Impact of the transport supercurrent on the density of states in the weak link

S. N. Shevchenko*

B. Verkin Institute for Low Temperature Physics and Engineering, 47 Lenin Avenue, Kharkov 61103, Ukraine

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The impact of the transport supercurrent on the quasiparticle density of states in a superconducting thin film with a weak link is investigated. At the weak link the order parameter is locally suppressed due to the order parameter phase difference ϕ. This results in the appearance of zero-energy states, which are strongly influenced by the supercurrent especially at ϕ close to π. The subgap density of states, dependent on both the supercurrent and the phase difference, is shown to modify the conductance characteristics of the structure.

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The properties of a mesoscopic superconductor are defined by its local density of states (LDOS). At the interface the LDOS can be probed with tunneling spectroscopy.[1] Particularly interesting are situations when there are states (so-called midgap states) within the bulk amplitude value of the order parameter Δ_0, which is at the quasiparticle energy $\varepsilon < \Delta_0$. This situation is ubiquitous in unconventional superconductors, where an interface results in the local suppression of the order parameter (due to its anisotropy) and in the appearance of zero-energy states.[2] These zero-energy states are responsible for several interesting phenomena. First, when there is either external magnetic field or supercurrent injected, these midgap states create a countercurrent, which flows counter to the diamagnetic current in the former case, leading to a paramagnetic response,[3,4] or counter to the injected supercurrent in the latter case.[5] Second, when the LDOS is probed with tunneling spectroscopy, they result in the zero-bias conductance peak.[3,6]

A weak link between two banks of conventional superconductors is similar to an interface of an unconventional superconductor in this context of local suppression of the order parameter[7] and of the creation of midgap states.[8] In recent years the impact of the supercurrent on the density of states, and in particular on the midgap states, has attracted renewed attention (see, e.g., Refs. 9–12). In Ref. 13 we studied the first of the above-mentioned effects in the weak link between current-carrying superconductors. In this paper we address the second problem of the appearance of the midgap states at the weak link between two current-carrying conventional superconductors. This subject is interesting both for further understanding of interference effects in mesoscopic superconducting structures and for possible applications for electronic devices, in which the current can be strongly controlled either electrodynamically (creating an order parameter phase difference) or thermodynamically (injecting transport supercurrent). In what follows we describe the model system to study the midgap states at the weak link between two current-carrying conventional superconductors.

We consider a superconducting film with an impenetrable partition along the y axis, shown with the thick line in Fig. 1. This partition (which can be, e.g, a scratch in the film) has a break, which plays the role of the weak link in the form of a slit between two superconducting banks S_L and S_R. The size of the weak link is assumed to be smaller than the coherence length (so-called pinhole model). The order parameter phase difference ϕ at the weak link is created by the magnetic flux Φ_e which pierces the loop connecting two banks. The magnetic flux can be created, e.g, via inductive coupling to a current-carrying coil. The supercurrent, tangential to the boundary between the banks S_L and S_R, is passed through the contacts, as in Ref. 12. The LDOS is assumed to be probed by the tip of a scanning tunneling microscope (STM) placed over the weak link. Another contact for the STM is not shown in Fig. 1 and we refer the reader to Ref. 12 where both the current injection and the STM measurement as in Fig. 1 are discussed in detail.

Thus, we consider a perfect point contact between two clean current-carrying singlet superconductors. The order parameter phase difference ϕ is assumed to drop at the contact plane at $x=0$. The homogeneous supercurrent flows in the banks of the contact along the y axis, parallel to the boundary. The sample is assumed to be smaller than the London penetration depth so that the externally injected transport supercurrent can indeed be treated as homogeneous far from the weak link. Such a system can be quantitatively described by the Eilenberger equations. Taking transport supercurrent into account leads to a Doppler shift of the energy variable by $\mathbf{p}_F\mathbf{v}_s$, where $\mathbf{p}_F$ is the Fermi momentum and $\mathbf{v}_s$ is the superfluid velocity. The standard procedure of matching the solutions of the bulk Eilenberger equations at the boundary gives the Matsubara Green's function $\hat{G}_{(0)}(\omega)$ at the contact at $x=0$.[5] The analytic continuation $(\omega \to -i\varepsilon+\gamma)$ of the component $G_{(0)}^{11}(\omega) \equiv g^{(0)}(\omega)$ of $\hat{G}_{(0)}(\omega)$ gives the retarded Green's function, which defines the LDOS at the boundary:

$$N(\varepsilon) = \langle N(\varepsilon, \mathbf{p}_F) \rangle = N_0 \langle \mathrm{Re}\, g^{(0)}(\varepsilon) \rangle, \tag{1}$$

$$g^{(0)}(\varepsilon) = g^{(0)}(\omega = -i\varepsilon + \gamma), \tag{2}$$

$$g^{(0)}(\omega) = \frac{\tilde{\omega}(\Omega_L + \Omega_R) - i\, \mathrm{sgn}(v_x)\Delta_L \Delta_R \sin\phi}{\Omega_L \Omega_R + \tilde{\omega}^2 + \Delta_L \Delta_R \cos\phi}; \tag{3}$$

here $\omega = \pi T(2n+1)$ are Matsubara frequencies, γ is the relaxation rate in the excitation spectrum of the superconductor, $N(\varepsilon, \mathbf{p}_F)$ is the angle-resolved DOS, N_0 is the density of states at the Fermi level, $\langle \cdots \rangle$ denotes averaging over the directions of the Fermi momentum $\mathbf{p}_F$, $\Delta_{L,R}$ stands for the order parameter in the left (right) bank, $\tilde{\omega} = \omega + i\mathbf{p}_F\mathbf{v}_s$, and $\Omega_{L,R} = \sqrt{\tilde{\omega}^2 + \Delta_{L,R}^2}$. The poles of the retarded Green's function $g^{(0)}(\varepsilon)$ define the energy of the interface bound states. The

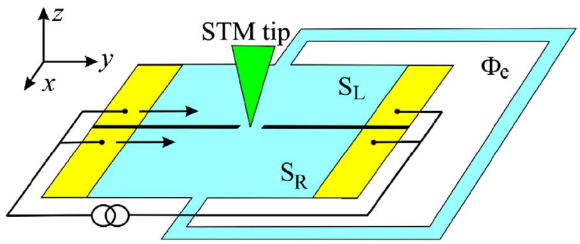

FIG. 1. (Color online) Scheme for probing LDOS at the point contact between superconducting banks S_L and S_R with an externally controlled order parameter phase difference $\phi=2\pi\Phi_e/\Phi_0$ and externally injected supercurrent in parallel to the boundary between S_L and S_R.

direction-dependent Doppler shift $\mathbf{p}_F\mathbf{v}_s$ results in the modification of the LDOS as it is discussed below.

The method of tunneling spectroscopy allows one to access the LDOS by measuring the tunneling conductance G of a superconductor–insulating barrier–normal metal (STM tip) structure. At zero temperature the dependence of the conductance on the bias voltage V is given by[1]

$$G(eV)=G_N\langle D(\mathbf{p}_F)N(eV,\mathbf{p}_F)\rangle,$$

where G_N is the conductance in the normal state; $D(\mathbf{p}_F)$ is the angle-dependent superconductor–insulator–normal metal barrier transmission probability. The barrier can be modeled, e.g, as in Ref. 3 with the uniform probability within the acceptance cone $|\vartheta|<\vartheta_c$, where ϑ is the polar angle and a small value of ϑ_c describes a thick tunneling barrier. In this paper for simplicity we do not take into account the angle dependence of the transmission probability, assuming that the conductance is proportional to the LDOS: $G(eV)\propto N(\varepsilon=eV)$.

