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CONTENTS

INVITED SPEAKERS

Spectral indefinite problems of hydrodynamics

Thomas Azizov, *Voronezh, Russia*
Nikolay Kopachevsky, *Simferopol, Ukraine*

This paper deals with the conservative and dissipative dynamical systems with infinite degrees of freedom that encountered in evolution problems of linear hydrodynamics, and corresponding spectral problems.

A common feature of the investigated spectral problems is a potential energy operator of the system with alternating signs. Because of that we use spaces with an indefinite metric (space of M. Krein and space of L. Pontryagin, see [1-6]), which allows us to prove the propositions on solvability of the considered problems, to study their spectral properties and to get conditions of dynamic stability (instability) of the investigated dynamic systems.

Using this approach we study some actual problems of fluid mechanics: the problems of the eigen-oscillations of a body with a cavity that is completely filled with an ideal or viscoelastic fluid, the problem of eigen-oscillations of a stratified fluid in a cylindrical container, the problem of S. Krein (on the normal oscillations of a heavy viscous fluid in an open vessel), the problem of the transverse vibrations of a viscoelastic rod with a weight at the end, the problem of the movements of the articulated gyrostats, etc.

[1] T. Ya. Azizov, N. D. Kopachevsky, *Introduction to the Theory of Pontryagin Spaces*, Ltd. "Form", Simferopol, 2008 (in Russian).

[2] T. Ya. Azizov, N. D. Kopachevsky, *Introduction to the Theory of Krein Spaces*, Ltd. "Form", Simferopol, 2010 (in Russian).

[3] T. Ya. Azizov, I. S. Iokhvidov, *Fundamentals of the Theory of Linear Operators in Spaces with Indefinite Metric*, Nauka, Moscow, 1986. (in Russian).

[4] L. S. Pontryagin, *Hermitian Operators in Spaces with Indefinite Metric*, News of AS of USSR, Ser. Math., 8 (1982), no. 6, 243–280. (in Russian).

[5] I. Tz. Gohberg, M. G. Krein, *Introduction to the Theory of Linear Non-Selfadjoint Operators*, Nauka, Moscow, 1965 (in Russian).

[6] N. D. Kopachevsky, S. G. Krein, Ngo Zuy Kan, *Operator Methods in Linear Hydrodynamics: Evolution and Spectral Problems*, Nauka, Moscow, 1989 (in Russian).

Invariant curves of rational functions

Alexandre Eremenko, *West Lafayette, Indiana, USA*

We consider Jordan analytic curves on the Riemann sphere which are invariant under a rational function. The study of such curves was suggested by Fatou.

Under the condition that the restriction of the rational function on the curve is not a homeomorphism, and some extra technical conditions, we show that such curves must be algebraic. The problem is closely related to solving certain type of functional equations (semi-conjugacy equations) in rational functions.

[1] A. Eremenko, *Invariant curves and semiconjugacies of rational functions*, Fund. Math. 219, 3 (2013) 787–804.

Limit sets of integrals

Anatolii Grishin, *Kharkiv, Ukraine*
Irina Poedintseva, *Kharkiv, Ukraine*

The main result is following.

Theorem 1 *Let the following conditions are satisfied.*

1. $\rho(r)$ *is arbitrary proximate order of special type.*
2. μ *is a Radon measure on the semiaxis* $(0,\infty)$ *of the class* $\mathfrak{M}(\rho(r))$.
3. $K(t)$ *is a Borel function on semiaxis* $(0,\infty)$ *such that* $t^{\rho-1}\gamma(t)K(t) \in L_1(0,\infty)$.
4. *The function* $\int\limits_0^\infty K(t)t^{\rho-1+i\lambda}dt$ *has not real zeros.*
5. *The measure* s, $ds(t) = \Psi(t)dt$, *where*
$$\Psi(t) = \int\limits_0^\infty K\left(\frac{u}{t}\right) d\mu(u)$$
is regular measure relatively proximate order $\rho(r)+1$.

