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of the National Academy of Sciences of Ukraine

V.N. Karazin Kharkiv National University

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dedicated to Vladimir A. Marchenko's 95th birthday
and the centennial anniversary of the National
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CONTENTS

INVITED SPEAKERS

Phase field models for material interfaces – the regularizing effect of interface energy

Hans-Dieter Alber, *Darmstadt, Germany*

Phase field models are popular in material science to simulate the propagation of interfaces in solids under the influence of stress and the electric and magnetic fields in the solid. The model should yield the propagation speed of the interface accurately and at the same time regularize the interface sufficiently to make the numerical simulation effective. These are conflicting requirements, since accurate propagation speed requires small interface width, effective simulation needs large interface width.

For interfaces carrying interface energy these requirements can be reconciled, since the naturally present interface energy increases the parabolicity of the model, and the propagation speed is accurate even if the interface width is large. However, for interfaces with low interface energy these properties can not be met at the same time. Engineers are almost never aware of this problem.

To show that these facts described above are really true one must determine the propagation speed of the interface with sufficiently small error. We discuss how this is done. One constructs an approximate solution, which is technical, but in principle, is a known procedure. The new mathematical problem is to derive an estimate for the error in the propagation speed and to show that this estimate is uniformly valid with respect to two parameters, the interface width and the interface energy.

[1] H.-D. Alber, *Asymptotics and numerical efficiency of the Allen-Cahn model for phase interfaces with low energy in solids.* Continuum Mech. Thermodyn. 29,3 (2017), 757 – 803. doi:10.1007/s00161-017-0558-x arxiv.org/abs/1505.05442

[2] H.-D. Alber, *An alternative to the Allen-Cahn phase field model for interfaces in solids – numerical efficiency.* In: Bettina Albers, Mieczysław Kuczma (eds.), Continuous Media with Microstructure 2. Springer, 2016, 121–136. doi:10.1007/978-3-319-2824-1_8 (Proceedings of the 2nd conference on Continuous Media with Microstructure, Łagów, Poland 2015.) arxiv.org/abs/1704.01092

The Steklov problem and estimates for orthogonal polynomials

Alexandr Aptekarev, *Moscow, Russia*

A Steklov's problem is to obtain the bounds on the sequence of orthonormal polynomials at the support of the weight of orthogonality. In 1921 [1], V.A.

Steklov made a conjecture that if weight of orthogonality is strictly positive then sequence of orthonormal polynomials (at the support of the weight) is bounded.

In 1979 [2] E.A. Rakhmanov disproved this conjecture by constructing the sequence of polynomials orthonormal with respect to a positive weight which has the logarithmical rate of growth. Then the Steklov's problem becomes: to obtain the maximal possible rate of growth for these sequences. The modern version of the Steklov's problem is intimately related with the following extremal problem. For a fixed $n \in \mathbb{N}$, find

$$M_{n,\delta} = \sup_{\sigma \in S_\delta} \|\phi_n\|_{L^\infty(\mathbb{T})},$$

where $\phi_n(z)$ is the orthonormal polynomials with respect to the measure $\sigma \in S_\delta$, and S_δ is the Steklov's class of probability measures σ on the unit circle, such that $\sigma'(\theta) \geqslant \delta/(2\pi) > 0$ at every Lebesgue point of σ.

There is an elementary bound

$$M_n \lesssim \sqrt{n}.$$

In 1981 [3] E.A. Rakhmanov have proved:

$$M_n \gtrsim \sqrt{n}/(\ln n)^{\frac{3}{2}}.$$

In our joint paper with S.A. Denisov and D.N. Tulyakov [4] we have proved, that

$$M_n \gtrsim \sqrt{n},$$

i.e. the elementary bound is sharp.

In our talk we discuss the history, statement of the problem and details of the construction of a sequence of the orthonormal polynomials from the Steklov class which have the maximal possible rate of growth. We also consider the Steklov problem in L_p spaces and give new upper estimates for the polynomials orthonormal with respect Muckenhoupt weights from the Steklov's class.

[1] W. A. Stekloff, *Une méthode de la solution du problême de développement des fonctions arbitraires en séries de polynômes de Tchébysheff indépendante de la théorie de fermeture. II*, Izv. Rus. Ac. Sci., 1921, 281–302, 303–326.

[2] E. A. Rahmanov, *On Steklov's conjecture in the theory of orthogonal polynomials*, Matem. Sb. 108(150) (1979), 581–608; English translation in: Math. USSR, Sb. 36 (1980), 549–575.

[3] E. A. Rahmanov, *Estimates of the growth of orthogonal polynomials whose weight is bounded away from zero*, Matem. Sb. 114(156):2 (1981), 269–298; English translation in: Math. USSR, Sb. 42 (1982), 237–263.

[4] A. Aptekarev, S. Denisov, D. Tulyakov, *On a problem by Steklov*, Journal of AMS 29(4) (2016), 1117–1165.

Phase field and free boundary models of cell motility

Leonid Berlyand, *State College, PA, USA*

We study two types of models describing the motility of eukaryotic cells on substrates. The first, a phase-field model, consists of the Allen-Cahn equation for the scalar phase field function coupled with a vectorial parabolic equation for the orientation of the actin filament network. The two key properties of this system are (i) presence of gradients in the coupling terms and (ii) mass (volume) preservation constraints. We pass to the sharp interface limit to derive the equation of the motion of the cell boundary, which is mean curvature motion modified by a novel nonlinear term. We establish the existence of two distinct regimes of the physical parameters and prove existence of traveling waves in the supercritical regime.

The second model type is a non-linear free boundary problem for a Keller-Segel type system of PDEs in 2D with area preservation and curvature entering the boundary conditions. We find an analytic one-parameter family of radially symmetric standing wave solutions (corresponding to a resting cell) as solutions to a Liouville type equation. Using topological tools, traveling wave solutions (describing steady motion) with non-circular shape are shown to bifurcate from the standing waves at a critical value of the parameter. Our bifurcation analysis explains, how varying a single (physical) parameter allows the cell to switch from rest to motion.

The work was done jointly with J. Fuhrmann, M. Potomkin, and V. Rybalko.

Conditional measures of determinantal point processes: the Gibbs property and the Lyons–Peres conjecture

Alexander Bufetov, *Moscow, Russia*

Determinantal point processes arise in many different problems: spanning trees and Gaussian zeros, random matrices and representations of infinite-dimensional groups. How does the determinantal property behave under conditioning? The talk will first address this question for specific examples such as the sine-process, where one can explicitly write the analogue of the Gibbs condition in our situation. We will then consider the general case, where, in joint work with Yanqi Qiu and Alexander Shamov, proof is given of the Lyons-Peres conjecture on completeness of random kernels.

The talk is based on the preprint arXiv:1605.01400 as well as on the preprint arXiv:1612.06751 joint with Yanqi Qiu and Alexander Shamov.

Asymptotic solutions to 2-D linear wave equation with degenerated velocity, Fock quantization of canonical transforms and the run-up problem in the framework of shallow water models

Sergey Dobrokhotov, *Moscow, Russia*
Vladimir Nazaikinskii, *Moscow, Russia*

We consider the Cauchy problem with localized initial data for the two-dimensional wave equation with variable velocity in a domain D, which describes long linear waves in shallow water. We assume that the velocity $C(x, y) = 0$ and $\nabla C(x, y)$ does not vanish on the boundary (shore) of D. We construct asymptotic solutions of this problem using a modified semiclassical approximation and the Maslov canonical operator. It is determined by the fronts, which are formed by the trajectories of the Hamiltonian system with Hamiltonian $H(p, x) = C(x, y)|p|$. When the trajectories reach the boundary of the domain D, the momentum p of the trajectories goes to infinity; in some sense one can view the boundary of D as a caustic of a special kind. We define a canonical transformation that leads to the compactification of the trajectories and allows one to continue the trajectories through this singularity. Near this boundary-caustic, the standard semiclassical approximation does not work, and we use the Fock quantization formulas for the canonical transformation, which results in the use of the Hankel transform in a neighborhood of this type, caustic, and gives an asymptotic solution to the original linear problem. It turns out that the solution on the boundary of D has the standard form for the canonical operator on some special curve in the two-dimensional phase space. Next, we consider the nonlinear shallow water equations. Using Pelinovskii-Masova ideas and the Carrier-Greenspan transformation, we derive formulas for the the run-up on the shore of long waves (e.g. tsunami waves) generated by localized sources. As a result, we obtain simple analytical formulas relating the magnitude of the uprush of long waves with the parameters of the generating source.

This work was supported by the Russian Science Foundation (project no. 16-11-10282).

Real solutions of Painleve VI equation and circular pentagons

Alexandre Eremenko, *West Lafayette, IN, USA*

We consider a class of Painleve VI equations with real parameters and real solutions $y(x)$ on the ray $x > 1$. A point $x > 1$ is called special if $y(x) = 0, 1, x$ or infinity. (These points are removable singularities of solutions.) We give an algorithm which determines the number and mutual position of the special points on the ray $x > 1$. The algorithm is based on a correspondence between such real solutions and special families of circular pentagons spread over the Riemann sphere.

Zakharov–Kuznetsov equation: well-posedness, controllability, scattering, regularity

Andrei Faminskii, *Moscow, Russia*

Initial-boundary value problems are considered for Zakharov–Kuznetsov equation $u_t + u_x + u_{xxx} + u_{xyy} + uu_x = 0$ on various plane domains of the type $I_1 \times I_2$, where I_1 and I_2 are certain intervals (bounded or unbounded) on $\mathbb{R}$ (in particular, the initial value problem) with different types of boundary conditions.

Consider the problem on the rectangle $\Sigma = (0, R) \times (0, L)$. Then besides the initial condition $u\big|_{t=0} = u_0(x, y)$, boundary conditions $u\big|_{x=0} = 0$, $u\big|_{x=R} = 0$, $u_x\big|_{x=R} = h(t, y)$ are set as well as boundary conditions on the sides $y = 0$ and $y = L$, which can be homogeneous Dirichlet, homogeneous Neumann or periodic.

Results on global well-posedness in the classes of weak and regular solutions, on boundary contollability and on large-time decay of small solutions are established.

Consider, for example, the case of periodic boundary conditions with respect to y. Let $T > 0$ be arbitrary, $B_T = (0, T) \times (0, L)$.

Theorem 1. *Let $u_0 \in L_2(\Sigma)$, $h \in L_2(B_T)$, Then the problem is well-posed in the space $C_w([0, T]; L_2(\Sigma)) \cap L_2(0, T; H^1(\Sigma))$.*

Theorem 2. *Let $u_0 \in H^3(\Sigma)$, $h \in H^{1,3}_{t,y}(B_T)$, $u_0(0, y) = u_0(R, y) \equiv 0$, $u_{0x}(R, y) \equiv h(0, y)$, $\partial_y^j u_0(x, 0) \equiv \partial_y^j u_0(x, L)$, $\partial_y^j h(t, 0) \equiv \partial_y^j h(t, L)$ for $j \le 2$. Then the problem is well-posed in the space $\{\partial_t^m u \in C([0, T]; H^{3(1-m)}(\Sigma)) \cap L_2(0, T; H^{3(1-m)+1}(\Sigma)), m \le 1\}$.*

Theorem 3. *Let $u_0 \in L_2(\Sigma)$, $h \equiv 0$, $bR^2 < 3\pi^2$, then there exist $\epsilon_0 > 0$ and $\kappa > 0$, such that if $\|u_0\|_{L_2(\Sigma)} \leq \epsilon_0$, the corresponding unique solution to the considered problem $u \in C_w([0,T]; L_2(\Sigma)) \cap L_2(0,T; H^1(\Sigma))$ $\forall T > 0$ satisfies an inequality $\|u(t,\cdot,\cdot)\|_{L_2(\Sigma)} \leq \sqrt{1+R}e^{-\kappa t}\|u_0\|_{L_2(\Sigma)}$ $\forall\, t > 0$.*

Consider also the inverse problem on Σ with additional condition $u\big|_{t=T} = u_T$, but with unknown function h.

Theorem 4. *Let for all integer $l \geq 0$ either $L \leq 2\pi l$ or $R \neq 2\pi L\big[(k^2+km+m^2)/(3L^2-12\pi^2 l^2)\big]^{1/2}$ for all natural k, m. Then there exists $\delta > 0$, such that if $\|u_0\|_{L_2(\Sigma)} \leq \delta$, $\|u_T\|_{L_2(\Sigma)} \leq \delta$, the inverse problem has a unique solution $h \in L_2(B_T)$, $u \in C([0,T]; L_2(\Sigma)) \cap L_2(0,T; H^1(\Sigma))$.*

Stark–Wannier resonances and cubic exponential sums

Alexander Fedotov, *Saint Petersburg, Russia*

The talk is based on a joint work with Frederic Klopp (University Paris VI). We discuss the Schrödinger operator $H = -\frac{\partial^2}{\partial x^2} + v(x) - \epsilon x$, where v is an entire 1-periodic function, and $\epsilon > 0$ is a constant. It describes the Bloch electron in a constant electric field. The ϵ is proportional to the electric field.

