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Coupling Energy Dependence of Spectral Properties in Weak Superconducting Contacts

By

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The sensitivity to the type of dependence between the supercurrent and the quantum phase is investigated for the $I_n(i)$ dependences of the zero-currents and current steps in the U–I curves of the Josephson point contacts on the amplitude of an external electromagnetic wave, i. Especially the discrepancies are considered between the calculated and the experimental data on the spectral properties of the Josephson contacts which rise, due to the use of a simple $j(\varphi) \sim \sin \varphi$ function independently of the coupling energy in the weak superconducting contact. A possible technique for the construction of the $j(\varphi)$ function by experimental $I_n(i)$ curves is proposed.

Экспериментально показана чувствительность зависимостей $I_n(i)$ нулевого тока и высоты ступеней тока на вольтамперных характеристиках точечных контактов Джозефсона от амплитуды i внешней электромагнитной волны к виду зависимости между током и квантовой фазой. Обращается внимание на расхождение между расчетными и экспериментальными данными по спектральным свойствам контактов Джозефсона, возникающие из-за использования простой функции $j(\varphi) \sim \sin \varphi$ независимо от энергии связи слабого сверхпроводящего контакта. Предлагается возможный метод восстановления функции $j(\varphi)$ по экспериментальным кривым $I_n(i)$.

1. Introduction

As it was already pointed out [1] the spectral properties (a spectrum of current steps in the U–I characteristics and the dependence of zero-currents I_0 and current steps I_n upon a microwave amplitude v) in the tunneling [2] and bridge [3] contacts are strongly dependent on the coupling energy and cannot be considered as being characteristic of a certain type of contacts. We managed to observe, for example, under certain experimental conditions the subharmonic current steps in the U–I curves of tunneling contacts [4]. The most convenient way to observe the changes of the spectral characteristics in a point contact [5] is to vary the normal resistance and the temperature of the latter. Recently analogous results have been obtained by varying resistance and temperature in thin-film bridges [6]. A method allowing to find the kind of current–phase dependence $j(\varphi)$ by means of the pattern of the critical current oscillations of the interferometer has been proposed in [7] as well. The experimental data [6, 8] are shown to be in poor consistency with the calculated ones. In the present paper we should like to emphasize the obvious causes of discrepancies between the experimental and the calculated data and to point out one more possibility to restore the $j(\varphi)$ dependence by experimental data.

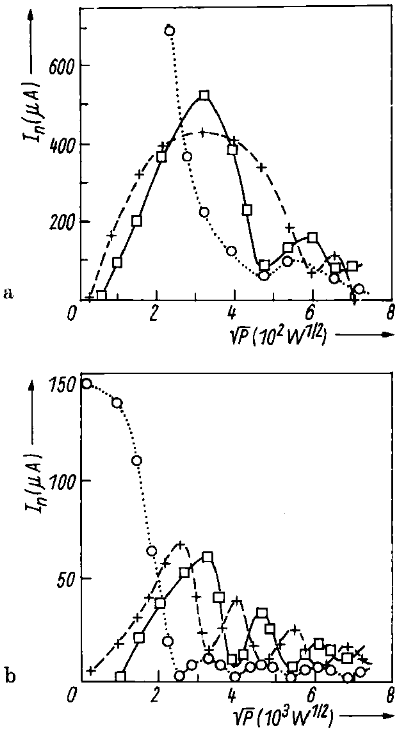


Fig. 1. $I_n(\sqrt{P})$ dependence of the critical currents and the first and the second current steps upon the amplitude of electromagnetic wave. Ta–Ta point contact is at 2.4 K, P is microwave power. Here and in Fig. 2 and 3 $\sqrt{P}$ is given in units of $10^{-3} \cdot W^{-1/2}$. a) Contact resistance $R = 0.08\ \Omega$; b) contact resistance $R = 0.50\ \Omega$; $\cdots \circ \cdots$ $n = 0$, $-+-$ $n = 1$, $-\square-$ $n = 2$

2. Experimental Technique

In our experiments the contacts were placed within an X-band rectangular waveguide in the electric field maximum. Taking into account the Josephson inductance, the contact impedance at ≈ 10 GHz was never found to exceed 2 to 3%. The multicoupled contacts have not been studied. In the experiments described here we have considered the contacts, exposed to external microwave radiation, with the coupling energy high enough to form the distinct current steps in the U–I curves. The behaviour of low coupling energy contacts and the fluctuation effects have been considered elsewhere [9].

