The theory of the reentrant effect in susceptibility of cylindrical mesoscopic samples

G. A. Gogadze[a)]

B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 47 Lenin Ave., Kharkov 61103, Ukraine
(Submitted February 9, 2006)
Fiz. Nizk. Temp. **32**, 716–728 (June 2006)

A theory has been developed to explain the anomalous behavior of the magnetic susceptibility of a normal metal–superconductor (NS) structure in weak magnetic fields at millikelvin temperatures. The effect was discovered experimentally [A. C. Mota *et al.*, Phys. Rev. Lett. **65**, 1514 (1990)]. In cylindrical superconducting samples covered with a thin normal pure metal layer, the susceptibility exhibited a reentrant effect: it started to increase unexpectedly when the temperature was lowered below 100 mK. The effect was observed in mesoscopic NS structures when the N and S metals were in good electric contact. The theory proposed is essentially based on the properties of the Andreev levels in the normal metal. When the magnetic field (or temperature) changes, each of the Andreev levels coincides from time to time with the chemical potential of the metal. As a result, the state of the NS structure experiences strong degeneracy, and the quasiparticle density of states exhibits resonance spikes. This generates a large paramagnetic contribution to the susceptibility, which adds to the diamagnetic contribution, thus leading to the reentrant effect. The explanation proposed was obtained within the model of free electrons. The theory provides a good description of the experimental results. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2215369]

I. INTRODUCTION

Mesoscopic systems[1–3] can exhibit surprising properties at comparatively low temperatures. For pure normal metals there is a length scale $\xi_N = \hbar V_F / k_B T$ (V_F is the Fermi velocity, T is the temperature, k_B is the Boltzmann constant) which has the meaning of a coherence length in a system with a disturbed long-range order. When this length is comparable with the characteristic dimensions of the system, the interference effects can come into play. Theoretically this was first demonstrated by Kulik[4] for a thin-wall normal pure-metal cylinder in the vector potential field. It appears that the magnetic moment of such a system is an oscillating function of the magnetic flux through the cross section of the cylinder, the oscillation period being equal to the flux quantum of the normal metal hc/e. The effect is generated by quantization of the electron motion and is due to the sensitivity of the states of the system to the vector potential field (Aharonov–Bohm effect).[5] Bogachek and this author showed the existence of an oscillating component with the period hc/e in the magnetic moment of a singly connected normal cylinder in a weak magnetic field. Oscillations with this period are produced by the magnetic surface levels of the cylindrical sample in a weak magnetic field.[6] The effect of flux quantization in a normal singly connected cylindrical conductor was first detected experimentally in 1976 by Brandt *et al.* when they were investigating the longitudinal magnetoresistance in pure Bi single crystals.[7,8] This was actually the first observation of the interference effect of flux quantization in nonsuperconducting condensed matter.

Recent advanced technologies of preparation of pure samples have enabled investigation of the coherence properties of mesoscopic structures taking proper account of the proximity effect.[9] The samples were superconducting Nb wires with a radius R of tens of μm coated with a thin layer d of high-purity Cu or Ag. The metals were in good contact and the electron mean free path exceeded the typical scale ξ_N. The magnetic susceptibilities of copper and silver were measured. The breakdown field H_b, the supercooled field H_{sc} and the superheated field H_{sh} were estimated as functions of temperature and normal metal thickness. While continuing their experiments on these samples, Mota and co-workers[10] detected surprising behavior of the magnetic susceptibility of a cylindrical NS structure (N and S are for the normal metal and the superconductor, respectively) at very low temperatures ($T < 100$ mK) in an external magnetic field parallel to the NS boundary.

Most intriguingly, a decrease in the sample temperature below a certain point T_r (at a fixed field) produced a reentrant effect: the decreasing magnetic susceptibility of the structure unexpectedly started growing. A similar behavior was observed with the isothermal reentrant effect in a field decreasing to a certain value H_r below which the susceptibility started to grow sharply. It is emphasized in Ref. 11 that the detected magnetic response of the NS structure is similar to the properties of the persistent currents in mesoscopic normal rings. It is assumed[9–12] that the reentrant effect reflects the behavior of the total susceptibility χ of the NS structure: the paramagnetic contribution is superimposed on the Meissner-effect-related diamagnetic contribution and nearly compensates it. Anomalous behavior of the susceptibility has also been observed in AgTa, CuNb, and AuNb structures.[11,13]

The reentrant effect revealed by Mota *et al.* is of great interest in the physics of the quantum proximity effect in NS sandwiches of ring geometry. We believe that the effect is not restricted only to NS structures with the ordinary electron–phonon interaction in the superconductors. A modification of the reentrant effect can well be expected if in place of Nb and Ta, high-T_c-superconductors with another type of pairing are used.

The possibility of the paramagnetic contribution to the susceptibility of the NS structure needs further clarification. The NS structure in question is essentially a combination of two subsystems capable of electron exchange, which corresponds to the establishment of equilibrium in a grand canonical ensemble (with fixed chemical potential). Assume that these systems are initially isolated with a thick dielectric layer. It is known that the superconductor response to the applied magnetic field generates superfluid screening current near the cylinder surface (Meissner effect). How does the normal mesoscopic layer respond to the weak magnetic field? Kulik[4] shows (see above) that in a weak magnetic field the magnetic moment of a thin-wall normal cylinder oscillates with the flux. The magnetic moment oscillations are equivalent to the existence of persistent current. Since the energies of the individual states and hence, the total energy are dependent on the flux, the average current is nonzero. The current state corresponds to the minimum free energy, therefore the inclusion of weak dissipation would not lead to the decay of the current state. When the N and S metals are isolated, the quantum states of the quasiparticles in the N metal are formed at the expense of specular reflection of the electrons from the dielectric boundaries. The amplitude of the magnetic moment oscillations in the N layer is small, which is determined by the smallness of the parameter $1/k_F R$ in the problem and by the paramagnetic character of the persistent current[4,6] (when the magnetic field tends to zero, the magnetic susceptibility is positive). Thus, in the absence of the proximity effect, the total susceptibility of the NS structure is only governed by the diamagnetic contribution of the S layer (the paramagnetic contribution is very small).

When the proximity effect is present in the NS structure, we assume that the probability of the electron transit from the superconductor to the N metal is close to unity. This significantly affects the properties of the NS structure. The diamagnetic response of the superconductor persists but new properties appear, that are brought about by the proximity effect. Now two kinds of electron reflection are observed in the normal film—a specular reflection from one boundary and the Andreev reflection from other. Along with the trajectories closed around the cylinder circle, new trajectories appear in a weak field, which "screen" the normal metal. The new trajectories of "particles" and "holes" confine the quantization area of the triangle whose base is a part of the NS boundary between the points of at which the quasiparticle collides with this boundary. This area is maximum for the trajectories touching the superconductor. It is shown below that at certain values of the flux through the triangle area, the electron density of states experiences flux-dependent resonance spikes. Thus, in the presence of the proximity effect, the periodic flux-induced oscillations of the thermodynamic values typical of the normal layer in the NS structure give way to periodic resonance spikes with a period equal to a superconducting flux quantum $hc/2e$.[16] The response of the normal mesoscopic layer to a weak magnetic field ($H \lesssim 10$ Oe) is paramagnetic and the susceptibility amplitude is large. The picture, however, changes when the quantized magnetic flux through the triangle area increases and its value divided by $hc/2e$ starts to exceed the highest Andreev "subband" number. A phase transition occurs at a certain field H_r. As a result, the N layer is now screened only by the trajectories of those quasiparticles that do not collide with the superconducting boundary. Their amplitudes are rather small (see above) against the large diamagnetic response. We can thus conclude that the resonance contribution to the paramagnetic susceptibility of the NS structure can only appear in comparatively weak magnetic fields. At this condition the reentrant effect may be generated. The conclusion correlates well with the experimental observations.[9–14]