In the case of a weak link in the form of a constriction between two conventional superconductors with $\Delta_L=\Delta_R=\Delta_0=\Delta_0(T,\mathbf{v}_s)$,[13] Eq. (3) can be rewritten

$$g^{(0)}(\omega)=\frac{\tilde{\omega}\Omega-i\frac{1}{2}\,\mathrm{sgn}(v_x)\Delta_0^2\sin\phi}{\tilde{\omega}^2+\Delta_\phi^2}, \tag{4}$$

where $\Delta_\phi=\Delta_0|\cos(\phi/2)|$ is the locally suppressed order parameter (so-called proximity gap).[7] The retarded Green's function, determined with Eqs. (2) and (4), gives for the energy of the Andreev bound states $\varepsilon_A=\pm\Delta_\phi-\mathbf{p}_F\mathbf{v}_s$. Thus, there are the zero-energy states, which are characterized by the values ϕ and v_s. The observation of the conductance characteristics of the system can be proposed as a test of the interface-induced transport-current-dependent quasiparticle states.[3,8,13]

For the sake of generality we also write down below the Green's function for the current-carrying d-wave superconductor. This can be easily done in the case of the specular reflection at the boundary, when the boundary between the current-carrying d-wave superconductor and the insulator can be modeled as the contact between two superconductors with the order parameters given by $\Delta_L=\Delta(\vartheta)$ and $\Delta_R=\Delta(-\vartheta)\equiv\bar{\Delta}$ and with $\phi=0$. Then from Eq. (3) we have the following expression, which together with Eq. (1) describes the LDOS in the current-carrying d-wave film as, e.g, in Refs. 3 and 12:

$$g^{(0)}(\omega)=\frac{\tilde{\omega}(\Omega+\bar{\Omega})}{\Omega\bar{\Omega}+\tilde{\omega}^2+\Delta\bar{\Delta}}, \tag{5}$$

where $\Omega=\sqrt{\tilde{\omega}^2+\Delta^2}$ and $\bar{\Omega}=\sqrt{\tilde{\omega}^2+\bar{\Delta}^2}$. This expression is valid for any relative angle χ between the a axis and the normal to the boundary; in particular, at $\chi=0$ we have: $g^{(0)}(\omega)=\tilde{\omega}/\Omega$, and at $\chi=\pi/4$ we have $g^{(0)}(\omega)=\Omega/\tilde{\omega}$. Analogously Eq. (3) can be used to describe the LDOS at the boundary between two current-carrying d-wave superconductors as in Fig. 1 (see also Ref. 5).

In what follows we consider the LDOS at the point contact between current-carrying conventional superconductors in details first analytically at $\gamma=0$ and then numerically for $\gamma\neq 0$.

In the absence of impurities, that is, in the limit $\gamma\rightarrow+0$, we obtain the LDOS depending on the energy ε and on the parameters $Q=p_Fv_s$ and ϕ:

$$\frac{N(\varepsilon,Q,\phi)}{N_0}=\left\langle\frac{|\tilde{\varepsilon}|\sqrt{\tilde{\varepsilon}^2-\Delta_0^2}}{\tilde{\varepsilon}^2-\Delta_\phi^2}\theta(|\tilde{\varepsilon}|-\Delta_0)+\frac{\pi}{2}\Delta_S\delta(|\tilde{\varepsilon}|-\Delta_\phi)\right\rangle, \tag{6}$$

where $\theta(\cdots)$ and $\delta(\cdots)$ are the theta and delta functions; $\tilde{\varepsilon}=\varepsilon-\mathbf{p}_F\mathbf{v}_s$; $\Delta_S\equiv\Delta_0|\sin(\phi/2)|$. This after integration results in

$$\begin{aligned}\frac{N(\varepsilon,Q,\phi)}{N_0}&=\sum_\pm(\pm1)\mathrm{sgn}(\varepsilon_\pm)\theta(\varepsilon_\pm^2-\Delta_0^2)\frac{\sqrt{\varepsilon_\pm^2-\Delta_0^2}}{2Q}\\&\quad+\frac{\Delta_S}{2Q}\sum_\pm\Bigg((\mp1)\mathrm{sgn}(\varepsilon_\pm)\theta(\varepsilon_\pm^2-\Delta_0^2)\\&\quad\times\arctan\frac{\sqrt{\varepsilon_\pm^2-\Delta_0^2}}{\Delta_S}+\frac{\pi}{2}\theta(Q-|\varepsilon\pm\Delta_\phi|)\Bigg).\end{aligned} \tag{7}$$

Here $\varepsilon_\pm\equiv\varepsilon\pm Q$; the first term describes the LDOS related to the continuum states, which coincides with the LDOS in the current-carrying homogeneous thin film,[14] and the second term gives the contribution of the bound states.