The spectrum of H is absolutely continuous and fills the real axis. The operator attracted attention after the discovery of the Stark-Wannier ladders that are ϵ-periodic sequences of resonances (the poles of the meromorphic continuation of the resolvent from the upper half of the complex plane across the spectrum), see [1, 3]. The ladders were studied in the case of small ϵ, see, e.g., [2]. The ladders non-trivially "interact" as ϵ changes, and physicists conjectured that they depend on the arithmetic nature of ϵ, see, e.g., [3].

For $v(x) = 2\cos(2\pi x)$, we study the reflection coefficient $r(E)$ in the lower half of the complex plane. There, r is an analytic ϵ-periodic function. Its zeros are resonances (and vice versa). Represent $1/r$ by its Fourier series, $1/r(E) = \sum_{m\in\mathbb{Z}} p(m) e^{2\pi i m E/\epsilon}$. Set $a(\epsilon) = \sqrt{2/\epsilon}\,\pi e^{i\pi/4}$. We prove that

$$p(m) = a(\epsilon)\,\sqrt{m}\,e^{-2\pi i\omega m^3 - 2m\log(2\pi m/e) + \delta(m)}, \quad \omega = \left\{\frac{\pi^2}{3\epsilon}\right\}, \qquad m \to \infty,$$

where $\{x\}$ denotes the fractional part of $x \in \mathbb{R}$, and $\delta(m) = O(\log^2 m/m)$, the estimate being locally uniform in $\epsilon > 0$.

The behavior of $1/r(E)$ as $\operatorname{Im} E \to -\infty$, is determined by the Fourier series terms with large positive m, and so, roughly, as $\operatorname{Im} E \to -\infty$,

$$\frac{1}{r(E)} \approx a(\epsilon)\,\mathcal{P}(E/\epsilon), \quad \mathcal{P}(s) = \sum_{m\geq 1} \sqrt{m}\, e^{-2\pi i\omega m^3 - 2m\log(2\pi m/e) + 2\pi i m s}.$$

As the cubic exponential sum $\sum_{n=1}^{N} e^{-2\pi i\omega n^3}$ studied for large N in the analytic number theory, see [4], the function $\mathcal{P}$ strongly on the arithmetic nature of ω. In the talk, we describe the resonances for rational ω.

[1] M. Gluck, A.R. Kolovsky and H.J. Korsch, Physics Reports 366 (2002), 103–182.

[2] V. Buslaev and A. Grigis, Journal of mathematical physics 39 (1998), 2520–2550.

[3] J.E. Avron, Annals of physics 143 (1982), 33–53.

[4] H. Davenport, *Analytic methods for Diophantine equations and Diophantine inequalities*, Cambridge University Press, 2004.

The Riemann–Hilbert approach to random matrices

Alexander Its, *Indianapolis, West Lafayette, IN, USA*

The Riemann–Hilbert approach in the theory of random matrices was introduced in the 90s as an adaptation of the method of the Inverse Scattering Transform to orthogonal polynomials. Since then, the techniques has been used in a big variety of problems and helped with the resolution of many long-standing question of the theory of orthogonal polynomials and random matrices. In the talk, an overview of the method will be presented with special focus on the recent results related to the double scaling limit of the sine-kernel determinant near the critical value of its spectral parameter. This determinant is one of the two most basic universal distribution functions of the random matrix theory.

The results which will be presented are obtained in the joint work with Thomas Bothner, Percy Deift, and Igor Krasovsky.

On the wellposedness of integrable PDEs: a survey of new results for the KdV, KdV2, and mKdV equations

Thomas Kappeler, *Zurich, Switzerland*

In form of a case study, I survey the "nonlinear Fourier" method to solve nonlinear dispersive equations such as the Korteweg-de Vries (KdV) equation or the nonlinear Schrödinger (NLS) equation. A key ingredient for desribing the solutions are the frequencies, associated to such type of equations. A novel approach of representing them allows to extend the solution map of such equations to spaces of low regularity and to study its regularity properties. This is joint work with Jan Molnar.

Homogenization in domain with holes revisited

Andrii Khrabustovskyi, *Karlsruhe, Germany*

Let Ω be a domain in $\mathbb{R}^n$, $\varepsilon > 0$ be a small parameter and Ω^ε be a domain, which is obtained by removing from Ω a lot of small sets. It is assumed that these sets are distributed evenly in Ω and, when $\varepsilon \to 0$, their diameters tend to zero, while their total number tends to infinity. We denote by $-\Delta_{\Omega^\varepsilon}$ the Dirichlet Laplacian in Ω. The goal is to describe the behaviour of its resolvent as $\varepsilon \to 0$. It turns out (see [1, 2]) that $-\Delta_{\Omega^\varepsilon}$ *strongly* resolvent converges to the limit operator $\mathcal{H}$ acting in $L_2(\Omega)$ for which three scenarios are possible:

- $\mathcal{H} = 0$ (large holes regime)
- $\mathcal{H} = -\Delta_\Omega$ is the Dirichlet Laplacian in Ω (tiny holes regime)
- $\mathcal{H} = -\Delta_\Omega + qu$, where $q(x)$ is a non-negative potential (borderline regime).

In this talk we present an improvement of this result by proving the *norm* resolvent convergence of the underlying operators. The proof is based on the abstract scheme proposed in [3]. We also discuss the convergence of spectra.

This is a joint work with O. Post (University of Trier).

[1] V.A. Marchenko, E.Ya. Khruslov, *Boundary-value problems with fine-grained boundary*, Mat. Sb. 65 (1964), 458–472.

[2] J. Rauch, M. Taylor, *Potential and scattering theory on wildly perturbed domains*, J. Funct. Anal. 18 (1975), 27–59.

[3] O. Post, *Spectral convergence of non-compact quasi-one-dimensional spaces*, Ann. Henri Poincaré 7 (2006), 933–973.

Determinantal formula of inverse spectral problem for Schrödinger operators and its application to KdV flow

Shinichi Kotani, *Osaka, Japan*

Sato discovered a determinantal formula to describe solutions to a class of integrable systems including KdV equation, which is called as τ -functions. Later his theory was developed by Segal–Wilson in the framework of Hardy space on the unit disc. However, their initial functions are restricted to a certain class of meromorphic functions on the entire complex plane $\mathbb{C}$, and the solutions may have singularities on the real axis $\mathbb{R}$. The purpose of the present speaker is to give a necessary and sufficient condition for the solutions not to have singularities on $\mathbb{R}$, and to obtain a new formula of the τ-functions by connecting Sato's theory with Weyl–Titchmarsh functions $m_{\pm}$ of 1d Schrödinger operator $H(q)$ with potential q. Define

$$m(z) = \begin{cases} -m_{+}\left(-z^{2}\right) & \text{for } \operatorname{Re} z > 0 \\ m_{-}\left(-z^{2}\right) & \text{for } \operatorname{Re} z < 0 \end{cases}.$$

Assume m is holomorphic on $\mathbb{C}\setminus([-r,r]\cup i[-r,r])$ for some $r>0$, which is equivalent to the reflectionless property of $H(q)$ on $[\sqrt{r},\infty)$ and $\operatorname{sp} H(q) \subset [-\sqrt{r},\infty)$. Let $\Gamma = \left\{g = e^{h};\ h \text{ is a real polynomial}\right\}$ and set

$$M_g(z,\xi) = \frac{g_o^{-1}(z)(gm)_e(\xi) + g_e^{-1}(z)(gm)_o(\xi) - m_o(\xi)}{\xi - z},$$

where $f_e(z) = \left(f(\sqrt{z}) + f(-\sqrt{z})\right)/2$, $f_o(z) = \left(f(\sqrt{z}) - f(-\sqrt{z})\right)/2\sqrt{z}$. For a simple closed smooth curve C surrounding $[-\sqrt{r},\sqrt{r}]$ define an integral operator on $L^2(C)$ by

$$(K_g u)(z) = \frac{1}{2\pi i}\int_C \frac{m_o(\eta)^{-1}}{\eta - z} d\eta\, \frac{1}{2\pi i}\int_C M_g(\eta,\xi)\, u(\xi)\, d\xi.$$

Then, the τ-function is

$$\tau_m(g) = \det\left(I + K_g\right).$$

Theorem 1. *$\tau_m(g) > 0$ is valid and the KdV flow can be written as*

$$(K(g)q)(x) = -2\partial_x^2 \log \tau_m(e_x g), \text{ where } e_x(z) = e^{xz} \in \Gamma.$$

Especially, $q(x) = (K(1)q)(x) = -2\partial_x^2 \log \tau_m(e_x)$, and for $g = e^{4tz^3}$, $K(g)q$ gives a solution to the KdV equation.

This determinantal formula for $\tau_m(g)$ makes it possible to extend its definition to non-reflectionless potentials if h is an odd polynomial, since g remains bounded by choosing C suitably. However, non-degeneracy of $\tau_m(g)$ is nontrivial and remains open.

Central spectral gaps of the almost Mathieu operator

Igor Krasovsky, *London, United Kingdom*

We consider the spectrum of the almost Mathieu operator H with an irrational frequency and in the case of the critical coupling. For frequencies admitting a power-law approximation by rationals, we show that the central gaps of H are open and provide a lower bound for their widths.

Small-amplitude solutions for space-multidimensional Hamiltonian PDEs under periodic boundary conditions

Sergei Kuksin, *Paris, France*

I will discuss the problem of studying the long-time behaviour of small solutions for nonlinear Hamiltonian PDEs on T^d. I will explain that the equations in question have abundance of small time-quasi-periodic solutions and that the behaviour of solutions for space-multidimensional equations $(d > 1)$ significantly differs from that for the 1d systems since in the 1d case the constructed solutions are linearly stable, while in the space-multidimensional case they exhibit the modulation instability.

The talk is based on my recent joint work with H.Eliasson and B.Grebert in GAFA 26 (2016), (arXiv 1604.01657)

On the almost sure location of the singular values of certain Gaussian block-Hankel large random matrices

Philippe Loubaton, *Marne la Vallée, France*

This presentation is devoted to the almost sure location of the eigenvalues of matrices $\mathbf{W}_N\mathbf{W}_N^*$ where $\mathbf{W}_N = (\mathbf{W}_N^{(1)T}, \ldots, \mathbf{W}_N^{(M)T})^T$ is a $ML \times N$ block-line matrix whose block-lines $(\mathbf{W}_N^{(m)})_{m=1,\ldots,M}$ are independent identically distributed $L \times N$ Hankel matrices with entries

$$\left(\mathbf{W}_N^{(m)}\right)_{i,j} = \frac{v_{m,i+j-1}}{\sqrt{N}}$$

for $1 \leq i \leq L$ and $1 \leq j \leq N$. Here, two-parameter sequence $(v_{m,n})_{m\geq 1, n\geq N}$ is independent identically distributed with $\mathcal{N}_c(0, \sigma^2)$ entries, where $\mathcal{N}_c(0, \sigma^2)$ represents the zero mean variance σ^2 complex Gaussian distribution (i.e real part and imaginary part are independent real zero mean variance $\sigma^2/2$ real Gaussian random variables).

It is shown that if $M \to +\infty$ and $\frac{ML}{N} \to c_*$ $(c_* \in (0, \infty))$, then the empirical eigenvalue distribution of $\mathbf{W}_N\mathbf{W}_N^*$ converges almost surely towards the Marcenko-Pastur distribution. More importantly, it is established using the Haagerup-Schultz-Thorbjornsen ideas that if $L = \mathcal{O}(N^\alpha)$ with $\alpha < 2/3$, then, almost surely, for N large enough, the eigenvalues of $\mathbf{W}_N\mathbf{W}_N^*$ are located in the neighbourhood of the Marcenko-Pastur distribution.

Phase transition for the variance of the ℓ_p^n-norm of the Gaussian vector and Dvoretzky's theorem

Anna Lytova, *Opole, Poland*

We study the variance of the ℓ_p^n-norm $||G||_p$ of the standard Gaussian vector G in $\mathbb{R}^n$ in the regime when p grows to infinity with n. It is known that for a fixed $p < \infty$, $\mathbf{Var}\|G\|_p \simeq v_p n^{2/p-1}$, where v_p depends only on p and not on n, while the variance of the $\|\cdot\|_\infty$–norm of G is of order $(\log n)^{-1}$. In [1], Paouris, Valettas and Zinn considered, in particular, the case when p grows to infinity with n and showed that $\mathbf{Var}\|G\|_p \simeq \frac{2^p}{p} n^{2/p-1}$ for $p \le c\log n$ and $\mathbf{Var}\|G\|_p \simeq \frac{1}{\log n}$ for $p \ge C\log n$ ($C, c > 0$ being universal constants). This result leaves the gap $c\log n \le p \le C\log n$ in which the behaviour of the variance was not clarified. We resolve this issue and determine the "phase transition window" in which the variance $\mathbf{Var}\|G\|_p$ changes from polynomially small in n to logarithmic. We also discuss an application of our result to Dvoretzky's theorem for ℓ_p^n.

This is a joint work with Konstantin Tikhomirov.