The origin of paramagnetic currents in NS structures has been discussed in several theoretical publications. Bruder and Imry[17] analyze the paramagnetic contribution to susceptibility made by quasiclassical ("glancing") trajectories of quasiparticles that do not collide with the superconducting boundary. The authors[17] point to a large paramagnetic effect within their physical model. However, their ratio between the paramagnetic and diamagnetic contributions is rather low and cannot account for the experimental results.[9–14]

Fauchere, Belzig, and Blatter[18] explain the large paramagnetic effect assuming a pure repulsive electron–electron interaction in noble metals. The proximity effect in the N metal induces an order parameter whose phase is shifted by π from the order parameter Δ of the superconductor. This generates the paramagnetic instability of the Andreev states, and the density of states of the NS structure exhibits a single peak near the zero energy. The theory in Ref. 18 essentially rests on the assumption of the repulsive electron interaction in the N metal. Is the reentrant effect a result of specific properties of noble metals, or does it display the behavior of any normal metal experiencing the proximity effect from the neighboring superconductor? Only experiment can provide answers to these questions. We just note that the theories of Refs. 17 and 18 do not account for the temperature and field dependences of the paramagnetic susceptibility and the nonlinear behavior χ of the NS structure. The current theories cannot explain the origin of the anomalously large paramagnetic reentrant susceptibility in the region of very low temperatures and weak magnetic fields.

It is worth mentioning the assumption made by Maki and Haas[19] that below the transition temperature (~ 10 mK) some noble metals (Cu, Ag, Au) can exhibit p-ware superconducting ordering, which may be responsible for the reentrant effect. This theory does not explain the high paramagnetic reentrant effect either.

In this paper a theory of the reentrant effect is proposed which is essentially based on the properties of the quantized levels of the NS structure. Levels with energies no more than Δ (2Δ is the gap of the superconductor) appear inside the normal metal bounded by the dielectric (vacuum) on one side and contacting the superconductor on the other side. The number of levels n_0 in the well is finite. Because of the Aharonov–Bohn effect,[5] the spectrum of the NS structure is a function of the magnetic flux in a weak field. The specific feature of the quantum levels of the structure is that in a varying field H (or temperature T) each level in the well periodically comes into coincidence with the chemical potential ξ of the metal. As a result, the state of the system suffers strong degeneracy, and the density of states of the NS sample experiences resonance spikes.

It is shown that the phenomenon of resonance appears in a certain interval of weak magnetic fields at temperatures no higher than a hundred of millikelvins. Resonance is realizable only in pure mesoscopic N layers under the condition of the Aharonov–Bohm effect. The resonance produces a large paramagnetic contribution χ^p to the susceptibility of the NS structure. When χ^p is added to the diamagnetic contribution χ^d produced by the Meissner effect, the total susceptibility displays the features of the reentrant effect.[20]

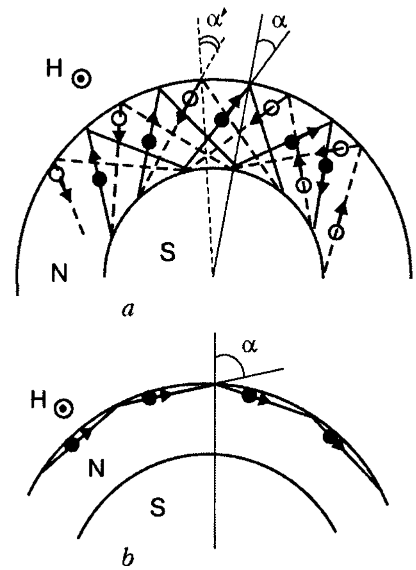

FIG. 1. Two classes of trajectories in the normal metal of an NS structure in a magnetic field: trajectories forming the Andreev levels (a); trajectories colliding only with the dielectric boundary (b).

II. SPECTRUM OF QUASIPARTICLES OF THE NS STRUCTURE

Consider a superconducting cylinder with the radius R which is covered with a thin layer d of a pure normal metal. The structure is placed in a weak magnetic field $\mathbf{H}(0,0,H)$ oriented along the symmetry axis of the structure. It is assumed that the field is weak to an extent that the effect of twisting of quasiparticle trajectories becomes negligible. It actually reduces to the Aharonov–Bohm effect,[5] i.e., allows for the increment in the phase of the wave function of the quasiparticle moving along its trajectory in the vector potential field.

We proceed from a simplified model of NS structure in which the order parameter magnitude changes stepwise at the NS boundary. It is also assumed that the magnetic field does not penetrate into the superconductor. The coherent properties observed in the pure normal metal can be attributed to its large "coherence" length ξ_N at very low temperatures.

One can easily distinguish two classes of trajectories inside the normal metal. One of them includes the trajectories which collide in succession with the dielectric and NS boundaries. The quasiparticles moving along these trajectories have energies $\varepsilon < \Delta$ and are localized inside the potential well bounded by a high dielectric barrier ($\simeq 1$ eV) on one side and by the superconducting gap Δ on the other side. On its collisions, the quasiparticle is reflected specularly from the dielectric and experiences the Andreev scattering at the NS boundary.[15] We introduce an angle α at which the quasiparticle hits the dielectric boundary. The angle is measured from the positive direction of the normal to the boundary (Fig. 1). In this case the first class contains the trajectories with α varying within $0 \lesssim \alpha \lesssim \alpha_c$ (α_c is the angle at which the trajectory touches the NS boundary). The other class includes the trajectories whose spectra are formed by collisions with the dielectric only, i.e., the trajectories with $\alpha > \alpha_c$.

The two groups of trajectories produce significantly different spectra of quasiparticles. The distinctions are particularly obvious in the presence of the magnetic field. The trajectories with $\alpha \lesssim \alpha_c$ form a spectrum of Andreev levels which contains a supplement in the form of an integral of the vector potential field. The spectrum characterizes the magnetic flux through the area of the triangle between the quasiparticle trajectory and the part of the NS boundary. It is also determines the magnitude of the screening current produced by "particles" and "holes" in the N layer. These states are responsible for the reentrant effect. The trajectories with $\alpha > \alpha_c$ do not collide with the NS boundary. The states induced by these trajectories are practically similar to the "whispering gallery" type of states appearing in the cross section of a solid normal cylinder in a weak magnetic field.[6,21] The size of the caustic of these trajectories is of the order of the cylinder radius, i.e., they correspond to high magnetic quantum numbers m. The spectrum thus formed carries no information about the parameters of the superconductor, and it is impossible to meet the resonance condition in this case. These states make a paramagnetic contribution to the thermodynamics of the NS structure but their amplitude is small ($\sim 1/k_F R$). It is therefore discarded from further consideration. Our interest will be concentrated on the trajectories with $\alpha \lesssim \alpha_c$.

The spectrum of quasiparticles of the NS structure can be obtained easily using the multidimensional quasiclassical method generalized for the case of the Andreev scattering in the system.[16,22] After collision with the NS boundary the "particle" transforms into a "hole." The "hole" travels practically along the path of the "particle" but in the reverse direction. In the strict sense, however, the path of the "hole" is somewhat longer because under the condition of Andreev elastic scattering the momentum of the "particle " exceeds that of the reflected "hole." According to the law of conservation of the angular momentum, the angle α' at which the "hole" comes up to the dielectric boundary and hence the distance covered by the "hole" are larger. Eventually, the trajectory of the quasiparticle becomes closed due to its displacement along the perimeter of the N layer. However, as the quasiparticle energy decreases and approaches the value of the chemical potential, the difference $\alpha - \alpha'$ starts tending to zero. Since our further interest is concerned with low-lying Andreev levels, we assume that the "hole" trajectory is strictly reversible. The distance covered by the "particle" ("hole") between two boundaries is $\mathcal{L}_0 \simeq 2d/\cos\alpha$.

