B.Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine

AMPH 2015

III International Conference

ANALYSIS AND MATHEMATICAL PHYSICS

June 15–19, 2015

Book of Abstracts

Kharkiv 2015

III International Conference "ANALYSIS AND MATHEMATICAL PHYSICS": Book of abstracts, Kharkiv: B.Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 2015, 32 p.

Program Committee

V. Marchenko, L. Pastur, E. Khruslov.

Organizing Committee:

G. Feldman, L. Fardigola, V. Vasilchuk.

CONTENTS

SENIOR PARTICIPANTS

Around the Van Daele–Schmüdgen theorem

Yury Arlinskii, *Severodonetsk, Ukraine*

For a *bounded* non-negative self-adjoint operator acting in a complex, infinite-dimensional, separable Hilbert space $\mathcal{H}$ and possessing a dense range $\mathcal{R}$ we propose a new approach to characterization of phenomenon concerning the existence of subspaces $\mathfrak{M} \subset \mathcal{H}$ such that $\mathfrak{M} \cap \mathcal{R} = \mathfrak{M}^{\perp} \cap \mathcal{R} = \{0\}$. We show how the existence of such subspaces leads to various *pathological* properties of *unbounded* self-adjoint operators related to von Neumann theorems [1]. We revise the von Neumann-Van Daele-Schmüdgen assertions [1], [3], [2] to refine them. We also develop a new systematic approach, which allows to construct for any *unbounded* densely defined symmetric/self-adjoint operator T *infinitely* many pairs $\langle T_1, T_2 \rangle$ of its closed densely defined restrictions $T_k \subset T$ such that $\mathrm{dom}\,(T^* T_k) = \{0\}$ $(\Rightarrow \mathrm{dom}\, T_k^2 = \{0\})$ $k = 1, 2$ and $\mathrm{dom}\, T_1 \cap \mathrm{dom}\, T_2 = \{0\}$, $\mathrm{dom}\, T_1 \dot{+} \mathrm{dom}\, T_2 = \mathrm{dom}\, T$.

This is a joint work with Valentin A. Zagrebnov, Département de Mathématiques, Université d'Aix-Marseille and Institut de Mathématiques de Marseille (LATP).

[1] J. von Neumann, *Zur Theorie der Unbeschränkten Matrizen*, J. Reine Angew. Math. **161** (1929), 208–236.

[2] K. Schmüdgen, *On domains of powers of closed symmetric operators*, J. Oper. Theory **9** (1983), 53–75.

[3] A. Van Daele, *On pairs of closed operators*, Bull. Soc. Math. Belg., Set. B **34** (1982), 25–40.

Quasiconformal surgery and linear differential equations

Walter Bergweiler, *Kiel, Germany*
Alexandre Eremenko, *West Lafayette, Indiana, USA*

We consider differential equations of the form $w'' + Aw = 0$, where A is an entire function. All solutions are entire functions. The product $E = w_1 w_2$ of two solutions normalized by $w_1 w_2' - w_1' w_2$ is called a Bank–Laine function. These are entire functions characterized by the property

$$E(z) = 0 \to E'(z) \in \{-1, 1\}.$$

An old question of Bank and Laine was what are the possible orders of growth of such functions E. All known examples had integer or infinite order. We construct functions E of any prescribed order ≥ 1. This completely solves the question because the only Bank-Laine functions of order < 1 are polynomials of degree 2.

Our method produces several other interesting examples, for example we show that the relations

$$\frac{1}{\rho(A)} + \frac{1}{\rho(E)} \leq 2, \quad \rho(E) \geq \rho(A)$$

for the orders of A and E are best possible: all values satisfying this inequality can occur.

The new method which we employ is based on quasiconformal deformation of entire functions of the kind which was used in 1950-s for solving the Inverse problem of Nevanlinna theory, but unlike these classical papers, we have to work with entire functions of infinite order and with Riemann surfaces with infinitely many ends.

[1] A. Goldberg and I. Ostrovskii, *Value distribution of meromorphic functions,* AMS, Providence, 2008.

[2] S. Bank and I. Laine, *Oscillation theory of $f'' + Af = 0$, where A is entire,* Bull. AMS **6** (1982), 95–98.

On the wave equation controlled by the Dirichlet boundary condition on a half-axis

Larissa Fardigola, *Kharkiv, Ukraine*

Necessary and sufficient conditions of approximate L^∞-controllability at a free time are obtained for the control system

$$z_{tt} = z_{xx} - q^2 z, \qquad x > 0, \quad t \in (0, T) \tag{1}$$

$$z(0, t) = v(t), \qquad t \in (0, T), \tag{2}$$

where $T > 0$ and $q \geq 0$ are constants, $v \in L^\infty(0, \infty)$ is a control [1]. The case $q = 0$ essentially differs from the case $q > 0$. In particular, if $q > 0$, then each initial state of (1), (2) is approximately L^∞-controllable at a free time. But, if $q = 0$, then an initial state of this system is approximately L^∞-controllable at a free time iff $z_t(\cdot, 0) = z_x(\cdot, 0)$. Using some transformation operator introduced and studied in [2], we see that the control system

$$w_{tt} = \frac{1}{\rho}\left(k w_x\right)_x + \gamma w, \qquad x > 0, \quad t \in (0, T), \tag{3}$$

$$w(0, t) = u(t), \qquad t \in (0, T). \tag{4}$$

replicates the controllability properties of (1), (2) and vise versa. Here ρ, k, and γ are given functions on $[0, +\infty)$; $u \in L^\infty(0, \infty)$ is a control; $T > 0$ is a constant. Thus we obtain necessary and sufficient conditions of approximate L^∞-controllability at a free time for (2).

Note that the Sobolev spaces $H^m(\mathbb{R})$, $m \in \mathbb{R}$, are the natural "environment" for solutions to hyperbolic equations with constant coefficients, in particular, to equation (1). Evidently, the growth of solutions to equations with variable coefficients depends on the properties of these coefficients at infinity. That is why control system (1), (2) is considered in Sobolev spaces $H^0(\mathbb{R}) \times H^{-1}(\mathbb{R})$, and control system (3) ,(4) is considered in special modified spaces of the Sobolev type introduced and studied in [2]. The growth of distributions from these spaces is associated with the equation data ρ and k.

[1] L.V. Fardigola, *Controllability problems for the 1-d wave equation on a half-axis with the Dirichlet boundary control*, ESAIM: Control, Optim. Calc. Var. **18** (2012), 748–773.

[2] L.V. Fardigola, *Transformation operators in controllability problems for the wave equations with variable coefficients on a half-axis controlled by the Dirichlet boundary condition*, MCRF **5** (2015), 31–53.

Fourier quasicrystals and Lagarias' conjecture

Sergii Favorov, *Kharkiv, Ukraine*

A Fourier quasicrystal is a pure point complex measure μ in $\mathbb{R}^p$ such that its spectrum (Fourier transform in the sense of distributions) $\hat{\mu}$ is also a pure point measure. For example, the sum μ of unit masses at the points of $\mathbb{Z}^p \subset \mathbb{R}^p$ is a Fourier quasicrystal, because $\hat{\mu}$ coincides with μ in this case.

J.C.Lagarias [1] conjectured that if μ is a measure with a uniformly discrete support and its spectrum is also a measure with a uniformly discrete support, then the support of μ is a subset of a finite union of shifts of some full-rank lattice. The conjecture was proved by N.Lev and A.Olevski [2] in the case p=1, i.e., for measures on the real axis, and in the case of an arbitrary p and a positive measure μ (or $\hat{\mu}$).

On the other hand, A.Cordoba [3] proved that under rather weak conditions on the spectrum $\hat{\mu}$ the support of μ is a finite union of shifts of *several* full-rank lattices.

In my talk I prove that Lagarias' conjecture does not valid in the general case and show the special case when the conjecture is true. Moreover, I give some extension of Cordoba's result.

[1] J.C.Lagarias, *Mathematical quasicrystals and the problem of diffraction*, Directions in Mathematical Quasicrustals, M. Baake and R. Moody, eds., CRM Monograph series, Vol. 13, AMS, Providence RI, 2000, 61-93.

[2] N. Lev and A. Olevskii, *Quasicrystals and Poisson's summation formula,* Invent.Math., to appear.

[3] A. Cordoba, *Dirac combs,* Lett.Math.Phis **17** (1989), 191–196.

Radial positive definite functions and Schoenberg matrices with negative eigenvalues

Leonid Golinskii, *Kharkiv, Ukraine*

A real-valued and continuous function f on the positive half-line $\mathbb{R}_+$, $f(0) = 1$, is called *n-radial positive-definite* (RPDF) if for each finite set $X = \{x_j\}_{j=1}^m \subset \mathbb{R}^n$ the Schoenberg matrix $\mathcal{S}_X(f) := \{f(\|x_i - x_j\|_n)\}_{i,j=1}^m \geq 0$.

I. Schoenberg [1] obtained an integral representation of RPDF's of the form

$$f(r) = \int_0^\infty \Omega_n(rt)\nu_n(dt), \quad \Omega_n(s) := \Gamma(q+1)\Big(\frac{2}{s}\Big)^q J_q(s), \quad q := \frac{n}{2} - 1,$$

J_q is the Bessel function of the first kind of order q, $\nu_n = \nu_n(f)$ is a probability measure on $\mathbb{R}_+$, called the *Schoenberg measure of f*. We write $f \in \Phi_n$.