Then the measure μ *is regular one relatively proximate order* $\rho(r)$. *If* $Fr[s] = \{ct^\rho dt\}$, *then* $Fr[\mu] = \left\{\frac{c}{c_1}t^{\rho-1}dt\right\}$ *where* $c_1 = \int\limits_0^\infty K(t)t^{\rho-1}dt$.

The theorem is essential improvement of the second Wiener Tauberian theorem. The first Wiener Tauberian theorem follows the theorem as well.

Now we explain some terms from the theorem. We assume that proximate order $\rho(r)$ has form $\rho(r) = \rho + \rho_1(r)$, $\rho \in (-\infty,\infty)$, where $\rho_1(r)$ is zero proximate order such that $\rho_1\left(\frac{1}{r}\right) = -\rho_1(r)$. This is the special type of proximate orders.

We define $V(r) = r^{\rho(r)}$. The function $\gamma(t)$ is defined as follows

$$\gamma(t) = \sup_{r>0} \frac{V(tr)}{t^{\rho} V(r)}, \qquad t \in (0, \infty).$$

It is proved that γ is continuous function that satisfies conditions:

$$\gamma(t_1 t_2) \leq \gamma(t_1)\gamma(t_2), \qquad \lim_{t\to\infty} \frac{\gamma(t)}{\ln t} = \lim_{t\to\infty} \frac{\gamma\left(\frac{1}{t}\right)}{\ln t} = 0.$$

If $\rho(r) \equiv \rho$, then $\gamma(t) = 1$.

A Radon measure μ on the semiaxis $(0, \infty)$ belongs to the class $\mathfrak{M}(\rho(r))$ if there exists $q > 1$ such that $\sup\limits_{r>0} \frac{|\mu|([r,qr])}{V(r)} < \infty$.

Let $\mu \in \mathfrak{M}(\rho(r))$. The measure μ_t is defined by the formula $\mu_t(E) = \frac{\mu(tE)}{V(t)}$.

Let $\mu \in \mathfrak{M}(\rho(r))$. Following Azarin we define limit set $Fr[\mu]$ of the measure μ relatively the proximate order $\rho(r)$ as follows: $\nu \in Fr[\mu]$ if $\nu = \lim\limits_{n\to\infty} \mu_{t_n}$ where $t_n \to \infty$.

Measure $\mu \in \mathfrak{M}(\rho(r))$ is called regular if $Fr[\mu]$ consists of an unique measure.

Complex random energy models: zeros and fluctuations

Zakhar Kabluchko, *Ulm, Germany*
Anton Klimovsky, *Leiden, The Netherlands*

Random energy models are paradigmatic models of disordered systems introduced by Derrida [1, 2]. These models played an important role in the mathematical progress on understanding the physics predictions of the hierarchical replica symmetry breaking in mean-field spin glasses [3].

We report on our recent work [4], where we analyze some *complex-valued generalizations* of Derrida's random energy models. Our motivation comes from (1) desire to understand the analytic mechanisms behind the phase transitions in disordered systems (cf., the Lee–Yang program); (2) problems of interference in random media, and (3) the quantum Monte Carlo method.

The partition function of the random energy model at inverse temperature β is a sum of random exponentials $Z_N(\beta) = \sum_{k=1}^{N} \exp(\beta\sqrt{n}X_k)$, where $X_1, X_2, \ldots$ are independent real standard normal random variables (= random energies), and $n = \log N$. We study the large N limit of the partition function viewed as an analytic function of the complex variable β. We identify the asymptotic structure of complex *zeros of the partition function* confirming and extending predictions made in the theoretical physics literature. We prove limit theorems for the random partition function at complex β, both on the logarithmic scale and on the level of *limiting fluctuations*. Our results cover also the case of the sums of independent identically distributed random exponentials with any given correlations between the real and imaginary parts of the random exponent.

Time permitting, we will discuss a generalization of the above results to the case of the complex-valued *generalized random energy model* which includes strong hierarchical correlations between the energies.

[1] B. Derrida, *Random-energy model: limit of a family of disordered models*, Phys. Rev. Lett., 45 (1980), 79–82.