According to the multidimensional quasiclassical method,[16,22] there are two congruences of "particle" rays — towards the dielectric (I) and in the opposite direction (II). There are also two congruences of "hole" rays — towards the NS boundary (III) and away from it (IV). The covering space is constructed of four similar NS structures whose edges are joined in accordance with the law of quasiparticle reflection from a dielectric and a NS boundary. At the dielectric boundary the congruences I and II are joined. The congruences III and IV are joined independently. The covering space consists of the outer ("particles") and inner ("holes") toroidal surfaces. Each surface contains only a part of the single independent integration contour. The path of the "particle " is $2d$. The "hole" travels the same length, whereupon the trajectory of the quasiparticle closes. The total length of the closed contour along the covering surface of the NS structure is $4d$.

It is possible to choose two independent integration contours within a torus that do not contract into a point. One condition of quantization relates the caustic radius to the magnetic quantum number m. We replace it with an angle of incidence of the quasiparticle on the dielectric boundary. The other condition of quantization introduces the radial quantum number n. Thus, the complete set of quantum numbers describing the motion of the quasiparticle includes n, α, q, where q is the quasimomentum component along the symmetry axis of the cylinder.

Assume that the condition $d \ll R$ is obeyed for the NS structure. We can then neglect the curvature of the cylinder boundary and assume that it is flat. The condition of quasiclassical quantization can be written as

$$\int_{\mathcal{L}_0}\left(\mathbf{p}_0-\frac{|e|}{c}\mathbf{A}\right)d\mathbf{s}-\int_{\mathcal{L}_0}\left(\mathbf{p}_1+\frac{|e|}{c}\mathbf{A}\right)d\mathbf{s} =2\pi\hbar\left(n+1-\frac{1}{\pi}\arccos\varepsilon/\Delta\right) \tag{1}$$

where $p_0(p_1)$ are the quasimomentum of the "particle" ("hole"), ε is the "quasiparticle" energy, $\mathbf{A}$ is the vector potential $(0,0,H_y)$, and $|\mathcal{L}_0|$ is the trajectory length covered by the "particle" ("hole"). The unity in the right-hand side of Eq. (1) appears when two collisions of the quasiparticle with the dielectric boundary are taken into account.[22] The term $(\mathrm{arcos}\,\varepsilon/\Delta)/\pi$ accounts for the phase delay of the wave function under the Andreev scattering of quasiparticles.[16] The quasimomentum p_0 and p_1 in Eq. (1) can be expanded in the parameter ε/ζ retaining the first-order terms and replacing $n-1$ by n. As a result, Eq. (1) furnishes the desired spectrum of the NS structure in a weak magnetic field ($\mathcal{L}$ is the quasiparticle trajectory):

$$\varepsilon_n(q,\alpha;\Phi)=\frac{\pi\hbar\, v_{\mathcal{L}}(q)\cos\alpha}{2d}\left(n+\frac{1}{\pi}\arccos\frac{\varepsilon}{\Delta}-\frac{\tan\alpha}{\pi}\Phi\right). \tag{2}$$

Here $v_{\mathcal{L}}(q)=\sqrt{p_F^2-q^2}/m^*$, p_F is the Fermi momentum, q is the quasiparticle momentum component along the cylinder axis, m^* is the effective mass of the quasiparticle, and $\Phi_0 = hc/2e$ is the superconducting flux quantum. The positive α values refer to "particles" ($n>0$), while the negative ones are for "holes" ($n<0$).

The last term in Eq. (2) has the meaning of "phase"

$$\Phi=\frac{2\pi}{\Phi_0}\int_0^d A(x)dx, \tag{3}$$

which is dependent on the vector potential field and varies with the angle α characterizing the trajectory of the quasiparticle.

The spectrum of Eq. (2) is similar to Kulik's spectrum[23] for the current state of an SNS contact. However, Eq. (2) includes an angle-dependent magnetic flux instead of the phase difference of the contacting superconductors.

The value of the "phase" (flux) controls the diamagnetic and paramagnetic currents in the NS structure. To calculate it, we should know the distribution of the vector potential field inside the normal metal.

The problem of the Meissner effect in superconductor–normal metal (proximity) sandwiches was solved by Zaikin.[24] It was shown that the proximity effect caused the Meissner effect bringing an inhomogeneous distribution of the vector potential field over the N layer of the structure: $A(x)=Hx+(4\pi/c)j(a)x(d-x/2)$. For convenience we introduce the notation $a=\int_0^d A(x)dx$. This expression can be obtained from the Maxwell equation curl $\mathbf{H}=(4\pi/c)\mathbf{j}$ with the boundary conditions $A(x=0)=0$ and $\partial_x A(x=d)=H$. The screening (diamagnetic) current $\mathbf{j}$ is a function of a, $j(a) = -j_s\varphi(a/\Phi_0)$, where j_s is the superfluid current and $\varphi(x)$ is the flux function. Thus, we can write down the self-consistent equation for a:[25,26]

$$a=\frac{Hd^2}{2}+\frac{4\pi}{3c}j(a)d^3. \tag{4}$$

The diamagnetic current $\mathbf{j}^d(a)$ was calculated in terms of the microscopic theory as a sum of currents of quasiparticles ("particles" and "holes") for all quasiclassical trajectories characterized by the angles θ and φ[24,26] (below the system of units $k_B=\hbar=c=1$ is used):

$$j^d(\Phi,T)=-AT\sum_{\omega_n>0}\int_0^{\pi/2}d\theta\int_0^{\pi/2}\frac{\sin^2\theta\cos\varphi\sin[2\Phi\tan\theta\cos\varphi]}{\left[\frac{\sqrt{\omega^2+\Delta^2}}{\Delta}\sinh\alpha_n+\frac{\omega_n}{\Delta}\cosh\alpha_n\right]^2+\cos^2(\Phi\tan\theta\cos\varphi)}d\varphi, \tag{5}$$

where $A=2ek_F^2/\pi^2$, $\omega_n=(2n+1)\pi T$, 2Δ is the superconductor gap, $\alpha_n=2\omega_n d/v_F \cos\theta$, and Φ is given by Eq. (3). The function $j^d(\Phi)$ is noted for interesting features. In small magnetic fields $(\Phi\ll 1) j^d\approx -j_s\Phi$. Such low fields can lead to the effect of extra screening of the external magnetic field (see Ref. 24). When the field increases $(\Phi\simeq 1)$, the current starts oscillating and for certain "phases" it turns to zero at regular intervals "phases" Φ. With high values of the inequality $(\Phi\gg 1)$, the current amplitude decreases.

III. RESONANCE SPIKES IN THE DENSITY OF STATES OF NS STRUCTURES IN WEAK MAGNETIC FIELDS

In the region of weak magnetic fields, the density of states of the quasiparticles that are described by the spectrum of Eq. (2) exhibits sharp singularities. The spectrum of Eq. (2) is formed by the trajectories of the quasiparticles which collide with the dielectric and superconducting boundaries. It encloses a certain area penetrated by a magnetic flux. At any instant when the magnetic flux becomes a multiple of the superconducting flux quantum, the density of states experiences resonance spikes.

Let us consider the cross section of a NS structure. Assume that the superconducting cylinder radius R and the normal layer thickness d have a mesoscopic scale. The density of states $\nu(\varepsilon)$ can be calculated proceeding from the expression

$$\nu(\varepsilon)=\sum_{n,\alpha,\sigma}\int dq\,\delta[\varepsilon-\varepsilon_n(q,\alpha)]. \tag{6}$$

The summation is taken over all quantum numbers n, q, α and spin σ. Since we are not interested in the contribution from the states formed by the trajectories of the quasiparticles with $\alpha>\alpha_c$, we can write down

$$\nu(\varepsilon)=\int_{-\alpha_C}^{\alpha_C} d\alpha\,\nu(\varepsilon;\alpha), \tag{7}$$

where $\nu(\varepsilon;\alpha)$ is the contribution to the density of states from the pre-assigned trajectory with a fixed α. Equation (2) for the low-lying Andreev levels $(\varepsilon\ll\Delta)$ is taken as a spectrum. After integration with respect to q and introduction of the notation $\beta=\pi\hbar/2dm^*$, we can pass on to the dimensionless energy $\varepsilon=\varepsilon/\beta p_F$. For $\nu(\varepsilon,\alpha)$ we have the expression

$$\nu(\varepsilon,\alpha)=\frac{2p_F}{\pi^2\beta d}\varepsilon^2\sum_n\frac{\sec^2\alpha\,\theta[|n+\kappa|-\varepsilon\sec\alpha]}{(n+\kappa)^2\sqrt{(n+\kappa)^2-\varepsilon^2\sec^2\alpha}}, \tag{8}$$

where $\kappa=1/2-\Phi\tan\alpha/\pi$, and $\theta(x)$ is the Heaviside step function. Equation (8) suggests two cases depending on the parameter $n+\kappa$.