[1] G. Paouris, P. Valettas, and J. Zinn, *Random version of Dvoretzky's theorem in ℓ_p^n*, Stochastic Processes and their Applications (2017). arXiv:1510.07284.

Discrete multichannel scattering with step-like potential

Yurii Lyubarskii, *Trondheim, Norway*

We study direct and inverse scattering problem for systems of interacting particles, having web-like structure. Such systems consist of a finite number of semi-infinite chains attached to the central part formed by a finite number of particles. We assume that the semi-infinite channels are homogeneous at infinity, but the limit values of the coefficients may vary from one chain to another.

Sharp uniqueness results for discrete Schrödinger evolutions

Eugenia Malinnikova, *Trondheim, Norway*

In a series of papers, L. Escauriaza, C.E. Kenig, G. Ponce and L. Vega have shown that the Hardy uncertainty principle is equivalent to a uniqueness property of solutions of the Schrödinger evolution. They also extended this uniqueness result to solutions of the Shrödinger equation with a potential.

We will discuss a discrete counterpart of this theory and prove that if a solution of a discrete time-dependent Schrödinger equation with bounded potential decays fast at two distinct times then the solution is trivial. For the case of time-independent potential a sharp analog of the Hardy uncertainty principle is obtained. The argument is based on the theory of entire functions and Complex Jacobi matrices. As a corollary we obtain sharp uniqueness theorem for non-linear discrete Shrödinger equation. The work is based on a joint works with Yu. Lyubarskii. A. Fernandez-Bertolin and Ph. Jaming.

The hidden landscape of localization of eigenfunctions

Svitlana Mayboroda, *Minneapolis, MN, USA*

Numerous manifestations of wave localization permeate acoustics, quantum physics, mechanical and energy engineering. It was used in construction of noise abatement walls, LEDs, optical devices, to mention just a few applications. Yet, no systematic methods could predict the exact spatial location and frequencies of the localized waves.

In this talk I will present recent results revealing a new criterion of localization, tuned to the aforementioned questions, and will illustrate our findings in the context of the boundary problems for the Laplacian and bilaplacian, $\operatorname{div} A\nabla$, and (continuous) Anderson and Anderson–Bernoulli models on a bounded domain. Via a new notion of "landscape" we connect localization to a certain multi-phase free boundary problem and identify location, shapes, and energies of localized eigenmodes. The landscape further provides estimates on the rate of decay of eigenfunctions and delivers accurate bounds for the corresponding eigenvalues, in the range where both classical Agmon estimates and Weyl law may fail.

"Irrational" Convexity

Vitali Milman, *Tel Aviv, Israel*

Do we have enough examples of Convex Bodies? Is diversity of our standard examples enough to understand Convexity?

In the talk we demonstrate many different constructions which are analogous to constructions of irrational numbers from rationals. We show, following Il. Molchanov, that the solutions of "quadratic" equations like $Z^o = Z + K$ always exists (where Z^o is the polar body of Z; Z and K are convex compact bodies containing 0 in the interior). Then we show how the geometric mean may be defined for any convex compact bodies K and T (containing 0 into their interior). We also construct K^a for any centrally symmetric K and $0 < a < 1$, and also $\operatorname{Log} K$ for K containing the euclidean ball D (and $K = -K$).

Note, the power a cannot be above 1 in the definition of power!

All these constructions may be considered also for the infinite dimensional setting, but this is outside the subject of the talk.

These results are joint with Liran Rotem.

Laguerre polynomials in asymptotics of the modified Korteweg–de Vries equation

Alexander Minakov, *Trieste, Italy*

We consider the modified Korteweg–de Vries equation

$$q_t + 6q^2 q_x + q_{xxx} = 0$$

with initial data $q_0(x) \to 0$ as $x \to +\infty$ and $q_0(x) \to c > 0$ as $x \to -\infty$. The asymptotics on a curve $x = 4c^2 t - C \ln t$, where C is a constant, was found by E.Ya. Khruslov and V.P. Kotlyarov. We show how to obtain this formula using the Riemann–Hilbert method. We also find asymptotics on a curve $x = 4c^2 t - C t^{\sigma} \ln t$, $\sigma \in (0, 1)$.

Exact solutions of Maxwell equations in Kerr space-time and their physical manifestations

Volodymyr Pelykh, *Lviv, Ukraine*
Yuri Taistra, *Lviv, Ukraine*

We have found for the first time in analytic form an exact general solution and solution with separated variables for null one-way Maxwell field $\varphi_{AB} = \varphi_2 o_A o_B$ on Kerr space-time background and have investigated some of their properties and physical consequences. This general solution generalizes the known solutions of other authors in Minkowski space-time. Solution with separable variables describes outgoing waves if $r > r_{cr.1} > r_+$, but for some Maxwell field parameters this solution describes ingoing, standing and outgoing waves on defined intervals in region $r > r_+$, where r_+ is Kerr outer horizon. Superradiation condition, established by Teukolsky, Starobinski and Churilov is equivalent to condition of existence of point $r_{cr.1}$ outside the horizon: $r_{cr.1} > r_+$. From solution with separated variables we deduce the damping of right-polarized electromagnetic waves (and rotation of plane of polarization) as well as expression for phase shift.

On the distribution of the largest real eigenvalue for the real Ginibre ensemble

Mihail Poplavskyi, *London, United Kingdom*

Distributions of extreme eigenvalues for different ensembles of random matrices are certainly some of the most striking results concerning the spectra of random matrices. For classical Gaussian ensembles and more generally for β Matrix Models scaled largest eigenvalue was shown to follow Tracy-Widom β distribution, which also surprisingly arises in many areas of modern mathematical physics: growth processes, simple exclusion processes, etc.

In this talk we discuss similar question for the Real Ginibre Ensemble, which consists of $N \times N$ matrices with i.i.d. $N(0,1)$ entries. With a probability close to 1 there is at least one real eigenvalue (see [1]). We study the distribution of the largest real one and based on the ideas developed in [2] we show in [3]

Theorem 1. *Let $\sqrt{N}+\lambda_{max}$ be the largest real eigenvalue of a random $N \times N$ matrix taken from the Real Ginibre ensemble and $\mathcal{T}$ be an integral operator with the kernel*

$$T(x,y) = \frac{1}{\pi}\int_0^\infty e^{-(x+u)^2} e^{-(y+u)^2} du.$$

Then,

$$\lim_{N\to\infty} \mathbb{P}\left\{\lambda_{max} < t\right\} = \sqrt{\det(I - \mathcal{T}\chi_t)(1-a_t)},$$

where Let χ_t be the indicator of (t,∞), $a_t = \int_t^\infty G(x)(I-\mathcal{T}\chi_t)^{-1} g(x)dx$ with $g(x) = \frac{1}{\sqrt{\pi}}e^{-x^2}$, $G(x) = \int_{-\infty}^x g(y)dy$. Moreover, in the limit when $N \to \infty$, for big enough positive t

$$\begin{aligned}
\mathbb{P}\left\{\lambda_{max} < t\right\} &= 1 - \frac{1}{4}\mathrm{erfc}(t) + O(e^{-2t^2}). \\
\mathbb{P}\left\{\lambda_{max} < -t\right\} &= \exp\left(-\frac{\zeta(3/2)}{2\sqrt{2\pi}}|t| + O(1)\right).
\end{aligned}$$

The talk is based on a joint work with R. Tribe and O. Zaboronski.

[1] E. Kanzieper, M. Poplavskyi, C. Timm, R. Tribe and O. Zaboronski, *Another fascinating article*, The Annals of Applied Probability 26 (2016), no. 5, 2733–2753.

[2] B. Rider and C. D. Sinclair, *Extremal laws for the real Ginibre ensemble*,The Annals of Applied Probability 24 (2014), no. 4, 1621–1651.

[3] M. Poplavskyi, R. Tribe and O. Zaboronski, *On the Distribution of the Largest Real Eigenvalue for the Real Ginibre Ensemble*,to appear in The Annals of Applied Probability.

Explicit solutions of inverse problems and of dynamical systems

Alexander Sakhnovich, *Vienna, Austria*

We present some latest applications of our GBDT version of Bäcklund-Darboux transformation. First, we consider the explicit recovery of discrete and continuous Dirac systems from rational Weyl matrix functions (which are not necessarily square) and the stability of those procedures [1, 2]. Another interesting application is connected with the explicit solving of important dynamical systems [3, 4, 5]. Our first works on GBDT [6, 7] were inspired by the well-known book [8] by V.A. Marchenko on explicit solution of nonlinear equations.

The research and participation in the conference are supported by the Austrian Science Fund (FWF) under Grant No. P29177.

[1] A.L. Sakhnovich, *Inverse problems for self-adjoint Dirac systems: explicit solutions and stability of the procedure*, Oper. Matrices 10 (2016), no. 4 Special issue in memory of Leiba Rodman, 997–1008.

[2] B. Fritzsche, B. Kirstein, I.Ya. Roitberg and A.L. Sakhnovich, *Stability of the procedure of explicit recovery of skew-selfadjoint Dirac systems from rational Weyl matrix functions*, arXiv:1510.00793 (Lin. Alg. Appl. to appear).

[3] A.L. Sakhnovich, *Dynamical canonical systems and their explicit solutions*, Discrete and Continuous Dynamical Systems A 37 (2017), no. 3, 1679–1689.

[4] A.L. Sakhnovich, *Dynamics of electrons and explicit solutions of Dirac-Weyl systems*, J. Phys. A: Math. Theor. 50 (2017) 115201.

[5] B. Fritzsche, B. Kirstein, I.Ya. Roitberg and A.L. Sakhnovich, *Continuous and discrete dynamical Schrödinger systems: explicit solutions*, arXiv:1701.08011.

[6] A.L. Sakhnovich, *Exact solutions of nonlinear equations and the method of operator identities*, Lin. Alg. Appl. 182 (1993), 109–126.

[7] A.L. Sakhnovich, *Dressing procedure for solutions of nonlinear equations and the method of operator identities*, Inverse problems 10 (1994), 699–710.

[8] V.A. Marchenko, *Nonlinear equations and operator algebras*, Mathematics and Its Applications (Soviet Series) 17, Reidel, Dordrecht, 1988.

PT-symmetric operators with parameter. Critical parameter values

Andrey Shkalikov, *Moscow, Russia*

We consider PT-symmetric Sturm–Liouville operators

$$T(\varepsilon) = -\frac{d^2}{dx^2} + \varepsilon P(x), \quad y(-1) = y(1) = 0, \quad \varepsilon > 0,$$

in the space $L_2(-a, a)$, $0 < a \leqslant \infty$, where P is subject to the condition $P(x) = -\overline{P(-x)}$. The spectra of these operators are symmetric with respect to the real axis and discrete, provided that the interval $(-a, a)$ is finite and P is not a singular potential. We will show that the spectrum of the operator $T(\varepsilon)$ is real for sufficiently small values of the parameter ε and in this case $T(\varepsilon)$ is similar to a self-adjoint operator. For large values of ε the complex eigenvalues do appear and the number of non-real eigenvalues increases as $\varepsilon \to \infty$.

The aim of the talk is to cast some light to the following problems: How the eigenvalues of $T(\varepsilon)$ do behave when the parameter changes? Is it possible to evaluate or to calculate the critical value ε_0 of the parameter, such that $T(\varepsilon)$ are similar to self-adjoint operators for all $\varepsilon < \varepsilon_0$? We will find an explicit answer for some particular potentials.

The talk is based on the joint papers with S.N.Tumanov.

Spectral theory sum rules, meromorphic Herglotz functions and large deviations

Barry Simon, *Pasadena, CA, USA*

After defining the spectral theory of orthogonal polynomials on the unit circle (OPUC) and real line (OPRL), I'll describe Verblunsky's version of Szego's theorem as a sum rule for OPUC and the Killip–Simon sum rule for OPRL and their spectral consequences. Next I'll explain the original proof of Killip–Simon using representation theorems for meromorphic Herglotz functions. Finally I'll focus on recent work of Gamboa, Nagel and Rouault who obtain the sum rules using large deviations for random matrices.

Spectra of stationary processes on Z

Mikhail Sodin, *Tel Aviv, Israel*

We will discuss a somewhat striking spectral property of finitely valued stationary processes on Z that says that if the spectral measure of the process has a gap then the process is periodic. We will give some extensions of this result and raise several related questions. Joint work with B. Weiss and A. Borichev.

The distribution of ζ'/ζ about a random point on the critical line

Sasha Sodin, *Tel Aviv, Israel, London, United Kingdom*

We shall discuss the properties of the logarithmic derivative of the Riemann zeta-function, rescaled about a point chosen at random on the critical line. The talk will be mostly self-contained.

Jacobi polynomials, Bernstein-type inequalities and dispersion estimates for the discrete Laguerre operator

Gerald Teschl, *Vienna, Austria*

I will talk about Bernstein-type estimates for Jacobi polynomials and their applications to various branches in mathematics. This is an old topic but we want to add a new wrinkle by establishing some intriguing connections with dispersive estimates for a certain class of Schrödinger equations whose Hamiltonian is given by the generalized Laguerre operator. More precisely, we show that dispersive estimates for the Schrödinger equation associated with the generalized Laguerre operator are connected with Bernstein-type inequalities for Jacobi polynomials. We use known uniform estimates for Jacobi polynomials to establish some new dispersive estimates. In turn, the optimal dispersive decay estimates lead to new Bernstein-type inequalities. This is based on joint work with Tom Koornwinder and Aleksey Kostenko.