[2] B. Derrida, *A generalization of the Random Energy Model which includes correlations between energies*, J. Physique Lett., 46 (1985), 197–211.

[3] D. Panchenko, *The Sherrington-Kirkpatrick model*, Springer Monographs in Mathematics, Springer, 2013.

[4] Z. Kabluchko, A. Klimovsky, *Complex random energy model: Zeros and fluctuations*, Prob. Theor. and Rel. Fields, Online First (2013), 1–38.

Abstract interpolation problem. Direct and inverse commutant lifting problem

Alexander Kheifets, *Lowell, USA*

Setting of the Abstract Interpolation Problem (Introduced by Katsnelson, Kheifets and Yuditskii) is extended so that solutions are nonnegative operator-valued (on a Hilbert space E) harmonic functions on the unit disk $\mathbb{D}$.

Theorem 1 *General solution is of the form (all functions of z, $|z| < 1$)*

$$\sigma = \sigma_0 + r_2\omega(\mathbf{1} - s\omega)^{-1}r_1 + r_1^*(\mathbf{1} - \omega^* s^*)^{-1}\omega^* r_2^*, \tag{1}$$

where ω is an arbitrary contractive analytic operator-function $\omega(z) : N_1 \to N_2$, coefficients (determined by the data of the problem) are such that

$$\Sigma_0 = \begin{bmatrix} \mathbf{1}_{N_1} & s & r_1 \\ s^* & \mathbf{1}_{N_2} & r_2^* \\ r_1^* & r_2 & \sigma_0 \end{bmatrix} : N_1 \oplus N_2 \oplus E \to N_1 \oplus N_2 \oplus E \tag{2}$$

is a nonnegative operator-function (of z), s, r_1, r_2 are analytic, $s(0) = 0$, σ_0 is harmonic.

Commutant Lifting Problem reduces to the Abstract Interpolation Problem. For this special type of data solutions are of the form

$$\sigma = \begin{bmatrix} \sigma'' & w \\ w^* & \sigma' \end{bmatrix} \geq 0, \tag{3}$$

where σ' and σ'' are fixed and w is called the symbol of the lift.

In the case of Commutant Lifting Problem $r_2 = \begin{bmatrix} 0 \\ s_2 \end{bmatrix}$, $r_1 = \begin{bmatrix} s_1 & 0 \end{bmatrix}$ and one gets description of the symbols of the lifts of a given contractive intertwiner as

$$w = s_0 + s_2\omega(\mathbf{1} - s\omega)^{-1}s_1. \tag{4}$$

Necessary and sufficient conditions on the coefficients s, s_1, s_2, s_0 of Commutant Lifting problems are obtained.

[1] A. Kheifets, *Abstract Interpolation Scheme for Harmonic Functions*, Operator Theory: Advances and Applications 134 (2002), 287-317.

[2] J. Ball, A. Kheifets, *The inverse commutant lifting problem. I: coordinate free formalism*, Integral equations and Operator Theory, 70 (2011) no. 1, 17–62.

[3] J. Ball, A. Kheifets, *The inverse commutant lifting problem. II: Hellinger functional-model spaces*, Complex Analysis and Operator Theory, Online First (2011), DOI 10.1007/s11785-011-0211-9

On the moment method in studies of large random matrices

Oleksiy Khorunzhiy, *Versailles, France*

Let $A^{(n)}$ be a n-dimensional real symmetric matrix with the elements $(A^{(n)})_{ij} = a_{ij}/\sqrt{n}$, where $\{a_{ij},\ 1 \le i \le j\}$ are jointly independent identically distributed random variables. We assume that the probability distribution of a_{ij} is symmetric and the variance is equal to v^2. Since the pioneering works by E. Wigner [3], it is known that if all moments of a_{ij} exist, then the mean values of the normalized moments of random matrix $A^{(n)}$,

$$M_l^{(n)} = \frac{1}{n}\mathbb{E}\,\mathrm{Tr}\left(A^{(n)}\right)^l, \quad l = 0, 1, 2, \ldots$$

converge as $n \to \infty$ to the moments of the limiting measure $\sigma_W(\lambda)$ known as the semicircle, or Wigner, distribution. This distribution describes the limiting distribution of the eigenvalues of $A^{(n)}$ for large values of n.