3. Results and Discussion

The subharmonic current steps and aperiodic $I_n(v)$ dependence were found to occur both when the temperature was decreased and when the resistance was lowered, which is very important in our opinions. The $I_n(\sqrt{P})$ dependences of zero currents, the first and the second current steps in a Ta–Ta contact at the same temperature but at two different resistance values (produced by varying the degree of a contact pressing) are shown in Fig. 1b. It is seen that when resistance is decreased the Bessel function periodicity has disappeared, being replaced by a nearly aperiodic $I_n(\sqrt{P})$ dependence. In contrast to the experiments, carried out with the contacts having external shunts [10], the resistance decrease in our case has led to an increase of the coupling energy in the contact. In a number of the theoretical papers [11] it was pointed out that with increasing coupling energy the simple dependence ($j \sim \sin\varphi$) was found to be changed by a more complex periodic function which can be written as ([1] and Bloch's paper [11])

$$j = \sum_n j_n \sin n\varphi\,. \tag{1}$$

It should be noted as well, that when the resistance R is changed the characteristic parameter of the theory $\gamma = \hbar\Omega/2\,eI_0R$ must not be varied (at least considerably) because one can consider a relation of the form $I_0 \sim 1/R$ to be

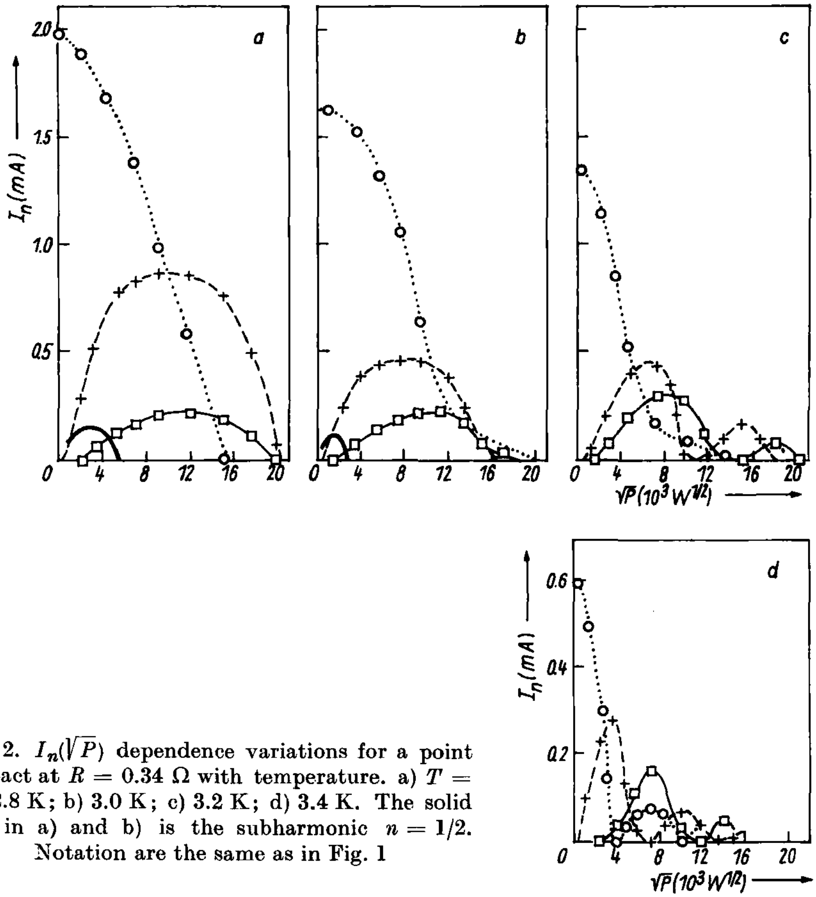
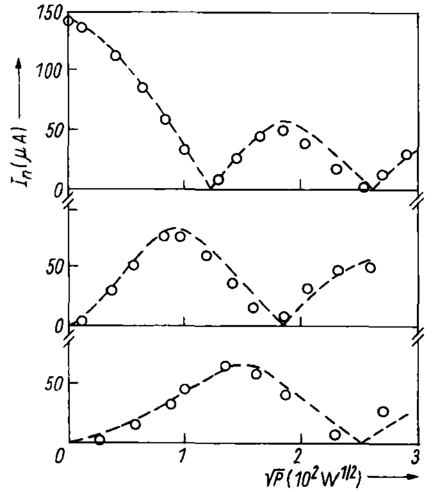