[1] T. Koornwinder, A. Kostenko, and G. Teschl, *Jacobi Polynomials, Bernstein-type Inequalities and Dispersion Estimates for the Discrete Laguerre Operator*, arXiv:1602.08626

Direct and inverse scattering theory for the matrix Schrödinger operator on the half line with the general selfadjoint boundary condition

Ricardo Weder, *Ciudad de México, México*

The matrix Schrödinger equation with a selfadjoint matrix potential is considered on the half line with the general selfadjoint boundary condition at the origin. When the matrix potential is integrable, the high-energy asymptotics are established for the Jost matrix, its inverse, and the scattering matrix. Under the additional assumption that the matrix potential has a first moment, it is shown that the scattering matrix is continuous at zero energy, and a explicit formula is provided for its value at zero energy. The small-energy asymptotics are established also for the Jost matrix, its inverse, and various other quantities relevant to the corresponding direct and inverse scattering problems . When the potential has a second moment more detailed results are obtained. Furthermore, Levinson's theorem is proven, the generalized Fourier maps are constructed, the stationary formulas for the wave operators are stablished and the existence and completeness of the wave operators are proven. Also, Krein's spectral shift function is studied and trace formulas of the Buslaev-Faddeev type are obtained. Finally a Marchenko theory is developed that gives a characterization of the scattering data.

[1] T. Aktosun, M. Klaus and R. Weder, *S*mall-energy analysis for the selfadjoint matrix Schrödinger operator on the half Line, J. Math. Phys. 52 (2011), 102101.

[2] T. Aktosun and R. Weder, *H*igh-energy analysis and Levinson's theorem for the selfadjoint matrix Schrödinger operator on the half line, J. Math. Phys. 54 (2013), 012108.

[3] T. Aktosun, M. Klaus and R. Weder, *S*mall-energy analysis for the selfadjoint matrix Schrödinger operator on the half line. II, J. Math. Phys. 55 (2014), 032103.

[4] R. Weder, *S*cattering theory for the matrix Schrödinger operator on the half line with general boundary conditions, J. Math. Phys. 56 (2015), 092103.

[5] R. Weder, *T*race Formulas for the matrix Schrödinger Operator on the half-line with general boundary conditions, J. Math. Phys., 57 (2016) 112101.

[6] T. Aktosun and R. Weder, *T*he characterization of the scattering data for the matrix Schrödinger operator on the half-line with the general selfadjoint boundary condition, to appear.

PARTICIPANTS

The Korteweg–de Vries equation with steplike initial data as a Hamiltonian system

Kyrylo Andreiev, *Kharkiv, Ukraine*

We construct an infinite series of the regularized integrals of motion for the Korteweg–de Vries equation

$$q_t - 6qq_x + q_{xxx} = 0,$$

with steplike initial profile $q_0(x)$, which belongs to the Schwartz type class in the following meaning:

$$\int_{\mathbb{R}_+} x^m(|q_0(-x) - c^2| + |q_0(x)|)dx + \int_{\mathbb{R}} |x|^m |q_0^{(s)}(x)| dx < \infty,$$

for all integer $m \geq 1$ and $s \geq 1$. Our work [1] is a generalization of the well known result by L.D. Faddeev and V.E. Zakharov [2] on the steplike case. We propose a representation of the integrals of motion via the scattering data of the initial profile. We also discuss the symplectic form and the canonical variables of the action–angle type.

[1] K.M. Andreiev, E.Ya. Khruslov *The Korteweg–de Vries equation with steplike initial data as a Hamiltonian system (in preparation).*

[2] V.E. Zakharov, L.D. Faddeev, *The Korteweg–de Vries equation - completely integrable Hamiltonian system, Functional Analysis and Its Applications, 5: 4 (1971), 18-27.*

On a class of equations with a difference kernel on a finite interval

Elena Arshava, *Kharkiv, Ukraine*

We have undertaken further work in the field of the theory for inversing the integro-differential operators in the Hilbert space.

We have studied the class of integral operators, the inverse to which can be found on the operator commutation relations basis. These operators include those with a difference kernel, which are the most general form of equations in space L_2.

Using the operator identities method made it possible to study various types of equations with a difference kernel of both the first and second kind within one frame of mind.

This method was applied for the study of systems of integral equations with a difference kernel, summatory equations with the Toeplitz matrix, a two-dimensional integral equations and operators with quasidifference kernels.

As an example we have considered the problem of filtrating and predicting non-stationary stochastic processes and signals.

Stabilizability and controllability of singular triangular systems

Maxim Bebiya, *Kharkiv, Ukraine*

We address the problems of stabilizability and controllability for a class of nonlinear systems of the form

$$\begin{cases} \dot{x}_1 = f_1(u, x_1, \ldots, x_n), \\ \dot{x}_i = f_i(x_{i-1}, \ldots, x_n), \quad i = 2, \ldots, n, \end{cases} \tag{1}$$

where $u \in \mathbb{R}$ is a control. System (1) is called a triangular system. The class of triangular systems was introduced and studied by V.I. Korobov in [1]. We consider the singular case in which $\frac{\partial f_n}{\partial x_{n-1}} = 0$ for $x = 0$.

Recall that system (1) is called stabilizable if there exists a feedback control law $u = u(x)$ such that the corresponding closed loop system has $x = 0$ as its asymptotically stable equilibrium point. System (1) is said to be controllable if for every $x_0 \in U \subset \mathbb{R}^n$ there exists a control $u = u(x)$ such that the corresponding solution of the closed loop system with initial condition $x(0) = x_0$ satisfies $x(T) = 0$ for some $T = T(x_0) < +\infty$.

We show that system (1) is globally 0–controllable and stabilizable if

$$\left|\frac{\partial f_i}{\partial x_{i-1}}\right| \geq a > 0, \quad i = 1, \ldots, n-1, \quad \left|\frac{\partial f_n^{\frac{1}{2k+1}}}{\partial x_{n-1}}\right| \geq a > 0$$

for any $x_0, x_1, \ldots, x_n$ ($x_0 = u$), where $a > 0$, $k \in \mathbb{N}$. Moreover, we construct the change of variables $z = F(x)$ and the change of control $v = G(x, u)$ reducing system (1) to the form

$$\begin{cases} \dot{z}_1 = v, \\ \dot{z}_i = z_{i-1}, \quad i = 2, \ldots, n-1, \\ \dot{z}_n = z_{n-1}^{2k+1}. \end{cases} \tag{2}$$

The case when $k = 0$ was investigated in [1]. The stabilizability and controllability problems for system (2) with $k > 0$ were solved in [2, 3]

[1] V.I. Korobov, *Controllability, stability of some nonlinear systems*, Differ. Uravn. 9 (1973), 614–619.

[2] M.O. Bebiya and V.I. Korobov, *On Stabilization Problem for Nonlinear Systems with Power Principal Part*, Journal of Mathematical Physics, Analysis, Geometry 12 (2016), No. 2, 113–133.

[3] M.O. Bebiya, *Global synthesis of bounded controls for systems with power nonlinearity*, Visnyk of V.N. Karazin Kharkiv National University, Ser. Math., Applied Math. and Mech. 81 (2015), 36–51.

A direct approach to the inverse scattering transform for the short pulse equation

Anne Boutet de Monvel, *Paris, France*
Dmitry Shepelsky, *Kharkiv, Ukraine*
Lech Zielinski, *Calais, France*

We develop a Riemann–Hilbert approach to the inverse scattering transform method for the short pulse (SP) equation [1]

$$u_{xt} = u + \tfrac{1}{6}(u^3)_{xx}$$

with zero boundary conditions (as $|x| \to \infty$). The SP equation was proposed as an alternative (to the nonlinear Schrödinger equation) model for approximating the evolution of ultra-short intense infrared pulses in silica optics. It was shown in [2] by numerical simulations that the SP equation can be successfully used for describing pulses with broad spectrum.

Our approach is directly applied to a Lax pair [3] for the SP equation. It allows us to give a parametric representation of the solution to the Cauchy problem. This representation is then used for studying the long-time behavior of the solution as well as for retrieving the soliton solutions. Finally, the analysis of the long-time behavior allows us to formulate, in spectral terms, a sufficient condition for the wave breaking [4].

[1] T. Schäfer and C.E. Wayne, *Propagation of ultra-short optical pulse in nonlinear media*, Phys. D 196 (2004), 90–105.

[2] Y. Chung, C.K.R.T. Jones, T. Schäfer and C.E. Wayne, *Ultra-short pulses in linear and nonlinear media*, Nonlinearity 18 (2005), 1351–1374.

[3] A. Sakovich and S. Sakovich, *The short pulse equation is integrable*, J. Phys. Soc. Japan 74 (2005), 239–241.

[4] A. Boutet de Monvel, D. Shepelsky and L. Zielinski, *The short pulse equation by a Riemann–Hilbert approach*, Lett. Math. Phys. (2017), DOI10.1007/s11005-017-0945-z.

Long-time asymptotics for Korteweg–de Vries equation with steplike initial data

Iryna Egorova, *Kharkiv, Ukraine*

We discuss the long-time asymptotics of the solutions for the Korteweg–de Vries equation $q_t(x,t) = 6q(x,t)q_x(x,t) - q_{xxx}(x,t)$, $(x,t) \in \mathbb{R} \times \mathbb{R}_+$, with steplike initial data $q(x,0) = q_0(x)$ satisfying

$$\begin{cases} q_0(x) \to 0, & \text{as } x \to +\infty, \\ q_0(x) \to \pm c^2 & \text{as } x \to -\infty. \end{cases} \tag{1}$$

The cases $-c^2$ and c^2 are known as the shock and the rarefaction problems respectively. The asymptotical behaviour of $q(x,t)$ as $t \to \infty$ is well understood for both cases on the physical level of rigour in all regions of the space-time half plane. Namely, there are three principal regions with different asymptotical behaviour of the solution. To the right of the leading wave front the solution is asymptotically given by a sum of solitons. Behind the back wave front it is close to the respective background $\pm c^2$. In the middle region, the rarefaction problem solution is asymptotically close to $\frac{x}{6t}$. The shock problem solution in its middle region is asymptotically close to an elliptic wave. These results were obtained by use of the Whitham method (cf. [4]), for pure step initial initial data $(q_0(x) = 0$ for $x > 0$ and $q_0(x) = \pm c^2$ for $x \leq 0)$.

We apply the nonlinear steepest descent method for oscillatory Riemann–Hilbert problem to rigorously justify the above mentioned asymptotics for arbitrary smooth initial data (1) which tend to their background constants sufficiently fast. Furthermore, for the rarefaction problem case we also compute the second terms in the asymptotic expansion ([1, 2, 3]).

[1] K. Andreiev, I. Egorova, T.L. Lange, G. Teschl, *Rarefaction waves of the Korteweg–de Vries equation via nonlinear steepest descent*, J. Differential Equations 261 (2016), 5371-5410.

[2] I. Egorova, Z. Gladka, V. Kotlyarov, G. Teschl, *Long-time asymptotics for the Korteweg–de Vries equation with steplike initial data*, Nonlinearity, 26 (2013), 1839-1864.

[3] I. Egorova, Z. Gladka, G. Teschl, *On the form of dispersive shock waves of the Korteweg–de Vries equation*, Zh. Mat. Fiz. Anal. Geom. 12 (2016), 3-16.

[4] V.E. Zakharov, S. V. Manakov, S. P. Novikov, L. P. Pitaevskii, *Theory of solitons. The inverse scattering method*, "Nauka", Moscow, 1980 (in Russian); English transl. : Contemporary Soviet Mathematics. Consultants Bureau , New York, 1984.

Transformation operators and wave equations

Larissa Fardigola, *Kharkiv, Ukraine*

In the talk, a transformation between the wave equations:

$$w_{tt} = \frac{1}{\rho}(kw_x)_x + \gamma w, \qquad x \in (0,l),\ t \in [0,t], \tag{1}$$

and

$$z_{tt} = z_{\xi\xi} - q^2 z, \qquad \xi \in (0,\infty),\ t \in [0,t], \tag{2}$$

is studied under some assumptions on the variable coefficients ρ, k, γ and the constant $q \geq 0$ where $l, T > 0$ are constants.

Equation (2) is considered in spaces $\widetilde{H}^{-m}(0,\infty)$ of the Sobolev type, and equation (1) is considered in some modified Sobolev spaces $\widetilde{\mathbb{H}}^{-m}(0,l)$ determined by the coefficients ρ and k, $m = 0,1,2$.

An isometric isomorphizm of the spaces $\widetilde{H}^{-m}(0,\infty)$ and $\widetilde{\mathbb{H}}^{-m}(0,l)$ transforming each solution to (2) into a solution to (1) (and vice versa) is constructed and studied, $m = \overline{-2,2}$. This isomorphism is called a transformation operator.