a. Nonresonance case. If $n+\kappa\neq 0$, the energy dependence under the radical sign in Eq. (8) can be neglected for small energies $(\varepsilon\to 0)$. Then, the nonresonance contribution to the density of states is

$$\nu^{(0)}\sim\frac{2p_F}{\pi^2\beta d}\varepsilon^2\int_0^{\alpha_c} d\alpha\sum_{n=-\infty}^{+\infty}\frac{\sec^2\alpha}{(n+\kappa)^3}. \tag{9}$$

The series in Eq. (9) is calculated readily by the formula in Ref. 27:

$$\sum_{k=-\infty}^{+\infty}\frac{1}{(k-\kappa)^n}=(-1)^{n-1}\frac{\pi}{(n-1)!}\frac{d^{n-1}}{d\kappa^{n-1}}\operatorname{cotg}\pi\kappa.$$

After calculation of the integral we obtain

$$\nu^{(0)}\sim\frac{p_F}{\beta d}\varepsilon^2\frac{\Phi_0}{a}\tan^2\left[\frac{2\pi a}{\Phi_0}\sqrt{\frac{2R}{d}}\right], \tag{10}$$

where $\sqrt{2R/d}\simeq\tan\alpha_c$.

b. Resonance case. Now we go back to Eq. (8). We find ν^{res} as

$$\upsilon^{\mathrm{res}}\sim\varepsilon^2\int_0^{\alpha_C} d\alpha\sum_n\frac{\sec^2\alpha\,\theta[|a_n-b\tan\alpha|-\varepsilon\sec\alpha]}{|a_n-b\tan\alpha|^2\sqrt{|\mathfrak{a}_n-b\tan\alpha|^2-\varepsilon^2\sec^2\alpha}} \tag{11}$$

where the notation $\mathfrak{a}_n=n+1/2$, $b=2a/\Phi_0$ is introduced. Equation (11) shows that at certain values of the flux (b), the radicand in the denominator turns to zero.

Prior to calculation of ν^{res}, let us discuss the question of the contribution of different angles α to the resonance amplitude. It is reasonable to assume that because of the factor $\sec^2\alpha$ in the numerator of Eq. (11), the angles $\alpha\sim\alpha_c$ are the main contributors to the integral. It is convenient to employ in the integral a new variable of integration $x=\tan\alpha$. Then the neighborhood of the upper limit $x_0=\tan\alpha_c$ is the main contributor to the integral. Introducing the notation $\tilde{a}=\mathfrak{a}_n-bx_0$ and the small deviation $\xi=x_0-x\ll 1$, we can write down the equation for the resonance condition as:

$$(b^2-\varepsilon^2)\xi^2+2(\breve{a}b+\varepsilon^2x_0)\xi+\tilde{a}^2-\varepsilon^2(1+x_0^2)=0. \tag{12}$$

The point of our interest is the asymptotics of $\nu(\varepsilon)$ at low $\varepsilon\to 0$. Eq. (12) is solved to the accuracy within first-order terms of $|\varepsilon|$:

$$\xi_{1,2}\simeq\frac{\tilde{a}}{b}+\frac{|\varepsilon|}{b}\sqrt{1-x_0^2}. \tag{13}$$

The expression in front of the radical in the denominator of Eq. (11) is of second order smallness in $|\varepsilon|$, i.e., $|\tilde{a}|^2\gtrsim|\varepsilon|^2(1+x_0^2)$, which leads to its cancellation with the similar small parameter in the numerator.

The remaining integral is estimated to be a constant of about unity. A resonance-induced spike of the density of states always appears when the Andreev level coincides with the Fermi energy at a certain flux in the N layer. In the vicinity of the chemical potential there is a strong degeneracy of the quasiparticle states with respect to the quantum number q. As a result, a macroscopic number of q states contribute to the amplitude of the effect. Near the resonance, the ratio of the resonance and nonresonance amplitudes of the density of states is

$$\frac{\nu^{\mathrm{res}}}{\nu^{(0)}}\sim\frac{1}{|\varepsilon|^2}\gg 1. \tag{14}$$

Thus it is shown that a change in the magnetic flux leads to resonance spikes in the density of states of the NS structure. The flux interval between the spikes is equal to the superconducting flux quantum Φ_0.

IV. CALCULATION OF SUSCEPTIBILITY OF AN NS CONTACT

To explain the reentrant effect, we need to have an expression for the susceptibility of the NS structure. We assume that in a weak magnetic field the total susceptibility of the NS sample consists of two contributions. First, the response of the superconductor to the applied magnetic field generates the Meissner effect. Note that the diamagnetic response is observed in all fields up to the critical field. The amplitude of the diamagnetic current increases monotonously with lowering temperature. On the other hand, the presence of a pure normal metal in the NS structure produces a paramagnetic contribution. In a weak magnetic field the contribution is due to the Aharonov–Bohm effect and the quantization of the quasiparticle spectrum of the mesoscopic system. When the penetrability of the barrier between the metals is small, the electrons of the normal metal are reflected specularly from its boundaries. As compared to the diamagnetic contribution from the superconductor, the paramagnetic contribution produced by the N layer has a small amplitude and can therefore be neglected. Thus, the paramagnetic and diamagnetic contributions cannot compete in the absence of the proximity effect in the NS structure. However, if the penetrability of the barrier is close to unity, the mechanism of the Andreev reflection becomes active at the NS boundary. The quasiparticle spectrum of the N layer undergoes a significant transformation, and resonance spikes appear in the amplitude of the density of states in a certain regions of magnetic fields and temperatures. Simultaneously, the distribution of the vector potential field in the normal layer becomes inhomogeneous. As shown below at certain values of the parameters of the problem, the paramagnetic contribution to the susceptibility of the NS structure can become equal to the diamagnetic contribution. This is the reason why the reentrant effect appears in pure mesoscopic NS structures.

Theoretically, the resulting susceptibility including the reentrant effect can be represented as a sum of the paramagnetic contribution χ^p of the NS structure caused by the Andreev scattering and the diamagnetic susceptibility χ^d of the system in which there is no proximity effect between the N and S metals. The temperature-induced behavior of the diamagnetic current in such a system is well known. As the temperature decreases, the diamagnetic current amplitude increases and becomes saturated at temperatures about several millikelvins. At high temperatures $k_BT \gg \hbar V_F/d$, the diamagnetic current decreases rapidly following the law $j \sim T^{-1}\exp(-4\pi k_B T d/\hbar V_F)$. Note that in a NS structure in which the electrons are reflected specularly at both boundaries of the normal metal, the susceptibility is negative (i.e., diamagnetic) in the whole interval of temperatures $0<T<T_c$. However, we will not use this approach to estimate the resulting susceptibility. Below we calculate the screening current of the NS structure. It naturally allows for the paramagnetic contribution at certain values of the magnetic field and temperature. We focus our attention on calculation of the paramagnetic contribution in structures with a pronounced proximity effect. This is important especially in the context of the recent statement[28] that no paramagnetic reentrance can occur in NS proximity cylinders in the absence of electron–electron interaction in the N layer.