It is well known that the classes $\{\Phi_n\}_{n\geq 1}$ are nested, and inclusion $\Phi_{n+1} \subset \Phi_n$ is proper. Given $f \in \Phi_n$ we suggest necessary and sufficient conditions in terms of the Schoenberg measure $\nu_n(f)$ for the relation $f \in \Phi_{n+k}$, $k \in \mathbb{N}$, to hold.

Define the *Schoenberg index of f* as

$$\mathrm{ind}(f) := \max\{n: \ f \in \Phi_n\}, \ \ n = 0, 1, \dots, \infty, \ \ \mathrm{ind}(f) = k \Leftrightarrow f \in \Phi_k \backslash \Phi_{k+1}$$

for finite k. We compute the Schoenberg indices of the kernels Ω_n and their products: $\mathrm{ind}(\Omega_n) = n$,

$$\mathrm{ind}(\Omega_n(a\cdot)\Omega_n(b\cdot)) = n, \quad a \neq b, \qquad \mathrm{ind}(\Omega_n^2) = 2n - 1.$$

Another problem we deal with concerns the number of negative squares of a function f with $\mathrm{ind}(f) = n$. The Schoenberg matrix $\mathcal{S}_Y(f)$ must have negative eigenvalues for certain test sets $Y \subset \mathbb{R}^{n+1}$. It turns out that given a function f with $\mathrm{ind}(f) = n$, $2 \leq n < \infty$, and an arbitrary positive integer N, there is a set $Y = \{y_k\}_{k=1}^p \subset \mathbb{R}^{n+1}$ so that the Schoenberg matrix $\mathcal{S}_Y(f)$ has at least N negative eigenvalues.

[1] I. Schoenberg, *Metric spaces and completely monotone functions,* Ann. Math. **39** (1938), 811–841.

Measurable selectors: positive results and open problems

Vladimir Kadets, *Kharkiv, Ukraine*

Let (Ω, Σ, μ) be a complete finite measure space, X be a Banach space. By $cl(X)$ and $wk(X)$ we denote, respectively, the families of non-empty closed and non-empty weakly compact subsets of X. A *multifunction* is a map $F : \Omega \to 2^X \setminus \{\emptyset\}$; a *selector* of F is a function $f : \Omega \to X$ with $f(t) \in F(t)$ for all $t \in \Omega$. A multifunction $F : \Omega \to 2^X \setminus \{\emptyset\}$ is said to be *scalarly measurable* if for each $x^* \in X^*$ the function $t \mapsto \sup\{x^*(x) : x \in F(t)\}$ is measurable. In particular a single valued function $f : \Omega \to X$ is scalarly measurable if the composition $x^* \circ f$ is measurable for every $x^* \in X^*$. If X is separable, then scalar measurability of $f : \Omega \to X$ is equivalent to its Borel measurability.

The most applicable theorem on existence of measurable selectors is the following classical Kuratowski and Ryll-Nardzewski result.

Theorem 1. *Let W be a separable complete metric space, and let $F : \Omega \to cl(W)$ be a multifunction satisfying the Effros measurability condition that $\{t \in \Omega : \ F(t) \cap C \neq \emptyset\} \in \Sigma$ for each open set $U \subset W$. Then F admits a $\Sigma \to$Borel(W) measurable selector.*

The problem of finding measurable selectors in non-separable case is difficult, because measurability is not stable under uncountable procedures. Jointly with Bernardo Cascales and José Rodríguez we obtained some results on measurable selectors, that do not require the separability of the space, and thus enable to build the integration of multifunctions theory in non-separable setting. The most important result is:

Theorem 2 [2]. *Every scalarly measurable multifunction $F : \Omega \to wk(X)$ admits a scalarly measurable selector.*

[1] B. Cascales, V. Kadets, and J. Rodríguez. *Measurable selectors and set-valued Pettis integral in non-separable Banach spaces*. Journal of Functional Analysis **256**, (2009), no. 3, 673-699.

[2] B. Cascales, V. Kadets, and J. Rodríguez. *Measurability and Selections of Multi-Functions in Banach Spaces*. Journal of Convex Analysis **17** (2010), no. 1, 229-240.

[3] B. Cascales, V. Kadets, and J. Rodríguez. *The Gelfand Integral for multi-valued functions*. J. Convex Anal. **18** (2011), No. 3, 873-895.

[4] B. Cascales, V. Kadets, and J. Rodríguez. *Radon-Nikodým Theorems for Multimeasures in Non-Separable Spaces*. Journal of Mathematical Physics, Analysis, Geometry **9** (2013), no. 1, 7-24.

Operator differential equations containing spectral parameter in Nevanlinna manner

Volodymyr Khrabustovskyi, *Kharkiv, Ukraine*

For operator differential equations of an arbitrary order containing spectral parameter in Nevanlinna manner we introduce and examine:

- characteristic operator, which is an analog of Weyl—Titchmarsh characteristic matrix; general case and the case of separated boundary conditions;
- analogue of the generalized resolvent;
- Weyl-type solutions;
- various cases of eigenfunction expansions, conditions implying the fulfillment of the Parseval equality, examples.

[1] V. Khrabustovskyi, *Eigenfunction expansions associated with an operator differential equation nonlinearly depending on a spectral parameter*, Methods Funct. Anal. Topol. **20** (2014), 68–91.

Two-dimensional photonic metamaterials with preassigned gaps in spectrum

Evgen Khruslov, *Kharkiv, Ukraine*

We study the Maxwell operator in two-dimensional dielectric medium with small heterogeneous inclusions which are periodically distributed with a small period ε. Media with such a structure are typical for photonic metamaterials, which are artificial composite with required electromagnetic properties. The spectrum of the Maxwell operator in such a medium has a band-gap structure, i.e. it consists of close intervals of continuous spectrum separated by gaps (if any). On the other hand, for applications in radio-engineering it is important to know location of gaps in the spectrum. That is why our main purpose is to construct inclusions which provide existence of preassigned gaps in the spectrum of the respective operator.

We consider trap-like inclusions in the form of annuli of a perfectly conducting material with slim slits. We prove that for sufficiently small period ε, the spectrum of the Maxwell operator has a finite number of gaps with edges converging to given points as $\varepsilon \to 0$. We establish a one-to-one correspondence between parameters of the trap-like inclusions and the edges of limiting spectrum.

Real Ginibre matrices with pure complex spectra

Mihail Poplavskyi, *Coventry, United Kingdom*

In this talk we consider the real Ginibre ensemble of random matrices whose matrix elements are i.i.d. random variables $\sim N(0,1)$. Starting from remarkable papers of Edelman, Kostlan and Shub the question about the number of real eigenvalues was stated and it was shown that typically it is $\sim \sqrt{\frac{2}{\pi}}N^{1/2}$.

Another interesting quantity introduced by the authors is

$$p_{N,k} = \mathbb{P}\left(G_N \text{ has exactly } k \text{ real eigenvalues}\right)$$

where G_N is randomly sampled matrix of size $N \times N$ from the real Ginibre ensemble. We are interested in asymptotic behaviour of $p_{N,k}$ when $N \to \infty$ while k either stays fix or grows with N. One expects to find a decay of $p_{N,k}$ as soon as k moves away from $\sqrt{N}$. It was proved very recently by Molino, Pakdaman, Touboul and Wainrib in the regime of $k = \alpha N$ that

$$\lim_{n\to\infty} \frac{\log p_{N,k}}{N^2} = -C_\alpha.$$

However we are interested in the case $k \ll \sqrt{N}$, i.e. small amount of real eigenvalues. Earlier Forrester suggested the right answer using the connection between Annihilating Brownian Motions and eigenvalues of real Ginibre ensemble. He used the non rigorous result of Derrida who has studied the probability of all particles annihilated after a certain time.

We use a determinant representation for the probability $p_{N,0}$ obtained by using orthogonal polynomials technique by Akemann and Kanzieper and prove

Theorem 1. *Let $\{k_n\}_{n=1}^{\infty}$ be a sequence of non-negative integer numbers such that*

$$\lim_{n\to\infty} k_n \left(n^{-1/2} \log n\right) = 0.$$

Then

$$\lim_{n\to\infty} \frac{\log p_{2n,2k_n}}{\sqrt{2n}} = -\frac{1}{\sqrt{2\pi}}\zeta\left(\frac{3}{2}\right)$$

The talk is based on a joint work with E. Kanzieper, C. Timm, R. Tribe, O. Zaboronski [1].

[1] E. Kanzieper, M. Poplavskyi, C. Timm, R. Tribe, and O. Zaboronski, *What is the probability that a large random matrix has no real eigenvalues?* arXiv:1503.07926.

Extremal cases for the log canonical threshold

Alexander Rashkovskii, *Stavanger, Norway*

The log canonical threshold of a plurisubharmonic singularity u at 0 is the infimum of all $\lambda > 0$ such that $\exp(-2u/\lambda)$ is locally integrable.

We show that any plurisubharmonic singularity with given Monge-Ampère mass and smallest possible log canonical threshold has essentially a logarithmic asymptotics.

The proof is based on a recent bound on the log canonical threshold obtained in [2]. We also study an equality case for that inequality.

[1] J.-P. Demailly, *Estimates on Monge-Ampère operators derived from a local algebra inequality*,Complex Analysis and Digital Geometry. Proceedings from the Kiselmanfest, 2006, ed. by M. Passare. Uppsala University, 2009, 131–143.