Applying this operator, it is proved that equation (1) replicates controllability properties of equation (2) and vice versa [1]. Thus controllability properties of (1) are obtained from the ones of (2) [1, 2].

[1] L.V. Fardigola, *Transformation operators in controllability problems for the degenerate wave equation with variable coefficients*, Ukr. Mat. Zh., submitted.

[2] L.V. Fardigola, *Controllability problems for the 1-D wave equation on a half-axis with the Dirichlet boundary control*, ESAIM: Control, Optim. Calc. Var. 51 (2012), 748-773.

Rational decay rates of solutions to some fluid-structure interaction models.

Tamara Fastovska, *Kharkiv, Ukraine*

We consider a sufficiently smooth domain $\mathcal{O} \in \mathbb{R}^3$. Assume that the boundary $\partial\mathcal{O}$ consists of two parts: $\partial\mathcal{O} = \overline{\Omega} \cup \overline{S}$, where $\overline{\Omega} \cup \overline{S} = \emptyset$. The domain $\mathcal{O}$ is filled with an inviscid fluid. Ω is a flexible part of the boundary, while S represents a rigid wall. For the sake of simplification we assume that Ω is flat, i.e.

$$\Omega \subset \{x = (x_1, x_2, 0) : x' = (x_1, x_2) \in \mathbb{R}^2\}$$

and its boundary Γ is smooth. We denote by n the exterior normal to $\mathcal{O}$. Let $\mathcal{O}$ is oriented in such a way, that $n = (0,0,1)$ on Ω. The motion of the fluid is described by the linear Euler equations for unknown fluid velocity $V(x,t) = (v_1(x,t), v_2(x,t), v_3(x,t))$ and pressure $p(x,t)$

$$V_t + \mu V + \nabla p = 0 \text{ in } \mathcal{O} \times (0,+\infty), divV = 0, \text{ in } \mathcal{O} \times (0,+\infty)$$

with non-penetration boundary conditions

$$(V,n) = 0 \text{ on } S; \ (V,n) = u_t \text{ on } \Omega$$

and initial conditions

$$V(0) = V_0,$$

where the transversal displacement $v(x',t)$ of the elastic boundary Ω satisfies the equation

$$u_{tt} - \alpha \Delta u_{tt} + \Delta^2 u = p|_{\Omega}$$

subjected to the Dirichlet boundary conditions

$$u = \nabla u = 0 \text{ on } \partial\Omega$$

and initial conditions

$$u(0) = u_0, \ u_t(0) = u_1.$$

Here $\alpha \geq 0$, $\mu > 0$ are constants denoting the rotational inertia and drag/frictional coefficient respectively. We investigate spectral properties of the problems considered and establish rational decay rates for strong solutions.

Large Fourier quasicrystals

Sergii Favorov, *Kharkiv, Ukraine*

A set $E \subset \mathbb{R}^d$ is $discrete$ if $E \cap B$ is finite for every ball $B \subset \mathbb{R}^d$, $uniformly$ $discrete$ if $\inf\{|x - x'| : x, x' \in E, x \neq x'\} > 0$, and a $finite\ type$ if the set $E - E$ discrete. A measure ν is $slowly\ increasing$ if its variation $|\nu|$ on the ball $B(r)$ of radius r with center at 0 has a polynomial growth as $r \to \infty$, and $translation\ bounded$ if its variations on every ball of radius 1 are uniformly bounded. Clearly, if a measure ν on $\mathbb{R}^d$ is translation bounded, then $|\nu|(B(r)) = O(r^d)$.

A discrete complex measure $\mu = \sum_{\lambda \in \Lambda} a_\lambda \delta_\lambda$ on $\mathbb{R}^d$ is a $Fourier\ quasicrystal$ if μ and its extended Fourier transform $\hat{\mu} = \sum_{\gamma \in \Gamma} b_\gamma \delta_\gamma$ are slowly increasing measures, and $spectrum$ Γ is countable (possibly nonclosed).

Theorem 1 (N. Lev, A. Olevskii, 2016). *If Λ, Γ and $\Lambda - \Lambda$ are uniformly discrete, then the support Λ is a subset of a finite union of translates of a* ***unique*** *full-rank lattice L.*

Theorem 2 (A. Cordoba, 1989). *If a Fourier quasicrystal $\mu = \sum_{\lambda\in\Lambda} a_\lambda \delta_\lambda$ has a uniformly discrete support Λ, values a_λ of all masses belong to a finite set $F \subset \mathbb{C}$, and $\hat{\mu}$ is a translation bounded measure, then the support Λ is a finite union of translates of* ***several*** *full-rank lattices.*

We say $\mu = \sum_{\lambda\in\Lambda} a_\lambda \delta_\lambda$ is *large Fourier quasicrystal* if $\inf_{\lambda\in\Lambda} |a_\lambda| > 0$.

Theorem 3 (F). *If μ is a large Fourier quasicrystal with a finite type support Λ, then Λ is a finite union of translates of a* ***unique*** *full-rank lattice L.*

Theorem 4 (F). *If μ_1, μ_2 are large Fourier quasicrystals with supports Λ_1, Λ_2 and the set $\Lambda_1 - \Lambda_2$ discrete, then Λ_1 and Λ_2 are finite unions of translates of a* ***unique*** *full-rank lattice L.*

Theorem 5 (F). *If μ is a large Fourier quasicrystal with a uniformly discrete support Λ and $|\hat{\mu}|(B(r)) = O(r^d)$ as $r \to \infty$, then Λ is a finite union of translates of* ***several*** *full-rank lattices.*

The proofs of Theorems 3–5 are based on Ronkin's theory of almost periodic measures and an appropriate variant of Wiener's Theorem on Fourier series.

Characterization theorems for Q-independent random variables with values in a locally compact Abelian group

Gennadiy Feldman, *Kharkiv, Ukraine*

It is well known that if a random variable ξ has a Gaussian distribution and ξ is a sum of two independent random variables (i.r.v.'s) $\xi = \xi_1 + \xi_2$, then ξ_j are also Gaussian (Cramér's theorem). Let ξ_j be i.r.v.'s. Then the independence of the linear forms $L_1 = \alpha_1\xi_1 + \cdots + \alpha_n\xi_n$ and $L_2 = \beta_1\xi_1 + \cdots + \beta_n\xi_n$, where α_j, β_j are nonzero real numbers, implies that ξ_j are Gaussian (Skitovich–Darmois's theorem). A theorem similar to the Skitovich–Darmois theorem was proved by Heyde, where a Gaussian distribution is characterized by the symmetry of the conditional distribution of the linear form L_2 given L_1.

On the one hand, in the article [1] A. Kagan and G. Székely introduced a notion of Q-independence and proved, in particular, that the classical Cramér and Skitovich–Darmois theorems hold true if instead of independence Q-independence is considered. On the other hand, the locally compact Abelian groups X for which group analogues of the Cramér, Skitovich–Darmois and Heyde theorems for i.r.v.'s with values in X where described (see [2]).

We introduce the notion of Q-independence for random variables taking values in a locally compact Abelian group. We prove that if we consider Q-independence instead of independence, then group analogues of theorems by Cramér, Kac–Bernstein, Skitovich–Darmois and Heyde hold true for the same classes of groups. The proofs of these theorems are reduced to solving some functional equations on the character group of the initial group in the class of continuous positive definite functions. These results are published in [3].

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Mixed initial-boundary value problem for nonlinear Maxwell–Bloch equations and Riemann–Hilbert problems

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We study the mixed problem for the Maxwell–Bloch equations

$$\frac{\partial \mathcal{E}}{\partial t}+\frac{\partial \mathcal{E}}{\partial x}=\rho,\quad \frac{\partial \rho}{\partial t}=\mathcal{N}\mathcal{E},\quad \frac{\partial \mathcal{N}}{\partial t}=-\frac{1}{2}\left(\overline{\mathcal{E}}\rho+\mathcal{E}\overline{\rho}\right),$$

which is defined by the initial and boundary conditions

$$\mathcal{E}(0,x)=\mathcal{E}_0(x),\quad \rho(0,x)=\rho_0(x),\quad \mathcal{N}(0,x)=\mathcal{N}_0(x),\quad \mathcal{E}(t,0)=\mathcal{E}_1(t),$$

where $x\in(0,l)$ ($l\le\infty$) and $t\in\mathbb{R}_+$. The function $\mathcal{E}_1(t)$ is a Schwartz type function. The functions $\mathcal{E}_0(x)$, $\rho_0(x)$, $1-\mathcal{N}_0(x)$ are smooth if $x\in[0,l]$ or of Schwartz type functions if $x\in\mathbb{R}_+$. The complex valued function $\rho(t,x)$ and the real function $\mathcal{N}(t,x)$ are related by the identity $|\rho(t,x)|^2+\mathcal{N}^2(t,x)\equiv 1$. If one choose the sign "minus" in $\mathcal{N}(0,x)=\mp\sqrt{1-|\rho(0,x)|^2}$, then the problem is considered in a stable medium (for example, it is a model of self-induced transparency). This problem was studied by different authors using essentially varied approaches. The sign "plus" corresponds to an unstable medium, for example, to a model of the two-level laser amplifier. This case is the subject of our study.

Our approach uses transformation operators whose existence is closely related with the Goursat problems with nontrivial characteristics. To prove their solvability a gauge transformation, which converts the Goursat problems into the problems of canonical type with rectilinear characteristics, is used. The same transformation is used to establish a gauge equivalence between two pairs of the Ablowitz–Kaup–Newel–Segur equations to construct their Jost type solutions with the well-controlled asymptotic behavior by a spectral parameter on the complex plane near the singular points. As a result, the well-posed regular matrix Riemann–Hilbert problem, which generates the solution of the mixed problem for the Maxwell–Bloch equations, is obtained.

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The generalized backward shift operator on $\mathbb{Z}[[x]]$, Cramer's formulas for solving infinite linear systems, and p-adic integers

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Let $a = (a_1, a_2, a_3, \ldots)$ be a sequence of positive integers, such that an infinite number of a_i more than 1 and for all prime p either p does not divide all of the a_i or p divides an infinite number of a_i. Define the operator on the ring of the formal power series with the integer coefficients, by the following rule: $A(f_0 + f_1x + f_2x^2 + \ldots) = a_1f_1 + a_2f_2x + a_3f_3x^2 + \ldots$. Let us consider the equation

$$(Ay)(x) + f(x) = y(x), \tag{1}$$

where $f(x) = f_0+f_1x+f_2x^2+\ldots \in \mathbb{Z}[[x]]$. We study solutions of this equation from $\mathbb{Z}[[x]]$.

Consider a sequence a' obtained from a through the deleting of 1. Let $\mathbb{Z}_{a'}$ be a ring of a'-adic integers with the standard metrics. First consider the question about solutions of equation $Ay + f(x) = y$ from the ring $\mathbb{Z}_{a'}[[x]]$.

Theorem 1. *The equation (1) has the following unique solution in $\mathbb{Z}_{a'}[[x]]$:*

$$y(x) = f(x) + (Af)(x) + (A^2f)(x) + (A^3f)(x) + \ldots \tag{2}$$

where the series on the right hand side converges in $\mathbb{Z}_{a'}[[x]]$ in the topology of coefficientwise convergence.

The next theorem is a criterion of the existence of a solution from $\mathbb{Z}[[x]]$.

Theorem 2. *Let $f \in \mathbb{Z}[[x]]$, $f(x) = f_0 + f_1 x + f_2 x^2 + \ldots$. Then the equation $Ay + f(x) = y$ has a solution from $\mathbb{Z}[[x]]$ if and only if the sum of the series*

$$y(0) = f_0 + a_1 f_1 + a_1 a_2 f_2 + a_1 a_2 a_3 f_3 + a_1 a_2 a_3 a_4 f_4 + \ldots$$

is integer. In this case, the solution of this equation is given by formula (2).

Example 1: if $a = (1, 2, 3, 4, \ldots)$, then A is the differentiation operator. Then the equation (1) is written as $y'(x) + f(x) = y(x)$.

Example 2: if $a = (b, b, b, b, \ldots)$, then $A = b \cdot S^*$, where S^* is a backward shift operator. Then the equation is written as $bS^*(y)(x) + f(x) = y(x)$ and for the integer coefficients y_n of the series y we have the difference equation $by_{n+1} + f_n = y_n$. Moreover, the solution of this system obtained with the aid of some analog of Cramer's rule is the only solution we need from $\mathbb{Z}[[x]]$.