Paramagnetic susceptibility of NS contact

The contribution of the states in Eq. (2) to the paramagnetic susceptibility of the normal layer in a NS contact can be calculated proceeding from the expression for the thermodynamic potential ($k_B=1$)

$$\Omega = -T \sum_{\substack{n,q,\alpha \\ \sigma}} \ln\left[1+\exp(-\varepsilon_n(q,\alpha)/T)\right], \tag{15}$$

where the summation is taken over the spin (σ) and all the states related to the trajectories of the quasiparticles with $a \lesssim a_c$. The expression for susceptibility (per unit volume V of the normal metal) is found using the formula

$$\chi = -\frac{1}{V}\frac{\partial^2 \Omega}{\partial H^2}.$$

After performing the summation over the spin and taking into account two signs of the angle α and of the quasimomentum component q, we arrive at the initial expression for paramagnetic susceptibility (ζ is the chemical potential of the metal):

$$\chi = \frac{d}{2Tm^{*2}\Phi_0^2}\int_{-\zeta}^{\infty}\frac{d\varepsilon \exp(\varepsilon/T)}{[\exp(\varepsilon/T)+1]^2}\sum_n \int_0^{\alpha_C} d\alpha \cos\alpha \sin^2\alpha \times \int_0^{p_F} dq (p_F^2-\eta^2)^{3/2}\delta(\varepsilon-\varepsilon_n(q,\alpha)). \tag{16}$$

In Ref. 20 we lost one of the radicals $(p_F^2-q^2)^{1/2}$ in the similar initial expression for χ. As a result, the amplitude of the paramagnetic contribution appeared to be underestimated. This mistake is corrected in this work.

It is convenient to present the spectrum in terms of $\beta = \pi\hbar/2m^* d$ and $\kappa = \frac{1}{2} - \frac{\tan\alpha}{\pi}\Phi$ as

$$\varepsilon_n(q,\alpha) = \beta \cos\alpha (n+\kappa)\sqrt{p_F^2-q^2}.$$

Now we introduce the dimensionless energy $\varepsilon = \varepsilon/(\beta p_F) = \varepsilon/\delta\varepsilon$, $\delta\varepsilon = \pi\hbar V_F/(2d)$ is the distance between the Andreev levels in the SN structure. Since $\zeta/\delta\varepsilon \gg 1$, the lower limit of the energy integral can be replaced with $-\infty$. By introducing the variable $x = \tan\alpha$ and the notation $\mathfrak{a}_n = n+1/2$, $b = b(H,T) = 2a/\Phi_0$,

$$a = \int_0^d A(x)dx, \quad x_0 = \tan\alpha_0 = \sqrt{2R/d},$$

and taking into account the parity of the integrand we obtain, instead of Eq. (16):

$$\chi = C\int_0^{\infty}\frac{\varepsilon^4 d\varepsilon}{\cosh^2(\eta\varepsilon/2)}\sum_{n=0}^{n_0}\int_0^{x_0}\frac{x^2 dx}{(\mathfrak{a}_n - bx)^4} \times \frac{\theta[\mathfrak{a}_\eta - bx - \epsilon\sqrt{1+x^2}]}{\sqrt{(\mathfrak{a}_n-bx)^2-\varepsilon^2(1+x^2)}}. \tag{17}$$

In Eq. (17) the summation is taken over the quantum numbers of the "particles." Here $C = \zeta^2 d/T\Phi_0^2$, $\eta = \delta\varepsilon/T$, n_0 is the number of Andreev levels in the potential well, and θ is the Heaviside step function. It is seen in Eq. (17) that for the given "subzone" n the amplitude of the paramagnetic susceptibility increases sharply whenever the Andreev level coincides with the chemical potential of the metal. The resonant spike of susceptibility occurs when $\mathfrak{a}_n - bx$ tends to zero on a change in the magnetic field (or temperature). Because of the finite number of Andreev levels, the existence region of the isothermal reentrant effect is within $0 < H \lesssim H_{\max}$.

Let us calculate the integral over x in Eq. (17). It contains a singularity under the radical $R(x) = \sqrt{Ax^2 + Bx - C}$, where $A = b^2 - \varepsilon^2$, $B = -2\mathfrak{a}_n b$, $C = \mathfrak{a}_n^2 - \varepsilon^2$. The singularity is determined by the roots of the quadratic equation

$$x_{1,2} = \frac{\mathfrak{a}_n b}{b^2 - \varepsilon^2} \pm \frac{|\varepsilon|}{b^2 - \varepsilon^2}\sqrt{b^2 - \mathfrak{a}_n^2 - \varepsilon^2}.$$

On introducing the notation $\alpha_0 = \mathfrak{a}_n / b$, the expression for the roots can be written with a linear accuracy with respect to ε as

$$x_{1,2} \simeq \alpha_0 \pm \frac{|\varepsilon|}{b}\sqrt{1 + \alpha_0^2}. \tag{18}$$

The main contribution to the integral over x, Eq. (17), is made by the vicinity of the point $\varepsilon \to 0$. If we exclude the singular points from the interval of integration, the indefinite integral over x can be calculated accurately (see the details in the Appendix). Because the θ function is present under the integral, the integration intervals $(0, x_1)$ and (x_2, x_0) make a finite contribution to the integral. On substituting the limits of integration, the expressions obtained have different powers of the parameter $|\varepsilon|^{-1}$. We retain only the most important terms in order $|\varepsilon|^{-4}$ that determine amplitude of the effect. The discarded terms have higher orders of ε-smallness. The intervals $(0, x_1)$ and (x_2, x_0) make contributions of the same order of ε-magnitude. The region (x_1, x_2) does not contribute to the integral at all.

The estimate for the integral over x is

$$\frac{4}{3}\frac{\alpha_0^2}{b(1 + \alpha_0^2)^2}\frac{1}{\varepsilon^4}. \tag{19}$$

On substituting Eq. (19) into Eq. (17), the parameter ε^4 drops out of the energy integral and we can take it quite easily. Taking into account the energy limits $\theta(\mathfrak{a}_n - |\varepsilon|)$ appearing in the process of calculation we can obtain the expression for the paramagnetic contribution to the susceptibility of the NS structure, which in dimensional units has the form

$$\chi^p \simeq \frac{16\zeta^2 d^2}{3\pi h V_F \Phi_0^2}\sum_{n=0}^{n_0}\frac{b(H,T)\tanh\left[\dfrac{\pi h V_F}{4 d k_B T}(n + 1/2)\right]}{(n + 1/2)^2\left[1 + \left(\dfrac{b(H,T)}{n + 1/2}\right)^2\right]^2}. \tag{20}$$

In Eq. (20) the summation over the quantum number n is taken within finite limits, where n_0 has the meaning of the maximum number of the Andreev levels inside the potential well of the NS structure. Its order of magnitude in $n_0 \sim \Delta / \delta\varepsilon$, where $\delta\varepsilon$ is the distance between the Andreev levels, $\delta\varepsilon = \pi\hbar V_F / 2d$, and 2Δ is the energy gap. The flux $b(H,T) = 2a/\Phi_0$ depends on both the magnetic field and temperature. In the pre-assigned field its value is dictated by the screening current of the NS structure $j = -j_s \varphi(a/\Phi_0)$ (see Eq. (4)). The obtained expression for χ^p manifests a more rapid decrease susceptibility on increasing parameter $b(H,T)$ than was evidenced by Eq. (5) in Ref. 20.

We first discuss the isothermal case of a very low temperature and clear up the qualitative behavior of susceptibility in Eq. (20). We shall proceed from the region of very strong magnetic fields $(a/\Phi_0 \gg 1)$ in which the second term in Eq. (4) is negligible. Then the dimensionless flux $b(H,T) \gg 1$ and the amplitude of the paramagnetic contribution in Eq. (20) decreases as $b(H,T)$ raised to the power 3. In comparatively weak magnetic fields $(a/\Phi_0 \sim 1)$ the function $\varphi(x)$ is actually an oscillating function of H, and here we can expect the reentrant effect. Indeed as the field decreases to a certain value and the parameter $b(H,T)/n_0$ becomes ~ 1 (n_0 is the number of the Andreev levels in the potential well), the amplitude of the paramagnetic susceptibility of the NS structure accepts for the first time an appreciable contribution from the highest Andreev "subband" (level). On a further decrease in this field, the contribution from the highest "subband" persists, but in a certain lower field an additional contribution appears from the neighboring lower-lying "subband" $n_0 - 1$. Finally, in a very weak field all the "subbands" of the NS structure start to contribute and the paramagnetic susceptibility amplitude reaches its peak. However, at $H \to 0 (a/\Phi_0 \to 0)$, the paramagnetic contribution turns to zero, as follows from Eq.(20). The reason is that the resonance condition for the Andreev levels (Eq. (2)) cannot be realized at zero field.