Results of the numerical calculation at nonzero relaxation rate γ are presented in Fig. 2. First, the LDOS in the homogeneous current state (that is, at $\phi=0$) is shown in Fig. 2(a) to experience the suppression of the peak at $\varepsilon=\Delta_0$ and gradual appearance of the midgap states with increasing the transport supercurrent.[14] Then the LDOS in the point contact without the supercurrent ($q=0$) is plotted in Fig. 2(b). With increasing phase difference ϕ from 0 to π the peak is shifted from $\varepsilon=\Delta_{\phi=0}=\Delta_0$ to $\varepsilon=\Delta_{\phi=\pi}=0$. Note that the peak at $\varepsilon=0$ would appear as the so-called zero-bias conductance peak (ZBCP) in the STM measurements (see also in Ref. 15 about the appearance of a ZBCP in point-contact Josephson junctions with ac biasing voltage). In Fig. 2(c) it is shown that the supercurrent results in the suppression and widening of this ZBCP similar to the ZBCP suppression in d-wave superconductors.[12] Note that in concrete realizations the de-

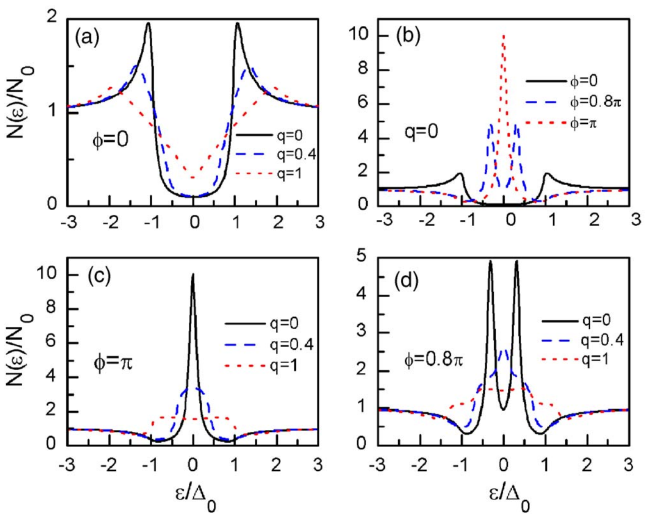

FIG. 2. (Color online) LDOS for the point contact between two current-carrying conventional superconductors for different values of the phase difference ϕ and the dimensionless supercurrent velocity $q=p_F v_s/\Delta_0$ at zero temperature $T=0$ and nonzero relaxation rate $\gamma/\Delta_0=0.1$.

tails of the modification of the ZBCP (widening, splitting) depend on several parameters, such as barrier anisotropy and surface roughness.[3,12,16] And finally in Fig. 2(d) we study the situation when the order parameter at the contact is significantly suppressed due to the phase difference at ϕ close to π. In the absence of the supercurrent there are two midgap peaks [see also in Fig. 2(b)]. With the gradual increase of the supercurrent these two peaks are shifted and modified so that first three-humped and then four-humped midgap structures appear, which are shown, respectively, for $q=0.4$ and for $q=1$. A similar humped structure of the LDOS in a homogeneous current-carrying superconductor (i.e., at $\phi=0$) was studied in Ref. 11. In the homogeneous situation, studied in Ref. 11, particularly interesting zero-energy states (which appear as the ZBCP) occur when the supercurrent is strong enough to depair electrons but smaller than the thermodynamic critical current, that is, in the rather narrow region $\Delta_0 < p_F v_s < 1.03\Delta_0$. We emphasize that in our case when the order parameter is locally suppressed (controlled) by the phase difference, the ZBCP appears at any value of the supercurrent at the phase difference defined by the relation $\Delta_\phi = \Delta_0 \cos(\phi/2) < p_F v_s$ [see Figs. 2(b) and 2(d)].

In conclusion, the LDOS at the weak link between current-carrying superconductors has been studied. We propose this LDOS to be visualized with the STM. Two controlling mechanisms—external magnetic flux and injected supercurrent—allow us to vary the LDOS, and correspondingly the characteristics of the structure in different ways in a wide range of the parameters ϕ and v_s.

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*Electronic address: sshevchenko@ilt.kharkov.ua

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