[2] J.-P. Demailly and Pham Hoàng Hiep, *A sharp lower bound for the log canonical threshold*, Acta Math. **212** (2014), no. 1, 1–9.

[3] A. Rashkovskii, *Extremal cases for the log canonical threshold*, C. R. Acad. Sci. Paris, Ser. I, **353** (2015), 21–24.

On the first eigenpair of singularly perturbed operators with Neumann boundary condition

Volodymyr Rybalko, *Kharkiv, Ukraine*

We consider the eigenvalue problem

$$\varepsilon\, a_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} + b_j(x)\frac{\partial u_\varepsilon}{\partial x_j} + c(x)u = \lambda u \qquad \text{in } \Omega \tag{1}$$

with the Neumann boundary condition $\frac{\partial u}{\partial \nu} = 0$ on $\partial\Omega$. We assume that $\varepsilon > 0$ in (1) is a small parameter, coefficients $a_{ij}(x)$ are uniformly elliptic and $\Omega \in \mathbb{R}^n$ is a bounded domain with a smooth boundary. According to the Krein-Rutman theorem the first eigenvalue λ_ε (the eigenvalue with the maximal real part) is real and simple, the corresponding eigenfunction u_ε is sign preserving. In the simplest case $c(x) = c = \mathrm{Const}$ one clearly has $\lambda_\varepsilon = c$ and $u_\varepsilon = \mathrm{Const}$. However, in general, the first eigenpair cannot be found explicitly when the coefficient $c(x)$ is not a constant, and we study the asymptotic behavior of λ_ε and u_ε as $\varepsilon \to 0$.

Representing u_ε as $u_\varepsilon = e^{-W_\varepsilon(x)/\varepsilon}$ we arrive at the additive eigenvalue problem for a singularly perturbed Hamilton-Jacobi PDE of the form $-\varepsilon a_{ij}(x)\frac{\partial^2 W_\varepsilon}{\partial x_i \partial x_j} + H(\nabla W_\varepsilon, x) + \varepsilon(c(x) - \lambda_\varepsilon) = 0$. We show that first derivatives of functions $W_\varepsilon(x)$ are uniformly bounded, this allows to pass to the limit $\varepsilon \to 0$ leading to a viscosity solution of the Hamilton-Jacobi PDE $H(\nabla W, x) = 0$ with the Neumann boundary condition on $\partial\Omega$. Assuming that the so-called Aubry set associated to the limit problem is a finite number of hyperbolic fixed points $\{\xi_k\}$ and hyperbolic limit cycles $\mathcal{C}_l$ of Skorohod dynamics of the ODE $\dot{x} = b(x)$ we establish the limit of eigenvalues λ_ε in terms of maximization of certain numbers $\sigma(\{\xi_k\})$, $\sigma(\mathcal{C}_l)$ associated to the fixed points and limit cycles. In the case of fixed points $\{\xi_k\}$ these numbers $\sigma(\{\xi_k\})$ are defined through eigenvalues of the the matrix of the linearized ODE and values $c(\xi_k)$, while in the case of of limit cycles $\mathcal{C}_l$ numbers $\sigma(\mathcal{C}_l)$ are defined via eigenvalues of the linearized Poincaré map and $\int c(\xi(t))\mathrm{d}t$, $\xi(t)$ being the trajectory of the limit cycle. We also select the solution $W(x)$ of the limiting problem corresponding to $\lim_{\varepsilon\to 0} W_\varepsilon(x)$ (this selection is nontrivial due to the fact that the limiting problem is not uniquely solvable when the Aubry set has several connected components).

Initial boundary value problems for integrable nonlinear equations

Dmitry Shepelsky, *Kharkiv, Ukraine*

By now, we have a well-developed theory of integrable evolution equations, by which we mean equations which can be analyzed via IST (inverse scattering transform) machinery. This formalism allows studying detailed aspects of solution behavior, which includes the long-time asymptotics of solutions and the central role played by the solitons. Heretofore, the IST methodology was pursued almost entirely for pure initial-value problems. However, in many laboratory and field situations, the wave motion is initiated by what corresponds to the imposition of boundary conditions rather than initial conditions. Hence, the study of initial-boundary value problems for integrable equations and their relatives presents itself naturally.

It is our purpose here to present a unified approach for analyzing initial-boundary value problems for linear and integrable nonlinear PDEs (sometimes called the Fokas method [1]) and to discuss some directions in its development, taking as example the nonlinear Schrödinger equation.

[1] A.S.Fokas, *A Unified Approach to Boundary Value Problems*, CBMS-NSF Regional Conference Series in Applied Mathematics, vol. 78, Philadelphia, SIAM, 2008.

Classification of symmetries on the Laurent extension of quantum plane

Sergey Sinel'shchikov, *Kharkiv, Ukraine*

The standard quantum plane is a unital algebra with the two generators x, y and a single relation $yx = qxy$. Our subject is the Laurent extension $\mathbb{C}_q\left[x^{\pm 1}, y^{\pm 1}\right]$ of this algebra, which is derived by letting the generators x, y to be invertible. A list of generic $U_q(\mathfrak{sl}_2)$-symmetries of $\mathbb{C}_q\left[x^{\pm 1}, y^{\pm 1}\right]$ (in other terminology, the structures of $U_q(\mathfrak{sl}_2)$-module algebra) is available [1]. The problem we solve now is that of producing the complete list of $U_q(\mathfrak{sl}_2)$-symmetries on $\mathbb{C}_q\left[x^{\pm 1}, y^{\pm 1}\right]$, with the degenerate (non-generic) series included. Here $U_q(\mathfrak{sl}_2)$ is the quantum universal enveloping algebra of $\mathfrak{sl}_2$, defined by its generators k, k^{-1}, e, f, together with the well known relations [2]. It turns out that $\mathbb{C}_q\left[x^{\pm 1}, y^{\pm 1}\right]$ is much more symmetric than the standard quantum plane. Here is an example of degenerate series.

Theorem 1. *Let* $a_0 \in \mathbb{C}\setminus\{0\}$, $a_2 \in \mathbb{C}$, $M, u, v, r, s \in \mathbb{Z}$ *such that* $rv - su = 1$. *There exists a family* π *of* $U_q(\mathfrak{sl}_2)$*-symmetries of* $\mathbb{C}_q[x^{\pm 1}, y^{\pm 1}]$ *with the above set of parameters, determined by*

$$\begin{aligned}
\pi(\mathsf{k})x =& q^{-2s}x \qquad , \qquad \pi(\mathsf{k})y = q^{2r}y \ , \\
\pi(\mathsf{e})x =& \frac{a_0\left(1 - q^{v+(M-2)s}\right)}{(q^2-1)^2} x^{u+1+Mr} y^{v+Ms} + \\
& + \frac{a_2\left(1 - q^{v+Ms}\right)}{(q^2-1)^2} x^{u+1+(M+2)r} y^{v+(M+2)s} \ , \\
\pi(\mathsf{e})y =& \frac{a_0\left(q^{u+Mr} - q^{2r}\right)}{(q^2-1)^2} x^{u+Mr} y^{v+1+Ms} + \\
& + \frac{a_2\left(q^{u+(M+2)r} - q^{2r}\right)}{(q^2-1)^2} x^{u+(M+2)r} y^{v+1+(M+2)s} \ , \\
\pi(\mathsf{f})x =& - a_0^{-1} q^{3+uv+Msu+Mrv+M^2rs}\left(q^{2s} - q^{-v-Ms}\right) x^{-u+1-Mr} y^{-v-Ms} \ , \\
\pi(\mathsf{f})y =& - a_0^{-1} q^{3+uv+Msu+Mrv+M^2rs}\left(q^{-u-(M+2)r} - 1\right) x^{-u-Mr} y^{-v+1-Ms} \ .
\end{aligned}$$

[1] S. Sinel'shchikov, *Generic symmetries of the Laurent extension of quantum plane,* J. Math. Phys., Anal., and Geom. (2015), to appear; arXiv:1410.8074 [math.QA]

[2] C. Kassel, *Quantum Groups,* Springer–Verlag, New York, 1995.

On the fluctuations of eigenvalues of multiplicative unitary invariant ensembles

Vladimir Vasilchuk, *Kharkiv, Ukraine*

We consider first the multiplicative ensemble of $n \times n$ random matrices $W_n = A_n U_n^* B_n U_n$, where A_n and B_n are unitary, having the limiting Normalized Counting Measure of eigenvalues, and U_n is unitary, uniformly distributed over $U(n)$.

We find the leading term of asymptotic expansion for the covariance of Stieltjes transform of W_n and establish the Central Limit Theorem for elements of sufficiently smooth statistic of eigenvalues of W_n as $n \to \infty$.

YOUNG PARTICIPANTS

Correlation function of two characteristic polynomials of diluted hermitian random matrices near the edge points of the spectrum

Ievgenii Afanasiev, *Kharkiv, Ukraine*

We consider the local edge regime for the second correlation function of the characteristic polynomials of diluted hermitian random matrices

$$M_n = (d_{jk} w_{jk})_{j,k=1}^n \tag{1}$$

where

$$d_{jk} = p^{-1/2} \begin{cases} 1 \text{ with probability } \frac{p}{n}; \\ 0 \text{ with probability } 1 - \frac{p}{n}; \end{cases} \tag{2}$$

and $\Re w_{jk}$, $\Im w_{jk}$ are i.i.d. Gaussian random variables with zero mean such that

$$2\mathbf{E}\{|\Re w_{jk}|^2\} = 2\mathbf{E}\{|\Im w_{jk}|^2\} = \mathbf{E}\{|w_{ll}|^2\} = 1, \quad j \neq k. \tag{3}$$

The second correlation function of the characteristic polynomials is

$$F_2(\Lambda) = \mathbf{E}\left\{\det(M_n - \lambda_1)\det(M_n - \lambda_2)\right\}, \tag{4}$$

where $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2\}$ are real parameters of the form $\lambda_j = \lambda_0 + \frac{x_j}{n}$, $j = 1, 2$.