The limiting moments $m_{2k} = \lim_{n\to\infty} M_{2k}^{(n)}$ have the form $m_{2k} = v^{2k}t_k$, where $t_k = \frac{(2k)!}{k!\,(k+1)!}$ is the Catalan number of k-edge rooted trees [4].

In the first part of the talk, we explain the origin of this nice combinatorial structure that appears in the proof of the semicircle law given by E. Wigner.

In the second part of the talk, we describe a generalization of the method of E. Wigner, developed by Ya. G. Sinai and A. B. Soshnikov to study the moments $M_{2k}^{(n)}$ in the limit $k, n \to \infty$ (see paper [2] and references therein).

In the third part of the talk, we discuss applications of the improved and completed version of the Sinai-Soshnikov method to the studies of the moments of dilute samples of random matrices $A^{(n)}$, where the randomly chosen elements are replaced by zeros [1]. We show that on certain critical asymptotic regimes, the limiting expressions are related not with numbers t_k of the Catalan rooted trees but with the numbers u_k of the ternary rooted trees given by $u_k = \frac{(3k)!}{k!\,(2k+1)!}$.

[1] O. Khorunzhiy, http://arxiv.org/abs/1107.5724
[2] A. Soshnikov, Commun. Math. Phys., 207 (1999), 697-733
[3] E. Wigner, Ann. of Math., 62 (1955), 548-564.
[4] wikipedia: http://en.wikipedia.org/wiki/Catalan_number

On the gaps in the spectrum of the Neumann Laplacian generated by a system of periodically distributed traps

Andrii Khrabustovskyi, *Karlsruhe, Germany*

It is well-known that the spectrum of self-adjoint periodic differential operators has a band structure, i.e. it is a union of compact intervals called *bands*. The neighbouring bands may overlap, otherwise we have a *gap* in the spectrum. In general the existence of spectral gaps is not guaranteed.

For applications it is interesting to construct the operators with non-void spectral gaps since their presence is important for the description of wave processes which are governed by differential operators under consideration. Namely, if the wave frequency belongs to a gap, then the corresponding wave cannot propagate in the medium without attenuation. This feature is a dominant requirement for so-called photonic crystals which are materials with periodic dielectric structure attracting much attention in recent years.

In the talk we discuss the effect of opening of spectral gaps for the Laplace operator in $\mathbb{R}^n$ $(n \geq 2)$ perforated by a family of periodically distributed traps on which we pose the Neumann boundary conditions. The traps are made from infinitely thin screens. In the case $n = 2$ this operator describes the propagation of the H-polarized electro-magnetic waves in the dielectric medium containing a system of perfectly conducting trap-like screens.

We also discuss the question of the controllability of the gaps, i.e. how to make the gaps close to predefined intervals via a suitable chose of geometry of the traps.

The talk is based on the results obtained jointly with E. Khruslov [1].

[1] A. Khrabustovskyi, E. Khruslov, *Gaps in the spectrum of the Neumann Laplacian generated by a system of periodically distributed traps*, arXiv:1301.2926 [math.SP], 2013.

Homogenization of differential equations on Riemannian manifolds with complex microstructure

Evgen Khruslov, *Kharkiv, Ukraine*

We consider manifolds M_ε of a special form depending on a small parameter ε. These manifolds are formed by gluing N $(N \geq 1)$ copies of the base manifold M with large number of thin tubes and small bubbles. The topological genus of M_ε increases as $\varepsilon \to 0$. The metric on M_ε depends on ε and is consistent with the metric on M.

We consider the Laplace-Beltrami operator Δ_ε on M_ε and study the asymptotic behavior as $\varepsilon \to 0$ of solutions of the Cauchy problem for evolution equations generated by Δ_ε. We prove that the limiting solutions are described by the homogenized equations on M, containing nonlocal terms with respect to space and time. We show that some fundamental equations of the quantum mechanics, such as the Schrödinger equation with a potential and the Klein-Gordon equation with a mass, can be obtained by homogenizing the Schrödinger equation without a potential and the wave equation respectively.