Now we change to the case when the temperature of the NS structure varies but the field is kept constant. We assume the field to be weak $(H \sim 2 \times 10^{-1}$ Oe). The second term in Eq. (4) for the flux is very important. It is highest at millikelvin temperatures. As a result, the parameter $b(H,T)$ has the lowest value. In this temperature region the hyperbolic tangent is close to unity and the paramagnetic contribution is dependent only on the parameter $b(H,T)$. Under this condition, all the "subbands" of the *NS* structure contribute to the amplitude of the effect. As the temperature rises, the parameter $b(H,T)$ increases smoothly. Simultaneously, the argument of the hyperbolic tangent decreases. At a certain temperature, when the condition $k_B T > \pi\hbar V_F / 4d$ is met, the contribution from the lowest "subband" starts dying down and its amplitude is decreasing linearly with growing T. On a further rise of the temperature, the contributions from the higher "subbands" of the spectrum die down in succession. Finally, at a very high temperature the paramagnetic contribution tends to zero.

Let us estimate the amplitude of the paramagnetic contribution. The parameter $b(H,T)$ is dependent on the value of the flux $a = \int_0^d A(x)dx$, which at constant T can be found by solving the self-consistent equation Eq. (4). In the region of millikelvin temperatures and magnetic fields $H \sim 2 \times 10^{-1}$ Oe the paramagnetic contribution has the largest amplitude. We obtain $b(H,T) \sim 10^{-4}$ in this region of T and H. The coefficient before the sum in Eq. (20) can be found by substituting $\zeta^{\mathrm{Ag}} \simeq 8.75 \times 10^{-12}$ erg, $d = 3.3 \times 10^{-4}$ cm, and $V_F^{\mathrm{Ag}} \sim 1.39 \times 10^8$ cm/s for the characteristic parameters of the normal Ag layer. We thus obtain $16\zeta^2 d^2 / 3\pi\hbar V_F \Phi_0^2 \simeq 2.418 \times 10^3$. The product of this coefficient and the parameter $b(H,T)$ yields the order of magnitude of the paramagnetic contribution amplitude. It is seen that the largest amplitude of the paramagnetic contribution exceeds that of the diamagnetic contribution in the vicinity of $T=0$.

Full magnetic susceptibility of NS structure in the presence of the proximity effect

Let us consider a structure in which the electrons experience Andreev scattering at the NS boundary. In the presence of magnetic field, a screening current is induced in the normal layer due to the Meissner effect. We estimate the susceptibility generated by this current.

The total current J is related to the magnetic moment M as

$$M=\frac{1}{c}JS_0, \tag{21}$$

where $S_0 \simeq \pi R^2$ is the cylinder cross section ($d \ll R$). Let the average current density be j. The total current is then $J=Sj$, where $S=dL$ (L is the cylinder generatrix). The density of the screening current in NS proximity sandwiches was calculated by Zaikin.[24,28] We reproduce the formula for the current density (see Eq. (5)), which is valid at arbitrary values of temperature and magnetic field. At $T \ll \hbar V_F/d$ it is

$$j(\Phi) \simeq -\frac{4ek_F^2T}{\pi^2}\sum_{\omega_n>0}\int_0^{\pi/2}d\theta\int_0^{\pi/2}\sin^2\theta\cos\varphi \times\frac{\sin[2\tan\theta\cos\varphi\Phi]}{\sinh^2\alpha_n+\cos^2[\tan\theta\cos\varphi\Phi]}d\varphi. \tag{22}$$

Here $\alpha_n=2\omega_n d/(V_F\cos\theta)$, $\omega_n=(2n+1)\pi T$ and the phase θ follows from Eq. (3). Near $T=0$ the summation of frequencies in Eq. (22) can be replaced with integration. For $\Phi \leq 1$ the response of the current is[28]

$$j(\Phi) \simeq -\frac{ek_F^2V_F}{\pi^3 d}\int_0^{\pi/2}d\theta\int_0^{\pi/2}\sin^2\theta\cos\theta\,\varphi\sin \times[2\Phi\tan\theta\cos\varphi]d\varphi. \tag{23}$$

If the field is small enough to meet the condition $\Phi \ll 1$, Eq. (23) reduces to the result that was obtained for the first time in Ref. 24:

$$J(\Phi)=-\frac{ek_F^2V_F}{6\pi^2 d}\Phi. \tag{24}$$

At "phases" $\Phi \gg 1$, the screening current of Eq. (23) turns to zero.

The current–phase relation at $T=0.092$ K is plotted in Fig. 2. The dependence is nonlinear and its amplitude has a maximum at a certain value of Φ. Knowing the current–phase dependence, we can determine the susceptibility of the NS structure using the equation $\chi=dM/dH$. It is seen in Fig. 2 that the susceptibility χ of the NS structure (the derivative of current with respect to field) changes its sign at a certain low value of the magnetic field H_r. The "paramagnetic" portion of the curve is due to the proximity effect at the NS boundary and to the Andreev levels in the N layer.

Let us estimate χ in the linear-response regime near $T=0$, when this dependence is described by Eq. (24). In such

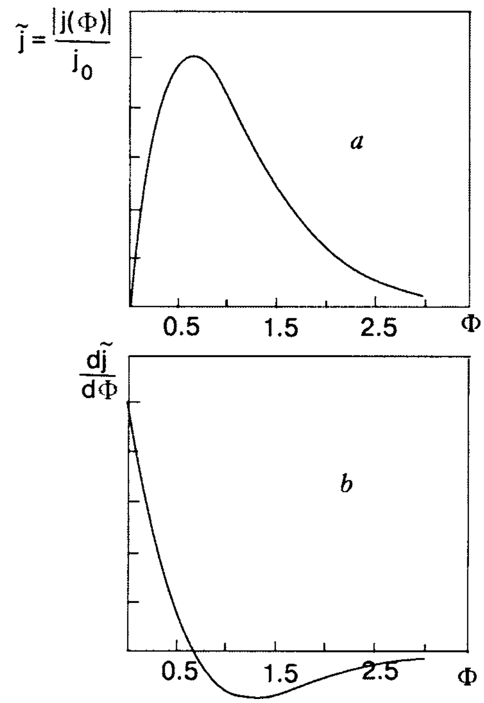

FIG. 2. Dependence of screening current Eq. (5) on "phase" Φ at $T=0.092$ K. Current is in arbitrary units (a). The derivative of current with respect to "phase" (magnetic field) changes its sign in the region of low Φ values (b). When the magnetic field tends to zero the total magnetic susceptibility of the NS structure is positive for $d=3.3\times10^{-4}$ cm.

weak fields we obtain $\Phi \simeq 3\pi H\lambda_N^2(T)/\Phi_0$, where the "penetration depth" λ_N into the normal metal is dependent on temperature:[26]

$$\lambda_N^{-2}(0)=\frac{4\pi ne^2}{m^*c^2} \quad (T\simeq 0);$$

$$\lambda_N^{-2}\sim\lambda_N^{-2}(0)\frac{6T}{T_A}\exp(-2T/T_A)\left(T\gg T_A=\frac{\hbar V_F}{2\pi d}\right). \tag{25}$$

The estimate of susceptibility in the millikelvin region is

$$\chi\sim-\frac{R}{4c}\frac{ek_F^2V_F}{\pi d}\frac{\lambda_N^2(0)}{\Phi_0}.$$

For the parameters of the problem

$$d=3.3\cdot10^{-4}\text{ cm},\quad R=8.2\cdot10^{-4}\text{ cm},$$

$$k_F^{Ag}\sim1.2\cdot10^{8}\text{ cm}^{-1},\quad V_F^{Ag}\sim1.39\cdot10^{8}\text{ cm/s},$$

$$\lambda_N(0)\sim2\cdot10^{-6}\text{ cm}$$

we obtain $\chi=-0.06$, which is close to $\chi=-3/4(1/4\pi)$.