Set

$$D_2(\Lambda) = \frac{F_2(\Lambda)}{\sqrt{F_2(\lambda_1 I) F_2(\lambda_2 I)}}, \tag{5}$$

where I is the unit matrix. The main result is

Theorem 1. *Let an ensemble of diluted random matrices be defined by (1), (2), (3), $p \to \infty$. Then the correlation function of two characteristic polynomials (4) satisfies the asymptotic relations*

(i) for $|\lambda_0| = 2$ and $n^{2/3}/p \to \infty$

$$\lim_{n\to\infty} D_2\left(\Lambda_0 + X/n\right) = 1;$$

(ii) for $n^{2/3}/p \to c$

$$\lim_{n\to\infty} D_2\left(2I + X/n^{2/3}\right) = \frac{\widetilde{Ai}(x_1 + 2c, x_2 + 2c)}{\sqrt{\widetilde{Ai}(x_1 + 2c, x_1 + 2c)\widetilde{Ai}(x_2 + 2c, x_2 + 2c)}},$$

with D_2 defined in (5), $X = \mathrm{diag}\{x_1, x_2\}$ and $\widetilde{Ai}(x, y) = \frac{Ai(x)Ai'(y) - Ai'(x)Ai(y)}{x - y}$, where $Ai(x)$ is the Airy function.

Regularized integrals of motion for the Korteweg–de Vries equation with steplike initial data

Kyrylo Andreiev, *Kharkiv, Ukraine*

In 1971 Fadeev, Zakharov [1] considered the KdV equation

$$\frac{\partial u}{\partial t} - 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0,$$

in the class of rapidly decreasing function as $x \to \pm\infty$ (belonging to the Schwartz class). It was showed that the equation has infinite series of integrals of motion, and was found the expression in terms of the scattering data. It was showed that the scattering data play a role variables action-angle type.

In this talk we consider the KdV equation in the class of functions with steplike initial data

$$\lim_{x\to-\infty} |u_0(x) - c^2| = \lim_{x\to+\infty} u_0(x) = 0,$$

belonging to the Schwartz class S on each half-line $[0, \pm\infty]$.

It was constructed an infinite series of regularized integrals of motion and founded their expression in terms of the scattering data for the Schrodinger operator with steplike potential.

[1] V.E. Zakharov and L.D. Faddeev, *The Korteweg–de Vries equation is a completely integrable Hamiltonian system,* Functional Analysis and Its Applications, **5**:4 (1971), 18–27.

Stabilization of systems with power principal part

Maxim Bebiya, *Kharkiv, Ukraine*

We solve the stabilization problem for a class of nonlinear systems with uncontrollable first approximation. Namely, we consider a nonlinear system of the form

$$\begin{cases} \dot{x}_1 = u, \\ \dot{x}_i = x_{i-1}^{2k_{i-1}+1} + f_{i-1}(t, x, u), \quad i = 2, \dots, n \end{cases} \tag{1}$$

where $k_i \in N$, $u \in R$ is a control. Functions $f_i(t, x, u)$ are continuous in (t, x, u) and satisfy the Lipschitz condition in x and u.

We assume that $k_1 = \dots = k_s = 0$ and $0 < k_{s+1} < \dots < k_{n-1}$ for some s such that $0 \le s \le n-2$. Besides, we assume that for some $\alpha_i > 0$ functions $f_i(t, x, u)$ satisfy the following estimates

$$|f_i(t, x, u)| \le \alpha_i(x_1^{2k_1+2} + x_2^{2k_2+2} + \dots + x_{n-1}^{2k_{n-1}+2}), \quad i = \overline{1, n-1}.$$

An approach for solving the stabilization problem for system (1) is based on stabilization with respect to nonlinear approximation. Consider the following system

$$\begin{cases} \dot{x}_1 = u, \\ \dot{x}_i = x_{i-1}^{2k_{i-1}+1}, \quad i = 2, \ldots, n \end{cases} \tag{2}$$

as a nonlinear approximation of system (1). Stabilization problem for system (2) in the case when $s = n - 2$ was solved in [1].

We show that a stabilizing control for system (2) can be found in the form

$$u(x) = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n + \sum_{i=s+1}^{n-1} a_{n-s+i} x_i^{2k_i+1},$$

and it is proved that this control solves the stabilization problem for original nonlinear system (1). A conditions on the coefficients a_i are obtained using the Lyapunov function method. A Lyapunov function $V(x)$ can be chosen in the form: $V(x) = (Fx, x)$, where F is a solution of a singular Lyapunov inequality.

[1] M. Bebya, *Stabilization of systems with power nonlinearity,* Visn. Khark. Univ., Ser. Math, Prykl. Mat. Mekh. **1120** (2014), 75–84 (in Russian).

On necessary and sufficient conditions for a function to belong to the Laguerre-Pólya class

Anton Bohdanov, *Kharkiv, Ukraine*
Anna Vishnyakova, *Kharkiv, Ukraine*

We study some necessary and sufficient conditions on an entire function to belong to the Laguerre-Pólya class. The subject is somewhat closely related to Riemann hypothesis (for detailed information we refer reader to [1]).

Definition 1. *Let us consider an entire function $f : \mathbb{C} \to \mathbb{C}$. Function f is said to belong to the Laguerre-Pólya class (denoted as $f \in \mathcal{L} - \mathcal{P}$), iff f can be expressed in a form of the following product:*

$$f(z) = z^m e^{\alpha + \beta z + \gamma z^2} \prod_{k=1}^{\omega} (1 - \frac{z}{z_k}) e^{-\frac{z}{z_k}},$$

where $0 \leq \omega \leq +\infty$; $\beta, \gamma, z_k \in \mathbb{R}$, $z_k \neq 0$, $\gamma \leq 0$, $m \in \mathbb{N}_0$, and $\sum_{k=1}^{\omega} \frac{1}{z_k^2} < \infty$.

Let us also remind that a polynomial $P \in \mathbb{C}[z]$ is called hyperbolic (denoted as $P \in \mathcal{HP}$), iff $\forall z \in \mathbb{C} : P(z) = 0 \Rightarrow z \in \mathbb{R}$.
In this study we present the following new theorem (as far as we are concerned):

Theorem 1. *Assume that an entire function* $f(z) = \sum\limits_{k=0}^{\infty} \gamma_k z^k$ *with positive coefficients satisfies the following conditions:*

$$2 \le q_2(f) < 3 \quad \text{and} \quad q_j(f) \ge 4 \quad \forall j \in \mathbb{N}, \quad j \ge 3,$$

where $q_k(f) = \frac{\gamma_{k-1}^2}{\gamma_{k-2}\gamma_k}, k \in \mathbb{N}, k \ge 2$. *Then* $f \notin \mathcal{L} - \mathcal{P}$.

We found such an approach concerning coefficients $q_k(f)$ in [2] and [3].
This paper also provides an answer to a question: when does the entire function $f^{(m,a)}(z) = \sum\limits_{k=0}^{\infty} \frac{z^k (k!)^m}{a^{k^2}}, a > 1, m > 1$ belong to the Laguerre-Pólya class with its partial sums $S_n^{(m,a)}(z) = \sum\limits_{k=0}^{n} \frac{z^2 (k!)^m}{a^{k^2}} \in \mathcal{HP}, n \in \mathbb{N}$.

[1] G. Csordas and R. S Varga, *Necessary and sufficient conditions and the Riemann hypothesis,* Adv. in Appl. Math. **11** (1990), 328–357.

[2] A.M. Vishnyakova, O. M. Katkova, and T. Lobova, *On entire functions having Taylor sections with only real zeros,* Journal of Math. Physics, Analysis, Geometry, **11** (2004), 449–469.

[3] G. Csordas and A. Vishnyakova, *The generalized Laguerre inequalities and functions in the Laguerre-Polya class,* Cent. Eur. J. Math. **11** (2013), 1643–1650.

On a characterization of idempotent distributions on discrete fields

Gennadiy Feldman, *Kharkiv, Ukraine*
Margaryta Myronyuk, *Kharkiv, Ukraine*

According to the classical Kac—Bernstein theorem if ξ and η are independent random variables and their sum $\xi + \eta$ and difference $\xi - \eta$ are independent, then the random variables ξ and η are Gaussian. This characterization of the Gaussian measure remains true if instead of the difference $\xi - \eta$ we consider its square $(\xi - \eta)^2$, but assume that the random variables ξ are η identically distributed. This characterization of the Gaussian measure is a particular case of the well-known Geary theorem: if $\xi_1, \xi_2, \dots, \xi_n,\ n \ge 2$, are independent identically distributed random variables such that the sample mean $\bar{\xi} = \frac{1}{n}\sum_{j=1}^n \xi_j$ and the sample variance $s^2 = \frac{1}{n}\sum_{j=1}^n (\xi_j - \bar{\xi})^2$ are independent, then the random variables ξ_j are Gaussian ([1, § 4.2.1]).