We also study the asymptotic behavior of the harmonic vector fields and tensors (harmonic differential 1- and 2-forms on M_ε). We prove that the homogenization of the Maxwell equations, which are free from charges and currents on M_ε, results in the Maxwell equations with non-zero charges and currents on M.

Matrix Riemann-Hilbert problems in the theory of nonlinear integrable equations

Vladimir Kotlyarov, *Kharkiv, Ukraine*

The matrix Riemann-Hilbert problem is a powerful tool in the theory of integrable nonlinear evolution equations, ordinary Painleve equations, models of random matrices and other areas modern mathematical physics. The inverse scattering method for integrable nonlinear equations was originally based on Marchenko approach to the theory of scattering on the line. This method allowed to develop a theory which includes a large number of significant results of modern mathematical physics. Alternative approach to the scattering theory on the line is a method of the matrix Riemann-Hilbert problem. This method has been very successful in the last two decades. In the world there are a few group of mathematicians who have received numerous interesting results using the matrix Riemann-Hilbert problem and so called nonlinear steepest descent method for them, originated by P.Deift and X.Zhou. Our institute also has a group of mathematicians who use and develop these powerful techniques for explore issues such as:

- the initial value problems for nonlinear integrable equations on the whole line with step-like initial data;
- the initial boundary value problems for the same equations on the half-line with decreasing and periodic boundary conditions;
- studying of asymptotic behavior of the corresponding solutions for a long or short time.

We present a brief overview of the results obtained in the Mathematical Division in the last few years. In more detail, we will examine how matrix Riemann-Hilbert problem generates compatible Ablowitz-Kaup-Newell-Segur equations that lead to Maxwell-Bloch equations for quantum laser amplifiers and attenuators with inhomogeneous broadening.

[1] Iryna Egorova, Zoya Gladka, Volodymyr Kotlyarov, Gerald Teschl, *Long-time asymptotics for the Korteweg–de Vries equation with step-like initial data* , Nonlinearity 26 (2013), no. 7, 1839–1864.

[2] Vladimir Kotlyarov, *Complete linearization of a mixed problem to Maxwell–Bloch equations by matrix Riemann-Hilbert problems*, J.Phys A: math and theor 46 (2013).

1D Schödinger and Dirac operators with local point interactions

Mark Malamud, *Donetsk, Ukraine*

Let $X = \{x_n\}_{n=1}^{\infty} \subset \mathbb{R}_+$ be a discrete set and let $d_* := \inf_{n\in\mathbb{N}} |x_{n+1} - x_n|$. We will discuss the Schrödinger operator $H_{X,\alpha,q}$ associated with the formal differential expression

$$H_{X,\alpha,q} = -\frac{d^2}{dx^2} + q(x) + \sum_{k=1}^{\infty} \alpha_k \delta(x - x_k), \quad \alpha_k \in \mathbb{R}, \tag{1}$$

and its counterpart $D_{X,\alpha,Q}$ associated with the Dirac differential expression

$$D^c := -i\,c\,\frac{d}{dx} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{c^2}{2} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{pmatrix}. \tag{2}$$

Here $c > 0$ denotes the velocity of light. Our approach to the spectral properties of operators $H_{X,\alpha,q}$ and $D_{X,\alpha,Q}$ with point interactions is based on a new connection of these operators with Jacobi matrices of certain classes.

We show that the spectral properties of $H_{X,\alpha}$ like self-adjointness, discreteness, and lower semiboundedness correlate with the corresponding spectral properties of Jacobi matrices of certain classes. Based on this connection, we will discuss necessary and sufficient conditions for the operators $H_{X,\alpha}$ to be self-adjoint, lower-semibounded, non-negative, and discrete in the case $d_* = 0$.

