing current density at $P=0$, agrees well with the experimental curve $I_c^P(T)$ (Fig. 2(▼)). On this basis it can be asserted that in the present case, as a result of the action of the microwave field, the sample becomes a narrow channel according to the absence of a vortex section on the IVC and the complete agreement of $I_c^P(T)$ with the Eq. (5) of Éliashberg's theory, which presumes that the superconducting current distribution over the cross section of the sample is uniform.

For the reasons presented above, when the radiation frequency is decreased ($f=9.2$ GHz), the power at which $I_c^P(T)$ reaches its maximum value and hence the irradiation has a stabilizing effect decreases. This results in a smaller decrease of T_{cros1}^P relative to T_{cros1}. It is important to note that even in this case the experimental curve $I_c^P(T)$ (Fig. 2(•)) agrees quite well with curve 4 in Fig. 2, constructed according to Eq. (5) for a narrow channel, right up to $T^{**}=3.760\ \mathrm{K}<T_{\mathrm{cros1}}=3.769$ K. In the temperature range $T_{\mathrm{cros1}}^P<T<T^{**}$ there is no vortex section on the IVC for the sample SnW10, but $I_c^P(T)$ deviates downward from the theoretical curve 4 in Fig. 2, constructed for a narrow channel and normalized so that to give the Ginzburg–Landau depairing current at $P=0$.

According to Eqs. (6) and (7), in Éliashberg's theory the expression for the micro wave-enhanced current becomes at $P=0$ the expression for the Ginzburg–Landau depairing current. Just as the entire theory, this is valid only for a narrow channel. At the same time, at temperatures below T_{cros1} the SnW10 film acts like a wide film (a vortex section appears on the IVC), the superconducting current distribution over its cross section becomes nonuniform, and the critical current $I_c(T)=5.9\cdot 10^2(1-T/T_c)^{3/2}$ mA of such a film at $P=0$ is less than the depairing current $I_c^{GL}(T)=7.07\cdot 10^2(1-T/T_c)^{3/2}$ mA, though the temperature dependence remains unchanged. In addition, it turns out that if a normalization factor is introduced into Eq. (5) in a manner so that at $P=0$ it gives not $I_c^{GL}(T)$ but $I_c(T)$, then this relation can be used to construct a curve (see Fig. 2, curve 6) which agrees well with the experimental temperature dependence $I_c^P(T)$. We note that it is possible to introduce such a universal normalization factor in the entire temperature range $T_{\mathrm{cros2}}<T<T_{\mathrm{cros1}}$ because the temperature dependence of I_c^P described by Eq. (5) is, despite its complexity, numerically very close to the law $\propto(1-T/T_c^P)^{3/2}$, which at $P=0$ becomes $I_c(T)\propto(1-T/T_c)^{3/2}$ for a wide film.

A similar situation is also observed for the wider SnW8 film. This sample is a narrow channel only very close to T_c. Consequently, for temperatures $T<T_{\mathrm{cros1}}=3.808$ K Eq. (5) gives values for the enhanced critical current which do not agree with the experimentally obtained values of $I_c^P(T)$. However, good agreement between theory and experiment is once again observed when Eq. (5) is normalized to the equilibrium critical current $I_c(T)=1\cdot 10^3(1-T/T_c)^{3/2}$ mA at $P=0$ (Fig. 4, curve 3).

We arrive at the conclusion that if the temperature dependence of the equilibrium critical current ($P=0$) of wide film is $I_c(T)\propto(1-T/T_c)^{3/2}$, characteristic for a narrow channel, then Éliashberg's formula normalized to $I_c(T)$ describes well the experimentally measured curves of the enhanced critical current $I_c^P(T)$, which numerically are very close to $(1-T/T_c^P)^{3/2}$. At temperatures $T<T_{\mathrm{cros2}}^P$, where the temperature dependence of the critical current of a wide film is linear $(I_c(T)\propto 1-T/T_c)$, the temperature dependence of the enhanced critical current is also linear: $I_c^P(T)\propto 1-T/T_c^P$. These facts indirectly confirm the conjecture that the mechanism of the superconductivity enhancement in wide films is that same as in narrow films.

There are additional considerations that can be given in support of the analogy between the enhancement mechanisms in narrow channels and wide films. As already stated in the Introduction, even though the current density increases near the edges of a wide film, the main current – transport and microwave-induced – is distributed over the entire width of the film. Thus, the nonequilibrium state of the quasiparticles in a wide film, just as in a narrow channel, is excited by the microwave field in the entire volume of the superconductor and, therefore, the enhancement effect in wide films is only slightly modified quantitatively as a result of the nonuniformity of the current distribution. In this connection we underscore the large difference in the conditions under which nonequilibrium is produced by a microwave field in wide films and bulk superconductors, where the enhancement effect still has not been observed. In the latter case the total current is concentrated in a thin Meissner layer of thickness $\lambda(T)$ near the surface of the metal, which results in the appearance of an additional relaxation mechanism – spatial diffusion of nonequilibrium quasiparticles, excited by the microwave field, away from the surface into the equilibrium volume. The strength of this mechanism is determined by the quasiparticle diffusion time $\tau_D(T)=\lambda^2(T)/D$ from the Meissner layer, which for typical temperatures turns out to be three or four orders of magnitude less than the inelastic relaxation time. Such a high efficiency of the diffusion mechanism of relaxation is probably the reason why the enhancement effect is suppressed in bulk superconducting samples.