A lot of research are devoted to generalizations of the classical characterization theorems of mathematical statistics to different algebraic structures, first of all to locally compact Abelian groups (see e.g. [2], where one can find references). In so doing, in all studying characterization problems only linear forms of independent random variables with values in a group, where coefficients of linear forms are either integers or topological automorphisms of the group, were considered. We study the simplest "nonlinear" characterization problem and prove the following theorem.

Theorem 1. *Let X be a countable discrete field of characteristic p. Let ξ and η be independent identically distributed random variables with values in X and distribution μ. Put $S = \xi + \eta$ and $D = (\xi - \eta)^2$. Then the following statements hold.*

1. Let $p = 0$ or $p = 2$. The random variables S and D are independent if and only if μ is a degenerate distribution.

2. Let $p > 2$. The random variables S and D are independent if and only if μ is a idempotent distribution.

[1] A.M. Kagan, Yu.V. Linnik, C.R. Rao, *Characterization problems in mathematical statistics*, Wiley Series in Probability and Mathematical Statistics, John Wiley & Sons, New York-London-Sydney, 1973.

[2] G.M. Feldman, *Functional equations and characterization problems on locally compact Abelian groups*, EMS Tracts in Mathematics, 5, Zurich: European Mathematical Society (EMS), 2008.

The global solvability and the Lagrange stability of semilinear singular differential-algebraic equations

Maria Filipkovskaya, *Kharkiv, Ukraine*

Research results of the concrete mathematical models of radio engineering, control theory, hydrodynamics, economics, robotics technology testify that the class of models with differential-algebraic equations (DAEs) is important for practice. Such equations contain a degenerate operator at the derivative and they are also called degenerate, algebraic-differential, descriptor. The Cauchy problem for the semilinear differential-algebraic equation

$$\frac{d}{dt}[Ax(t)] + Bx(t) = f(t,x) \tag{1}$$

is considered. It is assumed that $t \geq 0$, $x \in \mathbb{R}^n$, $f(t,x) \in C\left([0,\infty) \times \mathbb{R}^n, \mathbb{R}^m\right)$, A, $B : \mathbb{R}^n \to \mathbb{R}^m$ are linear operators to which $m \times n$ matrices A, B correspond, the operator A is degenerate, the characteristic pencil $\lambda A + B$ (λ is a complex parameter) is a singular operator pencil with the regular component of index 1. In particular, underdetermined and overdetermined systems of the differential-algebraic equations correspond to this DAE. DAEs with a singular characteristic pencil will be called singular DAEs.

The basic result of the work are the theorems on the global solvability and the Lagrange stability of the semilinear DAE (1). The Lagrange stability means that all solutions of an equation are bounded on the whole domain. The presence of a global solution guarantees a sufficiently long action term of the corresponding real system. It is important that the nonlinear function $f(t,x)$ may not satisfy the constraints of the global Lipschitz condition type. The rejection of such constraints is due to the fact that in many practical problems of radio engineering, electronics, mathematical economics the real nonlinearities aren't global Lipschitz. For example, the presence of cubic nonlinear resistances and conductivities in electric circuits, as a rule, admits the existence of global solutions.

To obtain the main results the extending solution method in terms of differential inequalities with Lyapunov and La Salle functions, the method of spectral projectors and the special block representations of the singular pencil are used.

The mathematical models of radio engineering filters with nonlinear elements are considered as applications. Check of the conditions of the proved theorems is carried out and it analysis have shown that the requirements of the theorem are physically feasible. The obtained numerical solutions verify the results of theoretical investigations.

An analogue of the Cauchy formula for the solution of a nonhomogeneous linear differential equation in the space of rational functions

Sergey Gefter, *Kharkiv, Ukraine*
Anna Goncharuk, *Kharkiv, Ukraine*

Let F be an arbitrary field of characteristic zero and $b \in F$. Consider the following differential equation,

$$y' = by + R(x), \tag{1}$$

where $R(x) = \dfrac{a_1}{x} + \dfrac{a_2}{x^2} + \dfrac{a_3}{x^3} + \cdots + \dfrac{a_m}{x^m}$, $a_1, a_2, \ldots, a_m \in F$, and $y \in F(x)$ is an unknown rational function.

We have found a condition for the existence of rational solutions of equation (1) and obtain an algebraic analogue of the Cauchy formula for such solutions (see, for example, [1, Ch.IV, §7]). Namely, we prove that equation (1) has a rational solution if and only if

$$a_1 - a_2 b + \frac{1}{2!}a_3 b^2 - \frac{1}{3!}a_4 b^3 + \cdots + (-1)^{m-1}\frac{1}{(m-1)!}a_m b^{m-1} = 0. \tag{2}$$

We also provide an explicit formula for the solution in terms of the *convolution* of two formal Laurent series. Here by the convolution of two formal series

$A(x) = \dfrac{\alpha_0}{x} + \dfrac{\alpha_1}{x^2} + \cdots$ and $B(x) = \dfrac{\beta_0}{x} + \dfrac{\beta_1}{x^2} + \cdots$ we call the formal Laurent series $(A * B)(x) = \alpha_0 B(x) - \dfrac{\alpha_1}{1!}B'(x) + \dfrac{\alpha_2}{2!}B''(x) - \dfrac{\alpha_3}{3!}B'''(x) + \cdots$.

It turns out that the unique rational solution of equation (1) has the form $y(x) = (\varepsilon * R)(x)$, where $\varepsilon = \dfrac{1}{b}\sum\limits_{k=1}^{\infty} \dfrac{(-1)^k (k-1)! \cdot b^k}{x^k}$.

[1] Yu.N. Bibikov *A Course of Ordinary Differential Equations*, M: Vysshaya shkola, 1991, 303 p. (in Russian).

Caushy problem for Korteveg-de Vries equation with steplike initial data

Zoya Gladka, *Kharkiv, Ukraine*

Let us consider the Cauchy problem for Korteveg-de Vries equation

$$q_t = -q_{xxx} + 6qq_x \tag{1}$$

with steplike initial data $q(x,0) = q(x) \to c_\pm$ as $x \to \pm\infty$, $c_- \neq c_+$, $c_\pm \in \mathbb{R}$. Unlike the case of decaying initial data PDE methods are not effective when we face steplike initial data. Nevertheless, the inverse scattering method appeared to be well-applicable. In [1] it was used finite-gap case and the following result was obtained

Theorem 1. *Let $p_\pm(x,t)$ be two finite-gap solutions of (1) corresponding to finite-gap initial data $p_\pm(x)$, $m \geq 8$ and $n \geq m+5$ are fixed integers and $\int_{\mathbb{R}_\pm} \left|\frac{\partial^s}{\partial x^s}\big(q(x) - p_\pm(x)\big)\right| (1+|x|^m)dx < \infty,\ 0 \leq s \leq n.$*

Then the Cauchy problem (1) plus described above initial data has a unique solution for all $t \in [-T,T]$ such that

$$\int_{\mathbb{R}_\pm} \left(\left|\frac{\partial^s}{\partial x^s}\big(q(x) - p_\pm(x)\big)\right| + \left|\frac{\partial}{\partial t}\big(q(x,t) - p_\pm(x,t)\big)\right| \right) (1+|x|^{[\frac{m}{2}]-2})dx < \infty,$$

when $0 \leq s \leq n-m-2$.

The inverse scattering method enabled us to clarify this result for steplike case specifically for the following class of functions.
Let $c_\pm \in \mathbb{R}$ and $m \geq 1$, $n \geq 1$ be fixed integers. We say that $q(x) \in \mathcal{L}_m^n(c_+, c_-)$ if $q^{(n)} \in L_2^{loc}(\mathbb{R})$ and for $i = 1,..,n$

$$\int_0^{+\infty} \Big(|q(x) - c_+| + |q(-x) - c_-| + |q^{(i)}(x)| + |q^{(i)}(-x)| \Big) (1+|x|^m)dx < \infty.$$

The main result was published in [2]. It is contained in the following theorem.

Theorem 2. *Let $m \geq 3$ and $n \geq m+3$ be fixed integers. Then the Cauchy problem (1) plus initial data $q(x) \in \mathcal{L}_m^n(c_+, c_-)$ has a unique solution $q(x,t)$ such that $q(\cdot,t) \in \mathcal{L}^{n-m}_{[\frac{m+1}{2}]-2}(c_+, c_-)$, $\frac{\partial}{\partial t} q(\cdot,t) \in \mathcal{L}^{n-m}_{[\frac{m+1}{2}]-2}(0,0)$ for all $t \in [-T,T]$.*

[1] I. Egorova and G. Teschl, *On the Cauchy problem for the Korteweg-de Vries equation with steplike finite-gap initial data II. Perturbations with finite moments,* J. d'Analyse Math. no. 115 (2011), 71–101.

[2] Z. Gladka *About solutions of the KdV equation with steplike initial data,* Dopov. Nats. Akad. Nauk Ukr. no. 2 (2015),(in Russian)

On Paulsen transformations of closed surfaces of rotation

Vasyl Gorkavyy, *Kharkiv, Ukraine*
Dmytro Kalinin, *Kharkiv, Ukraine*

We will discuss short deformations of closed surfaces of rotations in $\mathbb{R}^3$, which increase the volume, see [1]-[5].