Self-adjointness and discreteness of $D_{X,\alpha}$ on finite and infinite intervals will be discussed. The property of both operators $D_{X,\alpha}$ and $D_{X,\alpha}$ to have absolutely continuous or singular spectrum will be discussed too.

The talk is based on the results published in [1]-[4].

[1] A. Kostenko, M. Malamud, *1–D Schrödinger operators with local point interactions on a discrete set*, J. Differential Equations, 249 (2010), 253–304.

[2] M. Malamud, H. Neidhardt, *On the unitary equivalence of absolutely continuous parts of self-adjoint extensions*, J. Func. Anal., 260 (2011), no. 3, 613–638.

[3] M. Malamud, K. Schmuedgen, *Spectral theory of Schrödinger operators with infinitely many point interactions and radial positive definite functions*, J. Func. Anal., 263 (2012), no. 10, 3144–3194.

[4] R. Carlone, M. Malamud, A. Posilicano, *On the spectral theory of Gesztesy–Šeba realizations of 1–D Dirac operators with point interactions*, J. Dif Eq., 254 (2013), 3835-3902.

Hidden landscape of wave localization

Svitlana Mayboroda, *Minneapolis, USA*

Localization of vibrations is one of the most intriguing features exhibited by irregular or inhomogeneous media. A striking (but certainly not unique) example is the so called “Anderson localization” of quantum states by a random potential, that was discovered by Anderson in 1958 and that brought him the Nobel Prize in 1977. Anderson localization is one of the central topics in condensed matter physics, producing hundreds of papers each year. Yet, there exists up to now no theoretical framework able to predict exactly what triggers localization, where it happens and at which frequency.

We present a fundamentally new mathematical approach that explains how the system geometry and the differential operator intervening in the wave equation interplay to give rise to a “landscape" that reveals weakly coupled subregions inside the system, and how these regions shape the spatial distribution of the vibrational eigenmodes.

This is joint work with Marcel Filoche.

Geometric study of convex and quasi-concave functions in $\mathbb{R}^n$

Vitali Milman, *Tel-Aviv, Israel*

The talk deals with:

1. Duality and new structures on the family of convex (and log-concave) functions in $\mathbb{R}^n$.
2. Classical constructions in analysis which appear (uniquely) from elementary (simplest) properties.
3. Extension of Minkowski polarization result to the classes of log-concave and quasi-concave functions; Mixed integrals.

Pluricomplex Green functions with colliding poles

Alexander Rashkovskii, *Stavanger, Norway*

The talk is based on a joint paper with Pascal Thomas [3].

Given an ideal $\mathcal{I}$ of holomorphic functions on a bounded hyperconvex domain Ω in $\mathbb{C}^n$, the *pluricomplex Green function* $G_{\mathcal{I}}$ is the upper envelope of all negative plurisubharmonic functions u in Ω satisfying $u \leq \max_k \log|f_k| + O(1)$, where $\{f_k\}$ are local generators of the ideal $\mathcal{I}$ [2].

Let $\mathcal{I}_\varepsilon$ be a family of ideals vanishing at N distinct points all tending to a point $a \in \Omega$ as $\varepsilon \to 0$. As is known, convergence of the ideals $\mathcal{I}_\varepsilon$ to an ideal $\mathcal{I}$ does not guarantee the convergence of the Green functions $G_{\mathcal{I}_\varepsilon}$ to $G_{\mathcal{I}}$; moreover, the existence of the limit of the Green functions was unclear. Assuming that all the powers $\mathcal{I}_\varepsilon^p$ converge to some ideals $\mathcal{I}_{(p)}$, we prove that the functions $G_{\mathcal{I}_\varepsilon}$ do converge, locally uniformly away from a, and the limit function is essentially the upper envelope of the scaled Green functions $p^{-1}G_{\mathcal{I}_{(p)}}$ of the limit ideals $\mathcal{I}_{(p)}$, $p \in \mathbb{N}$.

As examples, we consider ideals generated by hyperplane sections of a holomorphic curve in $\mathbb{C}^{n+1}$ near a singular point. In particular, our result explains the asymptotics for 3-point models obtained, by a different method, in [1].