V. DISCUSSION

In this study we have investigated the behavior of a superconducting cylinder covered with a thin layer of a pure normal metal. It is assumed that the normal metal and superconductor are in good contact. The system was placed in a magnetic field directed along the NS boundary. The NS structure has mesoscopic scale dimensions. It is assumed that the mean free path of the quasiparticles in the N layer exceeds the characteristic length $\xi_N=\hbar V_F/k_BT$, which has the meaning of the coherence length for a system with disturbed long-range order. The goal of this study was to interpret the experiments in which A. C. Mota *et al.*[10–14] observed anomalous behavior of the magnetic susceptibility of an NS structure. This phenomenon was called a reentrant effect. Until recently it has not been explained adequately.

Earlier[20] the author clarified the nature of the reentrant effect. It was found that the origin of the paramagnetic contribution is closely connected with the properties of the quantized Andreev levels that are dependent on the magnetic flux varying with both temperature and magnetic field. Typically, the levels in the NS structure time from time (at certain values of the field H or temperatures) coincide with the chemical potential of the metal. As a result, the state of the system is highly degenerate and the density of states of the NS structure experiences resonance spikes. The response of the normal mesoscopic layer to a weak magnetic field is paramagnetic.

A theory of the reentrant effect has been developed in this study. We calculated the paramagnetic contribution separately and analyzed its behavior in a varying magnetic field and at varying temperature. In the course of this calculation we corrected the mistake made in Ref. 20 which led to underestimation of the amplitude of the effect. The paramagnetic response is determined only by the trajectories of the quasiparticles that collide with the NS boundary. It is shown that the reentrant effect can occur in a certain range of weak magnetic fields at temperatures no higher than 100 mK. We believe that paramagnetic reentrant effect is an intrinsic effect of mesoscopic NS proximity structures in the very low temperature limit.

Assume that the temperature of the NS structure is about 10^{-3} K and the magnetic field is increasing. As soon as the field exceeds a certain value H_r, the isothermal reentrant effect must vanish. In strong fields the Andreev levels cease to make a resonance contribution to the paramagnetic susceptibility. Now the paramagnetic contribution is made by the states formed by the trajectories of the quasiparticles that collide only with the dielectric boundary. However, their contribution to the resulting susceptibility of the structure is small because of the smallness of the quasiclassical parameter of the problem $1/k_F R$. Under this condition the susceptibility exhibits diamagnetic behavior in all strong fields up to the critical one.

A self-consistent calculation of the screening current of the NS structure was performed taking into account the contribution from the Andreev levels. The analysis of the derived expression suggests the paramagnetic contribution to current. For example, Fig. 2 illustrates the dependence of the current upon the phase (magnetic field). The values of the current j to the left of the extremum Φ_r account for the contribution of the Andreev levels. The derivative of this curve with respect to the field is proportional to the magnetic susceptibility of the NS structure. It is positive ("paramagnetic") in the region of low magnetic fields and negative ("diamagnetic") at high fields.

Similar behavior is observed when the susceptibility of the NS structure is measured as a function of temperature in a pre-assigned weak magnetic field: it is "paramagnetic" in the region $T<T_r$ and "diamagnetic" at $T>T_R$ up to the critical temperature. Temperature dependence of the magnetic susceptibility in the NS structure at fixed magnetic field will be investigated in detail in separate publication.

In the absence of the proximity effect in the NS structure, when the penetrability of the barrier between the S and N metals is small, the electrons of the normal metal are reflected specularly from its boundaries. In this case the SN structure is a total of two isolated subsystems (normal metal and superconductor) placed into a magnetic field. Because of the Meissner effect, diamagnetic current develops near the superconductor surface. In the normal metal, because of the Aharonov–Bohm effect, the quantized spectrum of quasiparticles is dependent on the magnetic flux through the cross section of the cylinder. The flux generates a paramagnetic contribution to the susceptibility whose quasiclassical parameter of the problem $1/k_F R$ is small. Hence, in the absence of the proximity effect no competition is possible between the paramagnetic and diamagnetic contributions in the NS structure, and the reentrant effect is unobservable in such an NS sample.

To conclude, it should be noted that the explanation proposed in this study for the reentrant effect was developed within a model which does not assume electron–electron interaction in the N layer of the NS structure. In terms of the free-electron model, a large paramagnetic contribution to the susceptibility of the NS structure appears in the region of very low temperatures in a weak magnetic field. If we increase the thickness d of the pre-assigned normal metal, this would lead to a greater number n_0 of Andreev levels in the potential well and would affect the solutions of the self-consistent equation for a. As a result, the shape of the curve of the paramagnetic susceptibility would be slightly "deformed," though its qualitative behavior would remain the same.

The author is sincerely grateful to A. N. Omelyanchouk for helpful discussions and support, and to S. I. Shevchenko for valuable comments. I am indebted to I. O. Kulik for reading through the text and useful comments.

APPENDIX

Let us calculate the integral taken over x in Eq. (15):

$$J=\int_0^{x_0}\frac{x^2\theta|\mathfrak{a}_n-bx-\epsilon\sqrt{1+x^2}|dx}{(\mathfrak{a}_n-bx)^4\sqrt{(\mathfrak{a}_n-bx)^2-\epsilon^2(1+x^2)}}. \tag{A1}$$

After introducing the notation $\alpha_0=\mathfrak{a}_n/b$, we can see that the function in front of the radical in the denominator has a singularity at the point $x=\alpha_0$. Besides, as was noted in the text, the integrand has singularities at the points x_1, x_2.

Integral (A1) can be written as a sum of four integrals

$$J=\int_0^{x_0}dx\ldots=\lim_{\epsilon\to 0}\left\{\int_0^{x_1-\epsilon}dx\ldots+\int_{x_1+\epsilon}^{\alpha_0-\epsilon}dx\ldots\right.$$
$$\left.+\int_{\alpha_0+\epsilon}^{x_2-\epsilon}dx\ldots+\int_{x_2+\epsilon}^{x_0}dx\cdots\right\}.$$

It is obvious that the presence of the θ function makes the second and the third integrals equal to zero. We first calculate the integral J_1:

$$J_1=\frac{1}{b^4}\lim_{\epsilon\to 0}\int_0^{x_1-\epsilon}\frac{x^2dx}{(\alpha_0-x)^4\sqrt{Ax^2+Bx+C}}, \tag{A2}$$

where $A=b^2-\varepsilon^2$, $B=-2\mathfrak{a}_n b$, $C=\mathfrak{a}_n^2-\varepsilon^2$. On substituting the variable $\alpha_0-x=1/t$, the indefinite integral becomes

$$\int\frac{t(\alpha_0 t-1)^2dt}{\sqrt{at^2+\beta t+A}},$$

where $\alpha=-(1+\alpha_0^2)\varepsilon^2$, $\beta=2\alpha_0\varepsilon^2$. It can be calculated by the method of undetermined coefficients:

$$\int\frac{f(t)dt}{\sqrt{\alpha t^2+\beta t-\gamma}}=(A_1t^{n-1}+A_2t^{n-2}+\cdots+A_n)\sqrt{\alpha t^2+\beta t+\gamma}+A_{n-1}\int\frac{dt}{\sqrt{\alpha t^2-\beta t-\gamma}}$$

if $f(t)$ is the polynomial to power n. Although the calculation is tedious, it is actually simple. The coefficients A_1, A_2, A_3, and A_4 are readily found as:

$$A_1=-\frac{\alpha_0^2}{3\epsilon^2(1+\alpha_0^2)},\quad A_2=\frac{\alpha_0(1+\alpha_0^2/\epsilon)}{\epsilon^2(1+\alpha_0^2)^2},$$

$$A_3=-\frac{2\mathfrak{a}_n^2}{3\epsilon^4(1+\alpha_0^2)^2}+\frac{-3-5\alpha_0^2+\alpha_0^4/2}{3\epsilon^2(1+\alpha_0)^3},$$

$$A_4=\frac{\mathfrak{a}_n^2(\alpha_0^2/2-1)}{\epsilon^2\alpha_0(1+\alpha_0^2)^2}+\frac{\alpha_0(2-\alpha_0^2/2)}{(1+\alpha_0^2)^3}.$$