Let F, $\tilde{F}$ be piece-wise smooth closed surfaces of rotations $\mathbb{R}^3$. A piece-wise smooth map $\Psi : F \to \tilde{F}$ is called a Paulsen transformation, and $\tilde{F}$ is called a Paulsen transform of F, if Ψ maps meridians and parallels of F to meridians and parallels of $\tilde{F}$ so that the length of meridians is preserved, whereas the lengths of parallels are non-increasing, c.f. [1]-[3]. Any Paulsen transformation Ψ is short, i.e. $l(\gamma) \leq l(\Psi(\gamma))$ holds true for any curve $\gamma \subset F$.

Given F, consider the family W_F all its Paulsen transforms. We are interested in the question of maximizing the volume $\mathrm{Vol}(\tilde{F})$, $\tilde{F} \in W_F$. This problem is completely solved, when F is supposed to be a two-covered disk, in this case the Paulsen transform of maximal volume is the so-called mylar balloon [1], [4]. On the other hand, if F is a sphere, then the volume of any its Paulsen transform is not greater then the volume of F due to the isoperimetric property of the sphere. Particular necessary and sufficient conditions for F to admit an increasing volume Paulsen deformation are derived in [2]-[3], [5].

Our aim is to present some results on the uniqueness of the Paulsen transform of maximal volume for an arbitrary given closed surface of rotation F. Besides, some concrete illustrative examples will be discussed.

[1] W. Paulsen, *What is the shape of the mylar balloon?*, Amer. Math. Monthly **101** (1994), 953–958.

[2] V.A. Gorkavyy, *On inflating closed mylar shells,* C.R. Mecanique **338** (2010), 656–662.

[3] V.A. Gorkavyy, *On increasing the volume of closed surfaces of rotation,* Contemporary problems of mathematics and mechanics **6** (2011), no. 2, 195–205 (in Russian).

[4] I. Mladenov and J. Oprea, *The mylar balloon revisited,* Amer. Math. Monthly **110** (2003), 761–784.

[5] D.D. Bleecker, *Isometric deformations of compact hypersurfaces,* Geom. Dedicata **64** (1997), 193–227.

Bishop-Phelps-Bollobás theorem for Lipschitz functionals in uniformly convex Banach spaces

Vladimir Kadets, *Kharkiv, Ukraine*
Mariia Soloviova, *Kharkiv, Ukraine*

Let X be a real Banach space, X^* be the dual of X. We denote S_X and B_X the unit sphere and the closed unit ball of X. The following sharp version of the Bishop-Phelps-Bollobás theorem was remarked in [1]:

Let $x \in S_X$ and $x^* \in S_{X^*}$ satisfy $x^*(x) \geq 1-\varepsilon$, where $\varepsilon \in (0,2)$. Then there exists $(y, y^*) \in X \times X^*$ with $\|y\| = \|y^*\| = y^*(y) = 1$, such that $\max\{\|x-y\|, \|x^*-y^*\|\} \leq \sqrt{2\varepsilon}$.

We are searching for possible extensions of the Bishop-Phelps-Bollobás theorem for non-linear Lipschitz functionals $f : X \to \mathbb{R}$. Recall that Banach space $Lip_0(X)$ consists of functions $f : X \to \mathbb{R}$ with $f(0) = 0$ which satisfy (globally) the Lipschitz condition. This space is equipped with the norm

$$\|f\| = \sup\left\{\frac{|f(x)-f(y)|}{\|x-y\|} : x, y \in X, x \neq y\right\}.$$

Definition 1. *Let $u \in S_X$. A functional $f \in Lip_0(X)$ attains its norm at direction u if there is a sequence of pairs $x_n, y_n \in X, x_n \neq y_n$ such that*

$$\lim_{n\to\infty} \frac{x_n - y_n}{\|x_n - y_n\|} = u \ \ and \ \ \lim_{n\to\infty} \frac{|f(x_n) - f(y_n)|}{\|x_n - y_n\|} = \|f\|.$$

We say that f attains its norm directionally, if it attains its norm at some direction $u \in S_X$.

Definition 2. *A Banach space X has directional Bishop-Phelps-Bollobás property for Lipschitz functionals , if for every $\delta > 0$ there is such an $\varepsilon > 0$, that for every $f \in Lip_0(X)$ with $\|f\| = 1$ and for every pair $x, y \in X, x \neq y$ with $\frac{|f(x)-f(y)|}{\|x-y\|} > 1-\varepsilon$ there is a $g \in Lip_0(X)$ with $\|g\| = 1$ and there is a $u \in S_X$ such that g attains its norm at direction u, $\|g - f\| < \delta$, and $\left\|\frac{x-y}{\|x-y\|} - u\right\| < \delta$.*

Theorem 1. *Every uniformly convex Banach space has directional Bishop-Phelps-Bollobás property for Lipschitz functionals.*

[1] M. Chica, V. Kadets, M. Martín, S. Moreno-Pulido, and F. Rambla-Barreno, *Bishop-Phelps-Bollobás moduli of a Banach space,* J. Math. Anal. Appl. **412** (2014), no. 2, 697–719.

Invariant measures on finite rank Bratteli diagrams

Olena Karpel, *Kharkiv, Ukraine*

In the talk, we will consider the following problem: let B be a Bratteli diagram of finite rank k. It is known that B can support at most k ergodic (finite and infinite) measures. Is it possible to determine under what conditions on the incidence matrices of B there exist exactly k ergodic measures? The following theorem (see [1]) is principally new in the context of Bratteli diagrams and is based on the careful study of incidence matrices.

Theorem 1. *Let $B = (V, E)$ be a Bratteli diagram of rank $k \geq 2$; identify V_n with $\{1, ..., k\}$ for any $n \geq 1$. Let $F_n = (f_{i,j}^{(n)})$ form a sequence of incidence matrices of B such that $\sum_{j \in V_n} f_{i,j}^{(n)} = r_n \geq 2$ for every $i \in V_{n+1}$. Suppose that $\mathrm{rank} F_n = k$ for all n. Denote*

$$z^{(n)} = \det \begin{pmatrix} \dfrac{f_{1,1}^{(n)}}{r_n} & \dots & \dfrac{f_{1,k-1}^{(n)}}{r_n} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \dfrac{f_{k,1}^{(n)}}{r_n} & \dots & \dfrac{f_{k,k-1}^{(n)}}{r_n} & 1 \end{pmatrix}.$$

Then there exist exactly k ergodic invariant measures on B if and only if $\prod_{n=1}^{\infty} |z^{(n)}| > 0$, or, equivalently, $\sum_{n=1}^{\infty} (1 - |z^{(n)}|) < \infty$.

This is a joint work with M. Adamska, S. Bezuglyi, and J. Kwiatkowski.

[1] M. Adamska, S. Bezuglyi, O. Karpel, and J. Kwiatkowski, *Subdiagrams and invariant measures on Bratteli diagrams*, arXiv:1502.05690

Homogenized conductivity tenzor and absorption function for a locally periodic porous medium

Larysa Khilkova, *Rubizhne, Ukraine*

Let Ω be a bounded domain in $R^n\,(n \geq 2)$, the set $F^\varepsilon = \cup_\alpha F^\varepsilon_{x^\alpha}$ is locally periodic and consist of disjoint connected subsets $F^\varepsilon_{x^\alpha}$. More precisely, we assume that all $F^\varepsilon_{x^\alpha}$ are located in parallelepipeds $\Pi^\varepsilon_{x^\alpha}$ with sides $O(\varepsilon)$, arranged periodically. For each α, the set $F^\varepsilon_{x^\alpha}$ is translation and homothetic contraction of set $F_x \in \Pi$ of diameter O(1) (for details, see [1]).

In the domain $\Omega^\varepsilon = \Omega \setminus F^\varepsilon$, which we call locally periodic, for some fixed ε consider the boundary value problem

$$\begin{cases} -\Delta u^\varepsilon = f^\varepsilon(x),\ x \in \Omega^\varepsilon, \\ \dfrac{\partial u^\varepsilon}{\partial \nu} + \varepsilon\sigma(x, u^\varepsilon) = 0,\ x \in \partial F^\varepsilon, \quad u^\varepsilon = 0,\ x \in \partial\Omega, \end{cases} \tag{1}$$

where ν is the exterior unit normal to the Ω^ε; the function $f^\varepsilon(x) \in L^2(\Omega^\varepsilon)$, the function $\sigma(x, u)$ is continuous with respect to $x \in \Omega^\varepsilon$ and satisfies conditions Lipschitz, monotony and "passive absorption" $(\sigma(x, 0) = 0)$ with respect to u.

Theorem 1. *Let the domains Ω^ε are locally periodic. Then the generalized solution of problem (1) $u^\varepsilon(x)$ for $\varepsilon \to 0$ converges in $L^p(\Omega^\varepsilon, \Omega)$ $\left(p \leq \frac{2n}{n-2}\right)$ to a function $u(x)$ that solves the following boundary value problem in Ω*

$$-\sum_{i,k=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ik}(x)\frac{\partial u}{\partial x_k}\right) + c_u(x, u) = f(x),\ x \in \Omega, \quad u(x) = 0,\ x \in \partial\Omega.$$

Here $\{a_{ik}(x)\}_{i,k=1}^n$ is a conductivity tensor of domain Ω^ε, function $c_u(x, u)$ characterizes the absorption properties of the boundary ∂F^ε and they are equal

$$c_u(x, u) = \frac{|\partial F_x|}{|\Pi|} \cdot \sigma(x, u),$$

$$a_{ik}(x) = \delta_{ik}\left(1 - \frac{|F_x|}{|\Pi|}\right) - \frac{1}{|\Pi|}\int\limits_{\Pi\setminus F_x} \sum_{j=1}^{n} \frac{\partial V_i(\xi, x)}{\partial \xi_j} \cdot \frac{\partial V_k(\xi, x)}{\partial \xi_j}\, d\xi,$$

where $|\partial F_x|$, $|F_x|$, $|\Pi|$ are surface and volumetric measures of the corresponding sets, the function $V_k(\xi, x)$ $(k = \overline{1, n})$ is a solution cell problem in Π.