[1] J. Magnússon, A. Rashkovskii, R. Sigurdsson, P. Thomas, *Limits of multipole pluricomplex Green functions*, Int. J. Math. 23 (2012), no. 6; available at http://arxiv.org/abs/1103.2296.

[2] A. Rashkovskii, R. Sigurdsson, *Green functions with singularities along complex spaces*, Int. J. Math. 16 (2005), no. 4, 333–355.

[3] A. Rashkovskii, P. Thomas, *Powers of ideals and convergence of Green functions with colliding poles*, Int. Math. Res. Notes, to appear; available at http://arxiv.org/abs/1208.2824.

Universal approach to β matrix models

Mariya Shcherbina, *Kharkiv, Ukraine*

We consider the distributions in $\mathbb{R}^n$, depending on the potential $V:\mathbb{R}\to\mathbb{R}_+$ and $\beta>0$

$$p_{n,\beta}(\lambda_1,...,\lambda_n)=Z_{n,\beta}^{-1}[V]e^{\beta H(\lambda_1,...,\lambda_n)/2},$$

where the function H (Hamiltonian) and the multiplier $Z_n[\beta,V]$ (partition function) have the form

$$H(\lambda_1,\dots,\lambda_n)=-\,n\sum_{i=1}^{n}V(\lambda_i)+\sum_{i\neq j}\log|\lambda_i-\lambda_j|, \tag{1}$$

$$Z_n[\beta,V]=\int e^{\beta H(\lambda_1,...,\lambda_n)/2}d\lambda_1\dots d\lambda_n,\quad V(\lambda)>(1+\varepsilon)\log(1+\lambda^2).$$

Assuming that V is smooth enough and the corresponding equilibrium density ρ has one-interval support $\sigma=[-2,2]$, we consider the change of variables $\lambda_i\to\zeta(\lambda_i)$ with $\zeta(\lambda)$ chosen from the equation

$$\zeta'(\lambda)=\frac{\rho_{sc}(\lambda)}{\rho(\zeta(\lambda))},\quad \zeta(-2)=-2,\ \text{with}\ \rho_{sc}(\lambda)=\frac{\sqrt{4-\lambda^2}}{2\pi}.$$

This gives us

$$H^{(\zeta)}(\lambda_1,\dots,\lambda_n)=-\,n\sum V(\zeta(\lambda_j))+\sum_{i\neq j}\log|\zeta(\lambda_i)-\zeta(\lambda_j)| \tag{2}$$
$$+\frac{2}{\beta}\sum\log\zeta'(\lambda_j),$$

and the partition function and all marginal densities of system (1) can be expressed in terms of system (2).

Then after some transformations we obtain that it suffices to study the partition function and marginal densities of the "gaussian" potential $V^*=\frac{\lambda^2}{2}$ with a "small perturbation" $h(\lambda)$

$$H^{(\zeta)}(\bar\lambda)=-\,n\sum\frac{\lambda_j^2}{2}+\sum_{i\neq j}\log|\lambda_i-\lambda_j|+\sum h(\lambda_i).$$

This reduces most of the problems of local and global statistics of β matrix models to the correspondent problems of the gaussian case with a small perturbation.

Universality of the second mixed moment of characteristic polynomials of the 1D Gaussian band matrices

Tatyana Shcherbina, *Princeton, USA*

We consider Gaussian random 1D band matrices (1D RBM), which are hermitian or real symmetric $n \times n$ matrices H_n, whose entries H_{ij} are random Gaussian variables such that $\mathbf{E}\{H_{ij}\} = 0$, $\mathbf{E}\{H_{ij}H_{lk}\} = \delta_{ik}\delta_{jl}J_{ij}$. Here $J_{ij} = \big(-W^2\Delta + 1\big)^{-1}_{ij}$, where Δ is the discrete Laplacian on $[1, n]$. The density of states of the ensemble is given by the Wigner semicircle law (see [1])