On stability in the Borg–Hochstadt theorem for periodic Jacobi matrices

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The problem we address here concerns a class of double infinite three-diagonal (Jacobi) matrices J with real numbers b's along the main diagonal and positive a's along the off-diagonal, which are periodic of period $p \in \mathbb{N}$, i.e., $a_{n+p} = a_n$, $b_{n+p} = b_n$, $n \in \mathbb{Z}$. The spectrum $\sigma(J)$ of the corresponding linear operator on the Hilbert space $\ell^2(\mathbb{Z})$ is known to have a banded structure, i.e., it is composed of p closed intervals (*spectral bands*)

$$\sigma(J) = \bigcup_{j=0}^{p-1} [\mu_j^+, \mu_{j+1}^-], \quad \mu_0^+ < \mu_1^- \le \mu_1^+ < \ldots < \mu_{p-1}^- \le \mu_{p-1}^+ < \mu_p^-,$$

interspersed with (interior) *gaps* $\gamma_j := (\mu_j^-, \mu_j^+)$, $j = 1, 2, \ldots, p-1$ of the length $|\gamma_j| = \mu_j^+ - \mu_j^-$, and $\mu_j^- = \mu_j^+$ means that the gap is closed (the adjacent bands merge).

The well-known result of Hochstadt [1] states, that a periodic Jacobi matrix J has all its gaps closed if and only if it is constant, i.e., $a_j = a_0$, $b_j = b_0$, for all integers j. In this case the spectrum $\sigma(J) = [b_0 - 2a_0, b_0 + 2a_0]$ is a single interval. The stability (or a quantitative version) of this result is the main problem under consideration [2]. We show that for periodic Jacobi matrices with "small" variations of the parameters a's and b's, the gaps in their spectra are "small", and vice versa.

To be more formal, given a bounded sequence $c = \{c_j\}_{j\in\mathbb{Z}}$ of real numbers, its *variation* is defined by $\omega_c := \sup_{i,j\in\mathbb{Z}}(c_i - c_j)$.

Let γ be a maximal gap in the spectrum of J, $|\gamma| = \max_{1\le j\le p-1}|\gamma_j|$.

Theorem 1. *Let J be a periodic Jacobi matrix of period p. Then*

$$\omega_b \le (p-1)|\gamma|, \qquad \omega_a \le p^2\sqrt{p}\,|\gamma|, \qquad \omega_b + \omega_a \ge \frac{|\gamma|}{4}.$$

In particular, $|\gamma| = 0$ (all gaps are closed) if and only if $\omega_a = \omega_b = 0$ (J is a constant Jacobi matrix), so the result of Hochstadt follows.

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Homogenization of the diffusion equation in domains with the fine-grained boundary with the nonlinear boundary Robin condition

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Let Ω be a bounded domain in R^3 and $F_i^\varepsilon = B(x^{i\varepsilon}, r_i^\varepsilon)$ be disjoint balls of radii $r_i^\varepsilon = a_i^\varepsilon\varepsilon^\alpha\,(\exists\, a, A \in R^1 :\ 0 < a \leqslant a_i^\varepsilon \leqslant A < \infty)$ with centers at the points $x^{i\varepsilon}$ $(i = 1, ..., N^\varepsilon)$. The small parameter ε characterizes average distance between the nearest balls, i.e. $N^\varepsilon = O(\varepsilon^{-3})$.

In the domain $\Omega^\varepsilon = \Omega \setminus \cup_{i=1}^{N^\varepsilon} F_i^\varepsilon$ we consider initial-value problem

$$\begin{cases} -\Delta u^\varepsilon(x) = f^\varepsilon(x),\ x \in \Omega^\varepsilon, \\ \dfrac{\partial u^\varepsilon(x)}{\partial\nu} + \sigma^\varepsilon(x^{i\varepsilon}, u^\varepsilon) = 0,\ x \in \partial F_i^\varepsilon,\ i = 1, ..., N^\varepsilon, \\ u^\varepsilon(x) = 0,\ x \in \partial\Omega. \end{cases} \tag{1}$$

where Δ is the Laplace operator, ν is the exterior unit normal with respect to ∂F^ε; the source function $f^\varepsilon(x) \in L^2(\Omega)$ is given and the density function of absorption $\sigma^\varepsilon(x,u)$ satisfies conditions: $\sigma^\varepsilon(x,u) = \varepsilon^\beta\sigma(x,u)$, here $\beta \in R^1$; $\sigma(x,u) \in C(\Omega, C^1(R^1))$ and $\sigma(x,0) = 0$; $\forall\, x \in \Omega :\ 0 < k_1 \leqslant \frac{\partial}{\partial u}\sigma(x,u) \leqslant k_2(1 + |u|^\nu)$, here $0 \leqslant \nu < 1$.

We study the asymptotic behaviour of the solutions $u^\varepsilon(x)$ of problem (1) as $\varepsilon \to 0$ for different value of the parameters $(\alpha,\, \beta) \in \Lambda = \{1 < \alpha \leqslant 3,\ \beta \geqslant 3 - 2\alpha\} \cup \{\alpha \geqslant 3,\ \alpha + \beta \geqslant 0\}$. It is described by the following theorem.

Theorem 1. *Let the generalized functions $c^\varepsilon(x,u)$, which are the functions of the space distribution of the absorption density in the domain Ω^ε, as $\forall\, u \in R^1$ converge in the weak topology of the space $\mathcal{D}'(\Omega)$ to the function $c(x,u) \in C(\Omega, C^1(R^1))$; the functions $f^\varepsilon(x)$ extended by zero to F^ε weakly converge in $L^2(\Omega)$ to the function $f(x)$.*

Then the solutions $u^\varepsilon(x)$ of problem (1) converge in $L^p(\Omega^\varepsilon,\Omega)$ $(p<6)$ to the function $u(x)$, which is a solution of the homogenized problem

$$\begin{cases} -\Delta u(x) + \dfrac{\partial}{\partial u} c(x,u) = f(x) \text{ in } \Omega, \\ u(x) = 0 \text{ on } \partial\Omega. \end{cases}$$

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On the difference equations with operator coefficients containing spectral parameter nonlinearly

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In this talk we consider in regular and singular cases various difference equations with operator coefficients containing the spectral parameter in Nevanlinna manner. Usually, relations (but not operators) correspond to the equations we investigate, even if the spectral parameter enters the equation linearly.

For this equations we establish analogues of the results obtained in [1, 2, 3, 4] for differential equations. In particular, we introduce and examine: characteristic operator, which is an analogue of Weyl–Titchmarsh characteristic matrix; Weyl type functions, solutions and equalities; analogue of the generalized resolvent; eigenfunction expansions; inverse problems.

We note that difference equations generated by Jacobi matrix with operator elements can be reduced to the considered equations.

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High-dimensional Gaussian fields with isotropic increments seen through spin glasses

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Studies of particles subjected to high-dimensional rugged potentials landscapes is a central topic in statistical physics and probability with a host of applications to complex systems performing a *search on rugged landscapes* such as those motivated by life sciences and artificial intelligence.

We focus on a generalization [2] of the toy model based on *Gaussian fields with isotropic increments* (= high-dimensional generalizations of fractional Brownian motions) suggested by [1] and derive a rather explicit *variational formula* for the *free energy*. The formula has a similar flavour to the celebrated Parisi formula [3] for the Sherrington-Kirkpatrick model of a spin glass. Depending on the strengths of correlations and a priori biases, the variational formula reveals *phase transitions* and rich scenarios for the effective complexity of the system. Some of the recent advances from the mathematical spin glass theory will be explained along the way.

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On the min-moment problem and the controllability function in completely controllable systems

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The min-moment problem consist in constriction a pair $(\Theta, u(t))$ such that a moment equalities hold

$$\int_0^{\Theta} g_k(t)u(t)dt = s_k, \quad k = 1, \dots, n, \quad |u(t)| \leq c,$$

where $g_k(t)$ are given functions, $s_k(t)$ are given numbers, the number Θ is the smallest number such that given equalities hold. This problem is proposed in [1]. On the bases of that the time-optimal control problem for linear completely controllable systems is solved in [2],[3].

In the case if completely controllable systems in general is not linear ($\dot{x} = f(x, u),\ u \in \Omega$), then the control and the time of motion is finding with the help of controllability function which was proposed in [4]. It is similarity to the Lyapunov function. For the solving the admissible and time-optimal synthesis problem we must construct the pair of functions $(\Theta(x) > 0, x \neq 0, u(x))$ such that for some $\alpha > 0$, $\beta > 0$ the following inequality holds:

$$\sum_{i=1}^{n} \frac{\partial \Theta(x)}{\partial x_i} f_i(x, u(x)) \leq -\beta \Theta^{1-\frac{1}{\alpha}}(x).$$

In the case if the time-optimal control problem the function $\Theta(x)$ is the time-optimal and the following inequality holds [5]:

$$\min_{u \in \Omega} \sum_{i=1}^{n} \frac{\partial \Theta(x)}{\partial x_i} f_i(x, u) = \sum_{i=1}^{n} \frac{\partial \Theta(x)}{\partial x_i} f_i(x, u(x)) = -1.$$

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On the feedback synthesis problem for one robust oscillating system

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The work deals with the synthesis problem for robust oscillating system of the fourth order with uncertain parameter (perturbation). The linearized equations of the motion of this system are of the form:

$$\begin{cases} m_1 \ddot{y}_1 + k y_1 - k y_2 = 0, \\ m_2 \ddot{y}_2 - k y_1 + k y_2 = u. \end{cases}$$

where m_1 and m_2 are masses of moving weights, k is the spring stiffness, y_1 and y_2 the deviations from the equilibrium position of the first and second weights, respectively. We assume that the force u satisfies the inequality $|u| \leq 1$. Suppose that the values of m_1 and m_2 are known. Suppose that the spring stiffness k is unknown.

Our approach is based on the controllability function method created by V.I. Korobov [1]. The case when the controllability function is the time of motion from the arbitrary initial point to the origin is discussed in detail. As a perturbation we consider the spring stiffness which is not known exactly, but lies on the segment, on the ranges of which we give the estimate. The solution of the synthesis problem is constructed, i. e. we find the segment where the perturbation can vary and we find the control which is independent from the perturbation and steers an arbitrary initial point from some neighborhood of the origin to the origin for any value of the perturbation from this segment. Ranges of perturbation are such that the total derivative of the controllability function due to the perturbed system is negative [2]. An estimate of the time of motion from below has been given. The solution is illustrated for a specific initial point.

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The boundary equations of the diffraction problem of oblique incidence electromagnetic waves on a periodic impedance lattice

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We are talking about the plane monochromatic electromagnetic wave diffraction [1].

The goal of our investigation is the analysis of possibility using a hypersingular integral equation for solving the mentioned diffraction problem.

The hypersingular integral equation in comparison with the singular integral equation [1] has the several advantages with respect to a numerical solution [2]. The modules of the main diagonal elements of the matrix that correspond to the hypersingular equation exceed modules of other row elements. The numerical solution of the hypersingular equation with given in advance accuracy requires less time. The hypersingular equation does not need the additional conditions.

Based on the parametric representation of hypersingular, singular and improper with logarithmic singularities integrals the diffraction problem is reduced to a pair of second kind boundary integral equations which are with logarithmic singularities in the kernel and hypersingular respectively. The obtained hypersingular equation is complete in the sense that it includes the singular integral along with the hypersingular and improper integrals.

The proposed result enables us to use the hypersingular equation for solving the diffraction problem of the oblique incidence electromagnetic waves on a periodic impedance lattice.

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A matrix Baker–Akhiezer function associated with the Maxwell–Bloch equations

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Let a constant real vector $\phi = (\phi_1, \ldots, \phi_N)$ and real function $n(\lambda)$ ($\lambda \in \mathbb{R}$) be given. Let $\Sigma_j := (E_j, E_j^*)$ $j = 0, 1, 2, \ldots, n$ be a set of vertical open intervals on the complex plane $\mathbb{C}$ which together with real line $\mathbb{R}$ constitute an oriented and symmetric with respect to $\mathbb{R}$ contour Σ.

Definition 1. *A 2×2 matrix $\Psi(t, x, z)$ is called the matrix Baker–Akhiezer function associated with the Maxwell–Bloch equations if*

- *for any $x, t \in \mathbb{R}$, $\Psi(t, x, z)$ is analytic in $z \in \mathbb{C} \setminus \overline{\Sigma}$, $\Sigma := \mathbb{R} \cup \cup_{j=0}^{N} (E_j, E_j^*)$;*
- *$\Psi(t, x, z)$ has square integrable singularities at the end points E_j and E_j^* and it is bounded at the points of self-intersection ($\Re E_j$, $j = 0, 1, \ldots, n$);*
- *$\Psi(t, x, z)$ satisfies the jump conditions:*
  $\Psi_-(t, x, z) = \Psi_+(t, x, z) J(x, z)$, $z \in \Sigma$, where

$$J(x,z) = \mathrm{e}^{-\frac{\pi x n(\lambda)\sigma_3}{2}}, \ \lambda \in \mathbb{R} \setminus \cup_{j=0}^{N} \Re E_j, \quad j = 0, 1, \ldots, n,$$
$$= \begin{pmatrix} 0 & \mathrm{i}\mathrm{e}^{-\mathrm{i}\phi_j} \\ \mathrm{i}\mathrm{e}^{\mathrm{i}\phi_j} & 0 \end{pmatrix}, \quad \phi_0 = 0, \quad z \in \Sigma_j = (E_j, E_j^*);$$

- *$\Psi(t, x, z) = \sigma_2 \Psi^*(t, x, z^*) \sigma_2$, where $\sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$ is the Pauli matrix;*
- *$\Psi(t, x, z) = \left(I + O(z^{-1})\right) e^{-\mathrm{i}z(t-x)\sigma_3}$ as $z \to \infty$.*

It is proved that such a matrix exists and is unique. It has an explicit representation by theta functions and Cauchy integrals. The function $\Psi(t, x, z)$

generates a finite-gap solution ($\mathcal{E} = \mathcal{E}(t,x)$, $\rho = \rho(t,x,\lambda)$, $\mathcal{N} = \mathcal{N}(t,x,\lambda)$ of the Maxwell–Bloch equations:

$$\frac{\partial \mathcal{E}}{\partial t} + \frac{\partial \mathcal{E}}{\partial x} = \int_{-\infty}^{\infty} n(\lambda)\rho(t,x,\lambda)d\lambda,$$

$$\frac{\partial \rho}{\partial t} + 2\mathrm{i}\lambda\rho = \mathcal{N}\mathcal{E}, \qquad \frac{\partial \mathcal{N}}{\partial t} = -\frac{1}{2}(\mathcal{E}^*\rho + \mathcal{E}\rho^*).$$

The functions $\mathcal{E}(t,x), \rho(t,x,\lambda), \mathcal{N}(t,x,\lambda$ have also an explicit form in theta functions and Cauchy integrals. In general, they are quasi-periodic in t and x.