[1] M.V. Goncharenko, L.A. Khilkova, *Homogenized conductivity tenzor and absorption function for a locally-periodic porous medium,* Ukrainian Mathematical Journal, submitted.

Feedback control for a robust canonical system

Valery Korobov, *Kharkiv, Ukraine*
Tetyana Revina, *Kharkiv, Ukraine*

In the talk we deal with the robust feedback synthesis of a bounded control for a system with an unknown perturbation. Namely, we consider the system

$$\begin{cases} \dot{x}_1 = r_{11}\, p(t, x)x_1 + (1 + p(t, x))x_2, \\ \dot{x}_i = \sum\limits_{1 \leq j \leq i+1} r_{ij}\, p(t, x)x_j + x_{i+1},\ i = 2, \ldots, n-1, \\ \dot{x}_n = \sum\limits_{1 \leq j \leq n} r_{nj}\, p(t, x)x_j + u. \end{cases} \tag{1}$$

Here $t \geq 0$, $x \in Q \subset \mathbb{R}^n$ is a state $(n \geq 2)$, Q is a neighborhood of the origin, $u \in \mathbb{R}$ is a control satisfying the constraint $|u| \leq 1$, r_{ij} are given numbers, and $p(t,x)$ is an *unknown* perturbation, which satisfies the constraint $d_1 \leq p(t,x) \leq d_2$. Our approach is based on the controllability function method conceived by V. I. Korobov in 1979 [1]. The *local robust feedback synthesis* consist in constructing a control of the form $u = u(x)$, $x \in Q$ such that:
1) $|u(x)| \leq 1$; 2) the trajectory $x(t)$ of the closed system, starting at an arbitrary initial point $x(0) = x_0 \in Q$, ends at the origin at some finite time for any perturbation $d_1 \leq p(t,x) \leq d_2$; 3) the control is independent of $p(t,x)$.

Let

$$F^{-1} = \left(\frac{(-1)^{2n-i-j}}{(n-i)!(n-j)!(2n-i-j+1)(2n-i-j+2)} \right)_{i,j=1}^{n},$$

$$D(\Theta) = \operatorname{diag}\left(\Theta^{-\frac{2n-2i+1}{2}}\right)_{i=1}^{n}, \quad F^1 = ((2n-i-j+2)f_{ij})_{i,j=1}^{n},$$

$$S(\Theta) = \Theta(FD(\Theta)RD^{-1}(\Theta) + D^{-1}(\Theta)R^*D(\Theta)F).$$

Suppose that for $\Theta(x) \leq 1$ we have $\sigma((F^1)^{-1}S(\Theta)) \subset [\lambda_1; \lambda_2]$. Let us choose $0 < \gamma_1 < 1$, $\gamma_2 > 1$. Put $\tilde{d}_1^0 = 1/\lambda_1, \tilde{d}_2^0 = 1/\lambda_2$, $d_1^0 = \max\{(1-\gamma_1)\tilde{d}_1^0; (1-\gamma_2)\tilde{d}_2^0\}$, $d_2^0 = \min\{(1-\gamma_1)\tilde{d}_2^0; (1-\gamma_2)\tilde{d}_1^0\}$.

Theorem 1. *For all d_1 and d_2 such that $d_1^0 < d_1 < d_2 < d_2^0$ the control of the form $u(x) = -\Theta^{-\frac{1}{2}}(x)FD(\Theta(x))x/2$, where the controllability function $\Theta(x)$ is a unique positive solution of equation*

$$2a_0\Theta = (D(\Theta)FD(\Theta)x, x), \quad x \neq 0, \quad \Theta(0) = 0, \quad 0 < a_0 \leq 2/f_{nn},$$

solves the local robust feedback synthesis for system (1).

[1] V.I. Korobov, *A general approach to the solution of the problem of synthesizing bounded controls in a control problem,* Mat. Sb. **109(151)** (1979), no. 4(8), 582–606 (in Russian).

A frequency analysis of the special form lattices: scattering capacity

Oleksii Kostenko, *Kharkiv, Ukraine*

We represent the results of applying the numerical-analytical method [1, 2] for solving the two-dimensional problem of the scattering plane monochromatic electromagnetic wave by the bounded lattices which consists of infinity ideal thin impedance strips with the disjoint generatrixes and parallel edges. The dependence on time is given by the factor $e^{-i\omega t}$.

We investigate the wave scattering by a lattice with the prefractal generatrix of different orders and consider the lattices with follows generatrix:

- generalized prefractal Cantor set with the different parameters;
- asymmetric Cantor set;
- prefractal Smith-Volterra-Cantor set.

The relations between the integral scattering characteristic and the product of the wave number and a half of the lattice width has been obtained. These relations are based on the results of the numerical experiments. They allow us to estimate the scattering capacities of these lattices and determine the better (worse) reflected waves.

[1] A.V. Kostenko, *A mathematical model of wave scattering by an impedance lattice*, Kibernetyka ta systemnyi analiz **51** (2015), no. 3, 25–43 (in Russian).

[2] A.V. Kostenko, *Numerical method for the solution of a hypersingular integral equation of the second kind*, Ukrayinskyi matematuchnyi zhurnal **65** (2013), no. 9, 1228–1236 (in Russian); English transl.: Ukrainian mathematical journal **65** (1980),no. 9, 1373–1383.

Asymptotics of the basic functions of generalized Taylor series for the classes $H_{s,\alpha}$

Victor Makarichev, *Kharkiv, Ukraine*

Consider the class

$$H_{s,\alpha} = \left\{ f \in C^{\infty}_{[-1,1]} : \|f^{(n)}\|_{C_{[-1,1]}} \leq c(f)\alpha^n 2^n (2s)^{n(n-1)/2}, n = 0, 1, 2, \ldots \right\},$$

where $s = 2, 3, 4, \ldots$ and $\alpha \in (1, 2s)$. We note that this class is non-quasianalytic.

It was shown in [1] that if $f \in H_{s,\alpha}$ and $s = 2, 3, 4, \ldots$, $\alpha \in (1, 2s)$, then

$$f(x) = \sum_{n=0}^{\infty} (\sum_{k \in N_{s,n}} f^{(n)}(x_{s,n,k})\varphi_{s,n,k}(x) + \sum_{p \in D_{s,n}} \triangle^2_{\frac{1}{s(2s)^n}} \left(f^{(n)}; x^*_{s,n,p} \right) \psi_{s,n,p}(x)),$$

where $N_{s,0} = \{-1, 0, 1\}$ and $N_{s,n} = \{-s(2s)^{n-1}, -s(2s)^{n-1}+1, \ldots, s(2s)^{n-1}\}$ for $n \in \mathbb{N}$, $D_{s,n} = \{1, 2, 3, \ldots, (2s)^{n+1} - 1\} \setminus \{k \cdot s\}$ for any $k \in \mathbb{N}$ and $n = 0, 1, 2, \ldots$; $x_{s,n,k} = \frac{k}{s(2s)^{n-1}}$ for $k \in N_{s,n}$ and $n \in \mathbb{N}$, $x_{s,0,k} = k$ for $k \in N_{s,0}$, $x^*_{s,n,p} = -1 + \frac{p}{s(2s)^n}$ for $p \in D_{s,n}$ and $n = 0, 1, 2, \ldots$; $\triangle^2_h(f, x) = f(x + h) - 2f(x) + f(x - h)$. Moreover, the series in the right part of this equality converges uniformly on the segment $[-1, 1]$.

In [2, 3], existence of the asymptotics of the functions $\varphi_{s,n,k}(x)$ and $\psi_{s,n,p}(x)$ was proved and the first term of asymptotic expansions of these functions was obtained.

In this talk we consider results about the second term of asymptotic expansion of the basic functions $\varphi_{s,n,k}(x)$ and $\psi_{s,n,p}(x)$ of the generalized Taylor series for the class $H_{s,\alpha}$.

[1] V.A. Rvachev and G.A. Starets, *Some atomic functions and their applications*, Proc. Ukr. SSR Acad. Sci. **11** (1983), 22–24 (in Ukrainian).

[2] V.A. Makarichev, *Asymptotics of the basis functions of generalized Taylor series for the class $H_{\rho,2}$*, Mat. Zametki 89 (2011), no. 5, 738–754 (in Russian); English transl.: Math. Notes **89** (2011), no. 5, 689–705.

[3] V.A. Makarichev, *On the asymptotics of the basic functions of generalized Taylor series for some classes of infinitely differentiable functions*, Dal'nevost. Math. J. **11** (2011), no. 1, 56–75 (in Russian).

Stability theorem for unconditional Schauder decompositions in ℓ_p spaces

Vitalii Marchenko, *Kharkiv, Ukraine*

In 1967 T. Kato [1] obtained one remarkable stability theorem for sequences of projections in Hilbert spaces. This theorem proved its importance in the study of spectral properties of various nonselfadjoint operators. The most recent results in this direction were obtained by J. Adduci and B. Mityagin for perturbed harmonic oscillator $-\frac{d^2}{dt^2}+t^2+B$, where $B=b(t)$, with domain in $L_2(\mathbb{R})$ [2], and for the perturbation of a selfadjoint operator with discrete spectrum [3]. We generalize a theorem of Kato to the case of unconditional Schauder decompositions in ℓ_p, $1\le p<\infty$, spaces in the following way.