In closing, we shall discuss the results of Ref. 12, where the temperature dependences of the enhanced critical current in aluminum films were investigated. As temperature decreased in some range $T^{**}<T<T_c$ quite good agreement was observed between the experimental values of the critical current $I_c^P(T)$ and the theoretical dependence calculated using Éliashberg's theory. However, for $T<T^{**}$ the authors of Ref. 12 observed a downward deviation of the experimental values of the critical current from the theoretical curve for $I_c^P(T)$ obtained from Éliashberg's theory. Interpreting their samples as narrow channels the authors attempted to attribute this discrepancy between theory and experiment to the different level of absorption of the microwave field as the enhanced gap width changed. From the standpoint of our approach the crux of this discrepancy is as follows. A detailed analysis of the results obtained in Ref. 12 shows that the deviation of the experimental values of the enhanced critical current of the aluminum sample ALT04 from Éliahsberg's theory starts at $T<T^{**}=1.157$ K, and the quantity $4\lambda_\perp(T)$ at this temperature is already comparable to the width of the sample. Therefore the deviation of the experimental values of the critical current from the predictions of the theory is due to the fact

that at $T < 1.157$ K the ALT04 sample behaves as a wide film with a nonuniform distribution of the superconducting current; this is the reason for the deviation from the theory constructed for a narrow channel with a uniform distribution of the superconducting current.

V. CONCLUSIONS

In summary, the following conclusion can be drawn from the results obtained. The mechanism of the superconductivity enhancement by a microwave field is the same in narrow channels and wide films – it is Éliashberg's mechanism. However, in a wide film the distribution of the superconducting current over the cross section of the sample differs from the uniform distribution in a narrow channel, which, of course, requires an elaboration of the theory as whole and the final equations in particular.

The main results of the present work can be formulated as follows:

1. It was found that when superconductivity is enhanced by a microwave field not only does the critical temperature increase $T_c^P > T_c$ but the temperature of the transition into the wide-film regime with a vortex section on the IVC decreases, $T_{\text{cros1}}^P < T_{\text{cros1}}$. As a result, the temperature range $T_{\text{cros1}}^P < T < T_c^P$ where the film behaves as a narrow channel becomes wider, i.e. irradiation with a high-frequency field stabilizes the current state of the film with respect to the entry of Abrikosov vortices into the film.
2. As the irradiation frequency increases, the temperature T_{cros1}^P below which the film starts to behave as a wide film (a vortex section appears on the IVC) decreases and therefore the stabilizing effect of the irradiation increases with frequency. The additional factor is probably the fact that the optimal pump power which maximizes the critical current increases with frequency.
3. In the narrow-channel regime the relation (5) obtained from Éliashberg's theory for a uniform current distribution describes well the experimentally measured temperature dependence of the critical current $I_c^P(T)$. This relation turns out to be numerically very close to the law $(1 - T/T_c^P)^{3/2}$ for the equilibrium critical depairing current with enhanced critical temperature T_c^P replacing T_c.
4. In the wide-film regime $(T < T_{\text{cros1}}, w > 4\lambda_\perp(T))$, where the current distribution in the film becomes nonuniform, the experimental temperature dependence $I_c^P(T)$ can be described well by Eq. (5) (or a function of the form $(1 - T/T_c^P)^{3/2}$ can used to obtain a good fit) if a numerical normalization factor is introduced into the relation to ensure agreement with the measured values of the equilibrium critical current at $P=0$. This dependence extends over the temperature range right up to the point T_{cros2}^P of the transition to a linear temperature dependence $I_c^P(T) \propto 1 - T/T_c^P$, following from the theory for very wide films $(w \gg \lambda_\perp(T))$ with T_c^P replacing T_c. Just as T_{cros1}^P, the crossover temperature T_{cros2}^P decreases with increasing frequency as compared with its value T_{cros2} in the absence of irradiation.

We thank T. V. Salenkova for preparing the samples and E. V. Khristenko for a helpful discussion of this work.

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[1]In calculating the maximum critical current the quantity α in Eq. (5), proportional to the radiation power, must be determined from the condition for $I_c^P(T)$ to have a maximum at a prescribed temperature. As a result, it should, generally speaking, depend on T. However, in practice, the optimal enhancement power is constant to within several percent in the entire temperature range, so that α can be approximated by a constant for the present sample and radiation frequency.

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Translated by M. E. Alferieff