Nonlinear non-stationary problems of heat conduction for three-component bodies

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We have proposed and developed a method for solving nonlinear non-stationary heat conduction problems for layered bodies (see, e.g., [1], [2]). In this work we continue the research by describing an approach based on analytical and numerical techniques and aimed to analyze one-dimensional temperature fields in three-component flat layered bodies. The method takes into account the temperature dependence of thermal characteristics and convective-radiant heat exchange with environment for a wide range of the components thickness. We use the Green function of a linear non-stationary heat conduction problem for three-component space in the form of a functional series [3].

We obtain new integral presentations for the solutions of corresponding problems [4, 5]. Assuming that the thermal conductivity depends weakly on temperature, we reduce the corresponding integral relations to a recurrence system of two (infinite rod), three (half-infinite rod) or four (layer) nonlinear algebraic equations. We present the results of numerical analysis and compare them with the results known in literature [1].

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Approximation of periodic functions by generalized Fup-functions

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Generalized Fup-functions were introduced in [1]. Further it was show in [2] that spaces of linear combinations of generalized Fup-functions shifts are asymptotically extremal for approximation of the class $\widetilde{W_2^r}$ that consists of functions $g \in C^{r-1}_{[-\pi,pi]}$ such that $g^{(k)}(-\pi) = g^{(k)}(\pi)$ for any $k = 0, 1, \dots, r-1$, $g^{(r-1)}$ is absolutely continuous and $\|g^{(r)}\|_{L_2[-\pi,\pi]} \le 1$. This convenient property is provided by almost-trigonometric basis theorem [2].

Consider an arbitrary even natural number N. Let V_N be the space of functions $f \in L_2[-\pi,\pi]$ such that $f(x) = \sum\limits_{p=0}^{N/2-1} (a_p v_{N,p}(x) + b_p w_{N,p}(x))$, where $v_{N,0}(x) = 1$,

$$v_{N,p}(x) = \sum_{k=0}^{\infty} (r_{N,p,k} \cos((p+kN)x) + q_{N,p,k} \cos((N(k+1)-p)x)),$$

$$w_{N,p}(x) = \sum_{k=0}^{\infty} (s_{N,p,k} \sin((p+kN)x) + t_{N,p,k} \sin((N(k+1)-p)x))$$

for $p = 1, 2, \dots, N/2-1$ and

$$w_{N,0}(x) = \sum_{k=0}^{\infty} \left(y_{N,k} \cos(N(2k+1)x/2) + z_{N,k} \sin(N(2k+1)x/2) \right),$$

$r_{N,p,0} = s_{N,p,0} = 1$ for any $p = 1, 2, \dots, N/2-1$.

Let the sequence of spaces $\{V_N\}$ satisfy conditions of theorem 7 [2] and $N = 2^{n+1}$. Moreover, if the functions $m(N)$ and $M(n)$ from this theorem satisfy the following conditions: $m(N) = n$ and $M(N) \to 0$ as $N \to \infty$, then there exists $n(r)$ such that for any $n \ge n(r)$ the space $V_{2^{n+1}}$ is extremal for approximation of the class $\widetilde{W_2^r}$ in the norm of the space $L_2[-\pi,\pi]$. This implies that generalized Fup-functions, which are extremal for approximation of periodic differentiable functions, can be constructed.

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Stability criterion for C_0-semigroups

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Stability theory for C_0-semigroups is the important direction of functional analysis and infinite-dimensional linear systems theory. The first result on the asymptotic stability of C_0-semigroups – the most subtle type of stability – on Banach spaces X was obtained by G. M. Sklyar & V. Ya. Shirman in 1982 as a development of the B. Sz.-Nagy & C. Foias theorem for contractions on a Hilbert space, see [1]. The result was obtained when A – the generator of the C_0-semigroup – is a linear bounded operator on X. In 1988 it was shown independently by two groups of mathematicians that this result is true for more general situation – when A may be unbounded but generates bounded C_0-semigroup on X, see [2, 3]. However, the assumption on the countability of the set $\sigma(A) \cap i\mathbb{R}$ was very essential in these studies. We obtain the following criterion for three types of stability of C_0-semigroups in a situation when A is an unbounded operator on a Hilbert space H with a Riesz basis of eigenvectors.

Theorem 1. *Let A be a closed operator on H with eigenvalues $\{\lambda_n\}_{n=1}^{\infty}$ (counting with multiplicity) and let A has no root vectors. Suppose that the corresponding eigenvectors $\{\phi_n\}_{n=1}^{\infty}$ form a Riesz basis of H. Then the following statements are true:*

1. *The operator A generates a stable C_0-semigroup if and only if $\sup_{n\in\mathbb{N}} \Re(\lambda_n) \leq 0$.*
2. *The operator A generates an exponentially stable C_0-semigroup if and only if $\sup_{n\in\mathbb{N}} \Re(\lambda_n) < 0$.*
3. *The operator A generates an asymptotically stable C_0-semigroup if and only if for each $n \in \mathbb{N}$ we have $\Re(\lambda_n) < 0$.*

The third point of this result complements studies, which are presented in [1, 2, 3], and includes the case when the set $\sigma(A) \cap i\mathbb{R}$ may be uncountable.

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Pseudospectral functions of symmetric systems with the maximal deficiency index

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We consider the symmetric differential system

$$Jy' - A(t)y = \lambda\Delta(t)y \tag{1}$$

with $n\times n$-matrix coefficients $J(= -J^* = -J^{-1})$ and $A(t) = A^*(t),\ \Delta(t) \geq 0$ defined on an interval $[a, b)$ with the regular endpoint a. Let $\varphi(\cdot, \lambda)$ be a matrix solution of this system of an arbitrary dimension and let

$$(Vf)(s) = \widehat{f}(s) := \int_{\mathcal{I}} \varphi^*(t, s)\Delta(t)f(t)\,dt$$

be the Fourier transform of the function $f(\cdot) \in L^2_\Delta(\mathcal{I})$. A pseudospectral function of the system is defined as a matrix-valued distribution function $\sigma(\cdot)$ of the dimension n_σ such that V is a partial isometry from $L^2_\Delta(\mathcal{I})$ to $L^2(\sigma;\mathbb{C}^{n_\sigma})$ with the minimally possible kernel.

It is assumed that the deficiency indices $N_\pm$ of the system satisfies $N_- \leq N_+ = n$. For this case we define the monodromy matrix $B(\lambda)$ as a singular boundary value of a fundamental matrix solution $Y(t, \lambda)$ at the endpoint b and parameterize all pseudospectral functions $\sigma(\cdot)$ of any possible dimension $n_\sigma \leq n$ by means of the the linear-fractional transform

$$m_\tau(\lambda) = (C_0(\lambda)w_{11}(\lambda) + C_1(\lambda)w_{21}(\lambda))^{-1}(C_0(\lambda)w_{12}(\lambda) + C_1(\lambda)w_{22}(\lambda))$$

and the Stieltjes inversion formula for $m_\tau(\lambda)$. Here $w_{ij}(\lambda)$ are the matrix coefficients defined in terms of $B(\lambda)$ and $\tau = \{C_0(\lambda), C_1(\lambda)\}$ is a Nevanlinna matrix pair (a boundary parameter) satisfying certain admissibility conditions. It turns out that the matrix $W(\lambda) = (w_{ij}(\lambda))^2_{i,j=1}$ has the properties similar to those of the resolvent matrix in the extension theory of symmetric operators.

The obtained results develop the results by Arov and Dym; A. Sakhnovich, L. Sakhnovich and Roitberg; Langer and Textorius.

The results of the talk are partially specified in [1], [2].

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On the Skitovich–Darmois theorem for the group of p-adic numbers

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The classical characterization theorems of mathematical statistics were extended to different algebraic structures such as locally compact Abelian groups, Lie groups, quantum groups, and symmetric spaces (see [1], where one can find necessary references). Much attention has been devoted to the study of the Skitovich–Darmois theorem, where a Gaussian distribution is characterized by the independence of two linear forms from n independent random variables. Coefficients of linear forms are topological automorphisms of a group.

The article [2] is devoted to the study of the distributions of two independent random variables with values in the group of p-adic numbers Ω_p which are characterized by the independents of two linear forms. The result of the article [2] can be considered as an analogue of the Skitovich–Darmois theorem for the group Ω_p and and for two independent linear forms from two independent random variables. We continue these investigations and obtain an analogue of the Skitovich–Darmois theorem for the group Ω_p and for three independent linear forms from three independent random variables.

Our results and results of [2] show that the Skitovich–Darmois theorem, generally speaking, fails for the group Ω_p. In particular, we give the complete descriptions of all automorphisms $\alpha_j, \beta_j, \gamma_j \in \mathrm{Aut}(\Omega_p)$ such that the independence of the linear forms $L_1 = \alpha_1\xi_1 + \alpha_2\xi_2 + \alpha_3\xi_3$, $L_2 = \beta_1\xi_1 + \beta_2\xi_2 + \beta_3\xi_3$ and $L_3 = \gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_3$ implies that random variables ξ_1, ξ_2, ξ_3 have idempotent distributions, i.e., shifts of the Haar distributions of compact subgroups of Ω_p. We note that since Ω_p is a totally disconnected group, the Gaussian distributions on Ω_p are degenerated. On the group Ω_p idempotent distributions are considered as an analog of Gaussian distributions.

The studing of the analogue of the Skitovich–Darmois theorem for the group Ω_p reduces to studying the solutions of a functional equation in the class of normalized continuous positive defined functions.

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The actions of the full symmetric group on von Neumann algebras

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Let M be separable w^*-algebra, and let $\operatorname{Aut} M$ be the automorphism group of M. Denote by $\overline{S}_\infty$ the infinite full symmetric group. Suppose that $\alpha : \overline{S}_\infty \mapsto \operatorname{Aut} M$ is the action of $\overline{S}_\infty$ on M. We will prove that there exists normal α-invariant weight on M.

The study of solitonic objects of the D-brane type by application of topological invariants

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The study of solitonic objects of the D-brane type is associated with a space of extra dimensions [1]. The ten-dimensional space of string theory can be represented as a direct product $M_{10} = M_4 \times K_6$, where the manifold M_4 is a four-dimensional space-time, and K_6 is an additional compact manifold. To obtain acceptable solutions, that are consistent with the Standard Model (SM), $SU(3) \times SU(2) \times U(1)$, Calabi-Yau manifolds or orbifolds are choosen as spaces of extra dimensions [2]. The Calabi-Yau space is Ricci-flat and has an SU (3) holonomy group. Additional selection criteria for agreement with physical observables are associated with the identification of the spinor and gauge connections, i.e. with the Bianchi identity $\operatorname{Tr} R \wedge R = \frac{1}{30} \operatorname{Tr} F \wedge F$ (on the left is the curvature form of the Riemannian manifold Calabi-Yau, and on the right is the Yang-Mills form). For description such a space are introduced the differential forms on complex manifolds with basic differentials $dz^j = dx^j + idy^j$. For (p, q) - forms

$$\omega = \omega_{ijk\ldots u\overline{i}\overline{j}\overline{k}\ldots\overline{u}} dz^i \wedge dz^{j\cdots} dz^u \wedge d\overline{z}^i d\overline{z}^{j\cdots} d\overline{z}^u$$

we define external derivatives

$$\partial\omega = \frac{\partial}{\partial z^i} \omega_{ijk\ldots\overline{ijk}} dz^i \wedge \ldots d\overline{z}^u \ , \ \overline{\partial}\omega = \frac{\partial}{\partial \overline{z}^i} d\overline{z}^i \omega_{ijk\ldots\overline{ijk}} dz^j \wedge \ldots d\overline{z}^u$$

The Dolbeault cohomology is determined as follows

$$H^{p,q} = \frac{\overline{\partial} - \text{closed}(p,q)\text{form}}{\overline{\partial} - \text{exact}(p,q)\text{form}}, \quad \text{dim} H^{p,q}_{\overline{\partial}}(M) = h^{p,q} \ ,$$

where $h^{p,q}$ are the Hodge numbers. The Euler characteristic for a Ricci-flat manifold is defined in terms of Hodge numbers

$$\chi = \sum_{p,q=n} (-1)^{p+q} h^{p,q} = 2(h^{1,1} - h^{2,1}) \ .$$

The number of generations for SM is equal to $3 = \frac{1}{2}|\chi|$.