Theorem 1. *Let $\{\mathfrak{M}_n\}_{n=0}^\infty$ be an unconditional Schauder decomposition of the space ℓ_p, $1\le p<\infty$, with constant M and the a.s.c.p. $\{P_n\}_{n=0}^\infty$, such that for every $x=(a_0,a_1,a_2,\dots)\in\ell_p$ one has $P_0x=(a_0,a_1,a_2,\dots,a_{m-1},0,0,\dots)$. Suppose that $\{J_n\}_{n=0}^\infty$ is a sequence of nonzero projections in ℓ_p, such that $J_nJ_m=\delta_n^mJ_n$ for $n,m\in\mathbb{Z}_+$. Also assume that $\dim P_0=\dim J_0=m<\infty$. Then the following statements are valid.*

1) If for each $x\in\ell_p$, $1\le p\le 2$, one has

$$\sum_{n=1}^{\infty}\|P_n(J_n-P_n)x\|^p\le\varsigma_1^p\|x\|^p,\ \text{where } 0\le\varsigma_1<\frac{1}{2M},$$

then $\{J_n(\ell_p)\}_{n=0}^\infty$ is also an unconditional Schauder decomposition of ℓ_p, isomorphic to $\{\mathfrak{M}_n\}_{n=0}^\infty$.

2) If for each $x \in \ell_p$, $p \geq 2$, one has

$$\sum_{n=1}^{\infty} \|P_n(J_n - P_n)x\|^2 \leq (\varsigma_2(p))^2 \|x\|^2,\ 0 \leq \varsigma_2(p) < \frac{1}{\sqrt{8}M}\left(\frac{\Gamma\left(\frac{p+1}{2}\right)}{\sqrt{\pi}}\right)^{-\frac{1}{p}},$$

then $\{J_n(\ell_p)\}_{n=0}^{\infty}$ is also an unconditional Schauder decomposition of ℓ_p, isomorphic to $\{\mathfrak{M}_n\}_{n=0}^{\infty}$.

[1] T. Kato, *Similarity for sequences of projections,* Bull. Amer. Math. Soc. **73(6)** (1967), 904–905.

[2] J. Adduci and B. Mityagin, *Eigensystem of an L^2-perturbed harmonic oscillator is an unconditional basis*, Open Mathematics (formerly Cent. Eur. J. Math.) **10(2)** (2012), 569–589.

[3] J. Adduci and B. Mityagin, *Root system of a perturbation of a selfadjoint operator with discrete spectrum,* Integral Equations Operator Theory **73(2)** (2012), 153–175.

Representation theorems of δ-subharmonic functions

Nguyen Van Quynh, *Kharkiv, Ukraine*

Representation theorems are important in the theory of subharmonic and δ-subharmonic functions. In this paper we sharpen Azarin's variant of representation theorem of δ-subharmonic functions of finite order in [1].

Let $v(z)$ is the δ-subharmonic function in $\mathbb{C}$ and $T(r,v)$ is the Nevanlinna's characteristic function of $v(z)$. The quantity

$$\rho = \overline{\lim_{r\to\infty}} \frac{\ln T(r, \mathsf{w})}{\ln r}$$

is called order of function $v(z)$.

Theorem 1. *Suppose $v(z)$ is the δ-subharmonic function of order ρ in the plane $\mathbb{C}$, μ is the Riesz measure of function v, $p=[\rho]$. We have the representation*

$$\mathsf{w}(z) = \int\limits_{B(0,1)} \ln|z-\zeta| d\mu(\zeta)$$

$$+ \int\limits_{CB(0,1)} \mathrm{Re}\left(\ln\left(1-\frac{z}{\zeta}\right) + \frac{z}{\zeta} + ... + \frac{1}{p}\frac{z^p}{\zeta^p}\right) d\mu(\zeta) + \sum_{n=0}^{p} \mathrm{Re}\, c_n z^n,$$

and c_n satisfies the equalities

$$c_n = \lim_{R\to\infty} \mathrm{Re}\left(\frac{1}{\pi R^n}\int\limits_0^{2\pi} \mathsf{w}(Re^{i\varphi})e^{-in\varphi}d\varphi \left(\frac{\overline{\zeta}^n}{R^{2n}} - \frac{1}{\zeta^n}\right) d\mu(\zeta)\right.$$

$$+\frac{1}{n}\int\limits_{B(0,R)\setminus B(0,1)}\left(\frac{\overline{\zeta}^n}{R^{2n}}-\frac{1}{\zeta^n}\right)d\mu(\zeta)\Bigg). \qquad (1)$$

Remark 1. *The existence of the limit (1) is not trivial. This is the main assertion of the theorem.*

[1] V.S. Azarin, *Growth theory of subharmonic functions,* Birkhanser, Basel, Boston, Berlin, 2009.
[2] W.K. Heyman and P.B. Kennedy, *Subbharmonic Functions,* London, New York, San Francisco, 1979.

Distribution of eigenvalues of some random matrices of hight order

Daria Tieplova, *Kharkiv, Ukraine*

We consider real symmetric and hermitian random matrices

$$M_n=\sum_{\mu=1}^{m}\Phi^\mu\otimes\Phi^\mu,$$

where $\Phi^\mu=Y^\mu\otimes Y^\mu$ and $\{Y^\mu\}_{\mu=1}^m$ are i.i.d. Gaussian random vectors of $\mathbb{R}^n$ (or $\mathbb{C}^n$) such that

$$\mathbf{E}\{Y_i^\mu\}=0,\ \mathbf{E}\{Y_i^\mu Y_k^\nu\}=n^{-1}\delta_{ik}\delta_{\mu\nu}$$

or in the complex case

$$\mathbf{E}\{Y_i^\mu\}=\mathbf{E}\{Y_i^\mu Y_k^\nu\}=0,\ \mathbf{E}\{Y_i^\mu\overline{Y_k^\mu}\}=n^{-1}\delta_{ik}.$$

Denote $-\infty<\lambda_1\le\ldots\le\lambda_{n^2}<\infty$ the eigenvalues of M_n and introduce their Normalized Counting Measure N_n, setting for any interval $\Delta\subset\mathbb{R}^n$ $N_n(\Delta)=\mathrm{Card}\{l\in[1,n^2]:\lambda_l\in\Delta\}/n^2$.

The main result is

Theorem 1. *Assume that $\{m_n\}$ is a sequence of positive integers such that*

$$c_n=m_n/n^2\to c\in[0,+\infty).$$

Then the Normalized Counting Measure N_n of eigenvalues M_n converges weakly with probability 1, as $n\to+\infty$, to a nonrandom probability measure N, and the Stieltjes transform f of N is uniquely determined by the equation

$$f=\frac{1+2f}{2c-z(1+2f)},$$

in the class of Stieltjes transforms of probability measures.

AUTHOR INDEX

Afanasiev, Ie.	afanasev_e@mail.ru
Andreiev, K.	andreiev@ilt.kharkov.ua
Arlinskii, Yu.	yury.arlinskii@gmail.com
Bebiya, M.	bebiya@rambler.ru
Bergweiler, W.	bergweiler@math.uni-kiel.de
Bohdanov, A.	phaust94@gmail.com
Eremenko, A.	eremenko@math.purdue.edu
Fardigola, L.	fardigola@ukr.net
Favorov, S.	sfavorov@gmail.com
Feldman, G.	feldman@ilt.kharkov.ua
Filipkovskaya, M.	fmarias@mail.ru
Gefter, S.	gefter@karazin.ua
Gladka, Z.	gladkazoya@gmail.com
Golinskii, L.	golinskii@ilt.kharkov.ua
Goncharuk, A.	a_ryska@ukr.net
Gorkavyy, V.	gorkaviy@ilt.kharkov.ua
Kadets, V.	v.kateds@karazin.ua
Kalinin, D.	kalinin@ilt.kharkov.ua
Karpel, O.	helen.karpel@gmail.com
Khilkova, L.	larahil@inbox.ru
Khrabustovskyi,V.	v_khrabustovskyi@ukr.net
Khruslov, E.	khruslov@ilt.kharkov.ua
Korobov, V.	vkorobov@univer.kharkov.ua
Kostenko, O.	kostenko@ilt.kharkov.ua
Makarichev, V.	victor.makarichev@gmail.com
Marchenko, V.	vitalii.marchenko@karazin.ua
Myronyuk, M.	m_margarita@bk.ru
Poplavskyi, M.	mpoplavskyi@gmail.com
Quynh, N.V.	quynhsonla1988@gmail.com
Rashkovskii, A.	alexander.rashkovskii@uis.no
Revina, T.	t.revina@karazin.ua
Rybalko, V.	vrybalko@ilt.kharkov.ua
Shepelsky, D.	shepelsky@yahoo.com
Sinel'shchikov, S.	sinelshchikov@ilt.kharkov.ua
Soloviova, M.	5_11_16@mail.ru
Tieplova, D.	dteplova@mail.ru
Vasilchuk, V.	vasilchuk@ilt.kharkov.ua
Vishnyakova, A.	anna.m.vishnyakova@univer.kharkov.ua