It is seen that the coefficients have different orders of ε^{-1}-magnitude: $A_1,A_2,A_4\simeq\varepsilon^{-2}$, $A_3\simeq\varepsilon^{-4}$. Finally, we have to calculate six integrals

$$J_1=\frac{1}{b^4}\lim_{\epsilon\to 0}\int_{t_0}^{t_1-\epsilon}\left\{2A_1t\sqrt{R(t)}+A_2\sqrt{R(t)}+A_1\alpha t^3\sqrt{R(t)}+(A_2\alpha+A_1\beta/2)\frac{t^2}{\sqrt{R(t)}}+\left(A_3\alpha+A_2\frac{\beta}{2}\right)\frac{t}{\sqrt{R(t)}}+\frac{(A_3\beta/2+A_4)}{\sqrt{R(t)}}\right\}, \tag{A3}$$

where $R(t)=\alpha t^2+\beta t-A$ and the designations $t_0=1/\alpha_0$, $t_1-\varepsilon=(\alpha_0-x_1+\varepsilon)^{-1}$ are introduced. All the six indefinite integrals in expression (A3) can be calculated exactly.[29] After substituting the limits of integration, integrals 1, 2, 3, 4 and 5 are bounded above in energy on account of the term $\sqrt{R(t)}\simeq\sqrt{\mathfrak{a}_n^2-\varepsilon^2}/\alpha_0$, i.e., $\theta(\mathfrak{a}_n-\varepsilon)$. Taking into account the determined coefficients A_i $(i=1,2,3,4)$, we can obtain the final expression for J_1:

$$J_1\simeq\frac{1}{b^4}\left\{\frac{b(\alpha_0^2+1/3)}{3\epsilon^2(1+\alpha_0^2)^2}-\frac{b(1+\alpha_0^2/6)}{\epsilon^2(1+\alpha_0^2)^2}+\frac{2\mathfrak{a}_n^2b}{3\epsilon^4(1+\alpha_0^2)^2}+\frac{b\left(1-\frac{5}{3}\alpha_0^2-\alpha_0^4/6\right)}{\epsilon^2(1+\alpha_0^2)^3}+\frac{\alpha_0b^2(\alpha_0^2/2-1)}{\epsilon^3(1+\alpha_0^2)^{5/2}}+\frac{\alpha_0(2-\alpha_0^2/2)}{\epsilon(1+\alpha_0^2)^{7/2}}-\frac{b(\alpha_0^2/2-1)}{\epsilon^2(1+\alpha_0^2)^2}-\frac{(2-\alpha_0^2/2)}{(1+\alpha_0^2)^3}\right\}. \tag{A4}$$

Of all the terms in (A4), the most significant contribution is made by the third term because there is a factor ε^4 in the numerator of the integral over the energy in Eq. (17). The contributions of the other terms are negligible. We thus obtain the estimate

$$J_1\simeq\frac{2\alpha_0^2}{3b(1+\alpha_0^2)^2}\frac{1}{\epsilon^4}. \tag{A5}$$

A similar calculation of the integral

$$J_4=\frac{1}{b^4}\lim_{\epsilon\to 0}\int_{x_2+\epsilon}^{x_0}\frac{x^2dx}{(x-\alpha_0)^4\sqrt{Ax^2+Bx+C}}$$

gives a contribution which is identical in order of magnitude with (A5). As a result, we obtain the J estimate presented in the text, Eq. (19).

[a]Electronic mail: gogadze@ilt.kharkov.ua

[1]Y. Imry, in *Directions in Condensed Matter Physics*, G. Grinstein and G. Mazenko (eds.), World Scientific, Singapore (1986), p. 101.
[2]S. Washburn and R. A. Webb, Adv. Phys. **35**, 375 (1986).
[3]A. G. Aronov and Yu. V. Sharvin, Rev. Mod. Phys. **59**, 755 (1987).
[4]I. O. Kulik, JETP Lett. **11**, 275 (1970).
[5]Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[6]E. N. Bogachek and G. A. Gogadze, Zh. Eksp. Teor. Fiz. **63**, 1839 (1972) [Sov. Phys. JETP **36**, 973 (1973)].
[7]N. B. Brandt, V. D. Gitsu, A. A. Nikolaeva, and Ya. G. Ponomarev, JETP Lett. **24**, 272 (1976); Zh. Éksp. Teor. Fiz. **72**, 2332 (1977) [Sov. Phys. JETP **45**, 1226 (1977)].
[8]N. B. Brandt, E. N. Bogachek, D. V. Gitsu, G. A. Gogadze, I. O. Kulik, A. A. Nikolaeva, and Ya. G. Ponomarev, Fiz. Nizk. Temp. **8**, 718 (1982) [Sov. J. Low Temp. Phys. **8**, 358 (1982)].
[9]A. C. Mota, P. Visani, and A. Pollini, J. Low Temp. Phys. **76**, 465 (1989).
[10]P. Visani, A. C. Mota, and A. Pollini, Phys. Rev. Lett. **65**, 1514 (1990).
[11]A. C. Mota, P. Visani, A. Pollini, and K. Aupke, Physica B **197**, 95 (1994).
[12]F. B. Muller-Allinger and A. C. Mota, Phys. Rev. Lett. **84**, 3161 (2000).
[13]F. B. Muller-Allinger and A. C. Mota, Phys. Rev. B **62**, R6120 (2000).
[14]R. Frassanito, P. Visani, M. Niderost, A. C. Mota, P. Smeibidl, K. Swieca, W. Wendler, and F. Pobell, *Proceedings LT-21, Part S4-LT; Properties of Solids 1*, Prague (1996), p. 2317.
[15]A. F. Andreev, Zh. Éksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **9**, 1228 (1964)].
[16]G. A. Gogadze, R. I. Shekhter, and M. Jonson, Fiz. Nizk. Temp. **27**, 1237 (2001) [Low Temp. Phys. **27**, 913 (2001)].
[17]C. Bruder and Y. Imry, Phys. Rev. Lett. **80**, 5782 (1998).
[18]A. L. Fauchere, W. Belzig, and G. Blatter, Phys. Rev. Lett. **82**, 3336 (1999).
[19]K. Maki and S. Haas, cond-mat/0003413 (2000).
[20]G. A. Gogadze, Fiz. Nizk. Temp. **31**, 120 (2005) [Low Temp. Phys. **31**, 94 (2005)].
[21]G. A. Gogadze, Fiz. Nizk. Temp. **9**, 1051 (1983) G. A. Gogadze [Sov. J.

Low Temp. Phys. **9**, 543 (1983)].
[22] J. B. Keller and S. I. Rubinow, Ann. Phys. (N.Y.) **9**, 24 (1960).
[23] I. O. Kulik, Zh. Éksp. Teor. Fiz. **57**, 1745 (1969) [Sov. Phys. JETP **30**, 944 (1969)].
[24] A. D. Zaikin, Solid State Commun. **41**, 533 (1982).
[25] W. Belzig, G. Bruder, and G. Schon, Phys. Rev. B **53**, 5727 (1996).
[26] A. L. Faucher and G. Blatter, Phys. Rev. B **56**, 14102 (1997).
[27] A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, *Integrals and Series*, Gordon and Breach, New York (1986), Nauka, Moscow (1984).
[28] A. V. Galaktionov and A. D. Zaikin, Phys. Rev. B **67**, 184518 (2003).
[29] I. S. Gradshtein and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, New York (1980), Nauka, Moscow (1971).

This article was published in English in the original Russian journal. Reproduced here with stylistic changes by AIP.