On constructing single-input non-autonomus systems of full rank

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The paper develops the method of constructing a system of full rank proposed in [Y. Kawano, Ü. Kotta, C.H. Moog. Any dynamical system is fully accessible through one single actuator, and related problems, Intern. J. of Robust and Nonlinear Control, – 2016. – 8. V.**26**. – P. 1748-1754.]. The problem is as follows: given a vector field $f(x)$, find a vector field $g(x)$ such that the resulting affine control system $\dot{x} = f(x) + g(x)u$ has full rank. In the mentioned paper it was shown that such $g(x)$ exists in a neighborhood of a point x if $f(x) \neq 0$, and a method of constructing $g(x)$ was proposed. As the main tool, the straightening theorem for vector fields was used; in fact, after straightening the vector field $f(x)$, one constructs a linear controllable system. However, only the case of real analytic vector fields was considered. In the present paper we consider two generalizations.

First, we study the question for vector fields $f(x) \in C^k$, $1 \leq k < \infty$. We show that the proposed method is applicable, however, the vector field $g(x)$, generally, belongs only to the class C^{k-1}. We give an example of the vector field $f(x) \in C^1$, namely, $f(x) = (0, 1/(1 + x_1|x_1|))^T$, for which the method yields a non-differentiable (though continuous) vector field $g(x)$. Second, we consider the case when $f(x)$ vanishes, and describe a method of constructing a vector field $g(t, x)$ which, in general, is non-autonomous, such that the system $\dot{x} = f(x) + g(t, x)u$ has full rank. Again, the straightening theorem for vector fields is applied, however, for an extended system in which the time is an additional coordinate.

We give an example of a linear vector field $f(x) = (0, x_1)^T$ in a neighborhood of the origin where the resulting vector field turns out to be autonomous, namely, $g(x) = (1, 0)^T$. Also we give an example of a nonlinear vector field

$f(x) = (x_1^2, x_2)^T$ in a neighborhood of the origin; the corresponding non-autonomous vector field has the form $g(t, x) = ((x_1 t + 1)^2, te^t)^T$.

References

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