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Exact null controllability and complete stabilizability of neutral type systems

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For abstract linear systems in Hilbert spaces we revisit the problems of exact controllability and complete stabilizability (stabilizability with an arbitrary decay rate), the latter property is related to exact null controllability. For controlled neutral type systems

$$\dot{z}(t) = A_{-1}\dot{z}(t-1) + \int_{-1}^{0} [A_2(\theta)\dot{z}(t+\theta) + A_3(\theta)z(t+\theta)]\,\mathrm{d}\theta + Bu(t) \quad (1)$$

we obtain the following characterization of complete stabilizability:

Theorem 1. *System (1) is completely stabilizable by a feedback law*

$$u(t) = F_{-1}\dot{z}(t-1) + \int_{-1}^{0} [F_2(\theta)\dot{z}(t+\theta) + F_3(\theta)z(t+\theta)]\,\mathrm{d}\theta,$$

if and only if

1. $\operatorname{rank}\begin{pmatrix}\Delta(\lambda) & B\end{pmatrix} = n$ *for all* $\lambda \in \mathbb{C}$,
2. $\operatorname{rank}\begin{pmatrix}\mu I - A_{-1} & B\end{pmatrix} = n$ *for all* $\mu \in \mathbb{C}$, $\mu \neq 0$,

where $\Delta(\lambda)$ *is characteristic matrix of (1), given by*

$$\Delta(\lambda) = \lambda I - \lambda e^{-\lambda}A_{-1} - \int_{-1}^{0} \left[\lambda e^{\lambda s}A_2(s) + e^{\lambda s}A_3(s)\right]\mathrm{d}s.$$

The conditions 1 and 2 of the Theorem are proved to be necessary for exact null controllability of neutral type systems (1). Moreover, we formulate the Conjecture that these conditions are also sufficient what means that exact null controllability is equivalent to complete stabilizability for neutral type systems. We illustrate the obtained results and Conjecture by several cases and examples.

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Long time asymptotics for the nonlocal nonlinear Schrödinger equation on the line

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We consider the initial value problem for a non-local modification of the nonlinear Schrödinger equation (NLS)

$$iq_t(x,t) + q_{xx}(x,t) + 2\sigma q^2(x,t)\bar{q}(-x,t) = 0, \qquad \sigma = \pm 1$$

with initial data vanishing as $x \to \pm\infty$. This equation was introduced by M. Ablowitz and Z. Musslimani, see [1] and [2]. Our main objective is to study the long-time asymptotics of the solution of this initial value problem. Our approach is based on the asymptotic analysis of the associated matrix Riemann–Hilbert problem using the ideas of the nonlinear steepest decent method [3]. We obtain the principal term of the asymptotics under certain assumptions on the index of the associated scalar Riemann–Hilbert problem, whose jump data are expressed in terms of two reflection coefficients. A principal difference of the Riemann–Hilbert approach for the non-local nonlinear Schrödinger equation being compared with that for the conventional (local) NLS is the lack of symmetry of the spectral functions w.r.t. the real axis (in the complex plane of the spectral parameter). One of the important consequence of this is the fact that the decay (in t) of the principal term of the asymptotics depends on the direction x/t (which is in sharp contrast with the case of the local NLS, where the decay is always of order $t^{-1/2}$).

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Classification of symmetries of the quantum disc

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Bounded quantum domains are normally considered together with their symmetries. The problem of classifying such symmetries in the case of quantum plane and its Laurent extension has been approached in [1], [2]. Another simplest quantum bounded symmetric domain is a quantum disc, which is a unital algebra $Pol(\mathbb{D})_q$ generated by z, z^* subject to the relation

$$z^* z = q^{-2} z z^* + 1 - q^{-2}$$

where q is not a root of 1. No involution is present here (z^* is treated as a single symbol). The 'algebra of symmetries' $U_q(\mathfrak{sl}_2)$ is the quantum universal enveloping algebra of $\mathfrak{sl}_2$, defined by its generators k, k^{-1}, e, f, together with the well known relations [3]. See also [3] for a definition of quantum symmetry (a structure of $U_q(\mathfrak{sl}_2)$-module algebra on $Pol(\mathbb{D})_q$).

Theorem 1. *A complete list of $U_q(\mathfrak{sl}_2)$-symmetries on the quantum disc is given by the following 4 two-parameter series, with the parameters $a, b \in \mathbb{C}$, $b \neq 0$, and $y = 1 - zz^* \in Pol(\mathbb{D})_q$ a distinguished element:*

1. $$\begin{array}{lll} \mathsf{k}(z) = q^2 z & \mathsf{e}(z) = bq^2 z^2 & \mathsf{f}(z) = -b^{-1} q^{-1} \\ \mathsf{k}(z^*) = q^{-2} z^* & \mathsf{e}(z^*) = -ay^2 - b & \mathsf{f}(z^*) = b^{-1} q z^{*2} \end{array}$$
2. $$\begin{array}{lll} \mathsf{k}(z) = q^2 z & \mathsf{e}(z) = b^{-1} q z^2 & \mathsf{f}(z) = -ay^2 - b \\ \mathsf{k}(z^*) = q^{-2} z^* & \mathsf{e}(z^*) = b^{-1} q^{-1} & \mathsf{f}(z^*) = bq^2 z^{*2} \end{array}$$
3. $$\begin{array}{lll} \mathsf{k}(z) = q^{-2} z & \mathsf{e}(z) = -bq^{-2} & \mathsf{f}(z) = -b^{-1} q^3 z^2 \\ \mathsf{k}(z^*) = q^2 z^* & \mathsf{e}(z^*) = 0 & \mathsf{f}(z^*) = b^{-1}((q + q^{-1})y - q) - aq^{-2} \end{array}$$
4. $$\begin{array}{lll} \mathsf{k}(z) = q^{-2} z & \mathsf{e}(z) = b^{-1}((q + q^{-1})y - q) - aq^{-2} & \mathsf{f}(z) = 0 \\ \mathsf{k}(z^*) = q^2 z^* & \mathsf{e}(z^*) = b^{-1} q^3 z^{*2} & \mathsf{f}(z^*) = bq^{-2} \end{array}$$

This list contains an uncountable family of isomorphism classes of $U_q(\mathfrak{sl}_2)$-symmetries.

The initial two series are non-disjoint, and their intersection (given by setting $a = 0$), after a suitable normalization of b, becomes a well-known $U_q(\mathfrak{sl}_2)$-symmetry on the quantum disc.

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Classically integrable models of massless particle and $D0-$brane on $AdS_4 \times \mathbb{CP}^3$ superbackground

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Methods elaborated to study integrable systems have nowadays become the basic tools for testing duality correspondences [1], [2] between superconformal gauge theories (CFT) in $D = 3, 4, 6$ dimensions and supersymmetric string/brane theories on the products of anti-de Sitter space-times AdS_{D+1} and compact manifolds $\mathcal{M}$, where $\mathcal{M} = S^7 = \mathbb{CP}^3 \times S^1$ for $D = 3$, $\mathcal{M} = S^5$ for $D = 4$, and $\mathcal{M} = S^4$ for $D = 6$ (for reviews see, e.g., [3], [4]). For the best explored case of AdS_5/CFT_4 correspondence the pivotal discovery on the string side was the proof of the integrability of the two-dimensional $AdS_5 \times S^5$ sigma-model equations [5]. In the case of AdS_4/CFT_3 duality there is an evidence [6] that $AdS_4 \times \mathbb{CP}^3$ sigma-model equations are also classically integrable but the proof is still lacking because of their much more complicated structure compared to that of the $AdS_5 \times S^5$ sigma-model. In the simplifying limit of infinite tension of the $AdS_4 \times \mathbb{CP}^3$ sigma-model its equations reduce to those of the massless superparticle [7] for which we have found Lax-type representation that also admits straight-forward generalization to the case of $D0-$brane [8].

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Spectral analysis of the Schrödinger operator with a periodic PT-symmetric potential

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I am going to give a talk about the Schrödinger operator $L(q)$ with a complex-valued PT-symmetric periodic potential q. A basic mathematical question of

PT-symmetric quantum mechanics concerns the reality of the spectrum of the considered Hamiltonian. First we consider the general spectral property of the spectrum of $L(q)$ and prove that the main part of its spectrum is real and contains the large part of $[0;\infty)$. Using this we find necessary and sufficient condition on the potential for finiteness of the number of the nonreal arcs in the spectrum of $L(q)$. Moreover, we find necessary and sufficient conditions for the equality of the spectrum of $L(q)$ to the half line and consider the connections between spectrality of $L(q)$ and the reality of its spectrum for some class of PT-symmetric periodic potentials. Finally we give a complete description, provided with a mathematical proof, of the shape of the spectrum of the Hill operator with optical potential $4\cos^2 2x + 4iV\sin 2x$.

Analytical forms of deuteron wave function

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The radial deuteron wave function in publications can be presented as a table: through respective arrays of values of proper radial wave functions. It is sometimes quite difficult and inconveniently to operate with such arrays of numbers at numerical calculations. And the program code for numerical calculations is huge and overloaded. Therefore, it is feasible to obtain simpler analytical forms of deuteron wave function (DWF) representation [1]. Cap's [2] and Dubovichenko [3] parameterizations can be generalized for the DWF approximation as such analytical forms:

$$\chi(r) = r^n \sum_{i=1}^{N} C_i \exp(-c_i r^3), \tag{1}$$

Given number N=11, search for an index of function of a degree r^n has been carried out, appearing as a factor before the sums of exponential terms of (1). The factors before the sums (1) can be chosen as $r^{3/2}$ and r^1 [4]:

$$u(r) = r^{3/2} \sum_{i=1}^{N} A_i \exp(-a_i r^3); \quad w(r) = r \sum_{i=1}^{N} B_i \exp(-b_i r^3). \tag{2}$$

Coefficients and DWFs (2) for NijmI, NijmII, Nijm93, Reid93 and Argonne v18 potentials it is resulted in papers [1, 4, 5]. A detailed comparison of the obtained values of deuteron tensor polarization $t_{20}(p)$. Calculated based for scattering angle θ=70^0. For these potentials with the up-to-date experimental data of JLAB t20 and BLAST collaborations.

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Dissipative model of vortex motion in the rotation Bose–Einstein condensate

Tetiana Zuieva, *Kharkiv, Ukraine*

We study a Bose–Einstein condensate in the presence of a strongly anisotropic trapping potential $V_{ext} = \frac{1}{2}m(\omega_r^2 r^2 + \omega_z z^2)$ with $\omega_r \ll \omega_z$. In this case the trapped BEC has a nearly planar pancake shape and can be consider in the frame of 2D model in the plane (x, y).

The non-dimensional Gross–Pitaevskii equation for the BEC rotating with the angular velocity Ω can be written as [1]

$$(i-\gamma_0)\frac{\partial\psi}{\partial t} = -\frac{1}{2}\nabla^2\psi + \frac{\Omega_{tr}^2}{2}r^2\psi + g^2\psi|\psi|^2 - \mu\psi + i(\mathbf{\Omega}\times\mathbf{r})\cdot\nabla\psi. \quad (1)$$

where $\Omega_{tr} = \omega_r/\omega_z$, g is the interaction parameter, μ is the chemical potential and $\gamma_0 > 0$ is the temperature dependent parameter describing the dissipation. We suppose that there are N vortices with intensities n_j that are situated at the points $\xi_j = \xi_j(t) = (\eta_j, \zeta_j)$, $j = 1...N$, and can move. The equations of this motion can be written in the form of system

$$\dot{\eta}_j - \Omega\zeta_j = -S_{kj} + \frac{\Omega_{tr}^2}{2\Xi_j^2}\zeta_j n_j\omega_j - n_j\gamma_0\dot{\zeta}_j\omega_j,$$

$$\dot{\zeta}_j + \Omega\eta_j = C_{kj} - \frac{\Omega_{tr}^2}{2\Xi_j^2}\eta_j n_j\omega_j + n_j\gamma_0\dot{\eta}_j\omega_j. \quad (2)$$

Parameters C_{kj}, S_{kj} depends on the positions of all N vortices, Ξ_j and ω_j are the functions of $\{\eta_j, \zeta_j\}$ and $j = 1...N$. To obtain these equations we use the method of matched asymptotic expansion of the core and far field solutions that was proposed by Rubinstein and Pismen in 1994 [2]. We are looking for a solution of Eq. (1) in the form of an expansion in the parameter ε, that is the vortex core size, in two different space scales.

We can draw the trajectories of the motion for any number of vortices if we start from the given initial positions. One can see that in the dissipative case the vortices move to their equilibrium points.

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