Fig. 2. $I_n(\sqrt{P})$ dependence variations for a point contact at $R = 0.34\ \Omega$ with temperature. a) $T = 2.8$ K; b) 3.0 K; c) 3.2 K; d) 3.4 K. The solid line in a) and b) is the subharmonic $n = 1/2$. Notation are the same as in Fig. 1

valid for point contacts. Thus, the nonlinearity of solutions $\varphi(t)$ in the equation for total currents (of phase relation) [13]

$$I + i \sin \Omega t = \frac{U}{R} + C \frac{dU}{dt} + I_0 \sin \varphi \tag{2}$$

(i is the amplitude and Ω the frequency of an external field) must not be changed significantly as well, which is not in agreement with the experimental data. We think, that the correct results, calculated by means of a computer can be obtained only when one takes into account (1) which affects the kind of $I_n(i)$ dependence stronger than the nonlinearity of the $\varphi(t)$ function being determined by the conditions of a contact involved in an external circuit and by the value of the γ parameter. The temperature decrease changes the γ parameter as well as the coupling energy. In this case the $I_n(i)$ curves were also seen to be abruptly changed (Fig. 2). It should be noted that Russer [8] has succeeded to obtain only the periodic $I_n(i)$ dependences in the limit $0.1 \leqq \gamma \leqq 1$ and that only the substitution of (1) can yield aperiodic $I_n(i)$ curves. Indeed, the experiments showed ([1, 12] and Fig. 3a) even under the given current conditions (direct



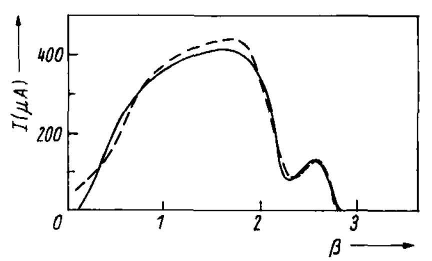


Fig. 3a. $I_n(\sqrt{P})$ dependence for $n = 0, 1, 2$ for a Ta–Ta contact at $R = 1.08$ and $T = 4.0$ K. Dashed line is the modulus of the Bessel function of 0,1, and 2 orders. b) Experimental (solid line) and calculated by a computer (dashed line) dependence of the first current step in U–I curve of a low-resistance Ta–Ta contact upon the microwave amplitude. Approximation is carried out by the sum of the Bessel function of the first eight orders with the coefficients

$j_1 = -1384\ \mu A$, $j_2 = 3233\ \mu A$,
$j_3 = -4464\ \mu A$; $j_4 = -1535\ \mu A$,
$j_5 = 1877\ \mu A$, $j_6 = 3307\ \mu A$,
$j_7 = 2444\ \mu A$, $j_8 = 617\ \mu A$

and microwave) the harmonic behaviour of the weakly coupled point contacts, which seems not to be consistent with the results of Waldram et al. [8].

The experiments we carried out indicate that the $I_n(i)$ dependences are very sensitive spectral characteristics from which the series (1) can be in principle obtained. If (1) is the case, the expansion for the frequency-modulated oscillations and for the dependences of the steps upon the incident wave amplitude $v \cos(\Omega t)$ is:

$$j(t) = \sum_{m=1}^{\infty} j_{0m} \sum_{n=0}^{\infty} J_n(m\beta) \left[\cos(m\omega + n\Omega)t + (-1)^n \cos(m\omega - n\Omega)t\right],$$

$$j_{n_0 m_0} = \left|\sum_{k=1}^{\infty} (-1)^{m_0 k} j_{m_0 k} J_{n_0 k}(m_0 k \beta)\right|, \qquad \beta = \frac{2ev}{\hbar\Omega}, \qquad \omega = \frac{2eU}{\hbar},$$

J_n is the Bessel function of order n.

The subharmonics are seen to appear at voltages

$$U_{n/m} = \frac{n}{m}\frac{\hbar\Omega}{2e}, \qquad n = n_0 k, \qquad m = m_0 k,$$

where m_0, n_0 are minimum integers, which meet the condition of the current steps appearing in the U–I curves. Now the $I_{nm}(v)$ curves can be expressed by a sum of Bessel functions. The $I_1(\sqrt{P})$ dependence of a low-resistance Ta–Ta contact, approximated by the Bessel function of the first eight orders is shown in Fig. 3b. The expansion coefficients are harmonic amplitudes of the Josephson currents in (1). Substituting $\sin\varphi$ in (2) by the series (1) with the coefficients obtained by the approximation of the experimental $I_n(\sqrt{P})$ curve, we get the equation, its numerical solutions, as we hope, should be in a better agreement with the experiments. If the currents but not the voltages are given (2) has no analytical solutions, however, as the calculations of Russer indicate even in this case the $I_n(i)$ curves are close to the simple Bessel functions at high γ-values and are not very different from them at low γ-values.

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