$$\rho(\lambda) = (2\pi)^{-1}\sqrt{4-\lambda^2}, \quad \lambda \in [-2, 2].$$

The physical conjecture of Mirlin, Fyodorov (see [2]) states that for 1D RMB there is a phase transition: for $W \gg \sqrt{N}$ the local behavior of the eigenvalues is the same as for GUE or GOE (which corresponds to delocalized states), and for $W \ll \sqrt{N}$ we get another behavior, which determines by the Poisson statistics (and corresponds to localized states). Here we prove the first part of this conjecture for the second correlation function of the characteristic polynomials, which is defined by

$$F_2(\Lambda) = \mathbf{E}\big\{\det(\lambda_1 - H_n)\det(\lambda_2 - H_n)\big\}.$$

The main result is

Theorem 1 *For 1D Gaussian RBM with $W^2 = n^{1+\theta}$, $0 < \theta < 1$ we have*

$$\lim_{n\to\infty} D_2^{-1} F_2\Big(\Lambda_0 + \frac{\hat{\xi}}{2n\rho(\lambda_0)}\Big) = \begin{cases} \dfrac{\sin(\pi\xi)}{\pi\xi}, & \textit{hermitian case.} \\ \dfrac{\sin(\pi\xi)}{\pi^3\xi^3} - \dfrac{\cos(\pi\xi)}{\pi^2\xi^2}, & \textit{real symmetric case.} \end{cases}$$

Here $D_2 = F_2(\lambda_0, \lambda_0)$, $\Lambda_0 = \lambda_0 \cdot I_2$, $\hat{\xi} = \xi \cdot \mathrm{diag}\,\{1, -1\}$, $\lambda_0 \in (-2, 2)$.

The result is a manifestation of the universality, that can be compared with the universality of the local bulk regime for correlations function.

[1] S. A. Molchanov, L. A. Pastur, A. M. Khorunzhii. *Distribution of the eigenvalues of random band matrices in the limit of their infinite order*, Theor. Math. Phys., 90 (1992), 108–118

[2] Y. V. Fyodorov, A. D. Mirlin. *Scaling properties of localization in random band matrices: a σ-model approach*, Phys. Rev. Lett., 67 (1991), 2405–2409.

TBA

Mikhail Sodin, *Tel-Aviv, Israel*

Geometrization of branched covering maps and decidability of Thurston equivalence

Michael Yampolsky, *Toronto, Canada*

A basic object in holomorphic dynamics is a rational map of the Riemann sphere with finite critical orbits. A general postcritically finite branched covering maps of the sphere is known as a Thurston map. Two such maps are equivalent in the sense of Thurston if they can be homotopically deformed into one another, up to a conjugacy, without moving the postcritical sets. A celebrated theorem of Thurston tells when a Thurston map is equivalent to a rational map. It is an old problem whether such an equivalence can be decided constructively. I will describe joint works with M. Braverman, S. Bonnot, and N. Selinger which, in particular, answer this question.

Kotani-Last problem and Hardy spaces on surfaces of Widom type

Peter Yuditskii, *Linz, Austria*

We present a small theory of non almost periodic ergodic families of Jacobi matrices with pure (however) absolutely continuous spectrum. And the reason why this effect may happen: under our "axioms" we found an analytic condition on the resolvent set that is responsible for (exactly equivalent to) this effect.

Joint work with Alexander Volberg. In the framework of Austrian Science Fund (FWF) project P25591-N25.

YOUNG RESEARCHERS

On the motion of the gas in a rotating cylindrical channel

Allaberdi Ashirov, *Ashgabat, Turkmenistan*
Zera Sitshayeva, *Simferopol, Ukraine*

Let homogeneous ideal compressible fluid of density ρ_0 moves at speed $\overrightarrow{v}_0 = v_0\overrightarrow{e}_3$, $v_0 > 0$, in a cylindrical tube evenly rotating about the axis $O'x'_3$ of the cylinder at angular velocity $\overrightarrow{\omega}_0$, $\omega_0 > 0$. We study the small oscillations of the fluid particles in the moving coordinate system $O'x'_1x'_2x'_3$ with pole O', moving at speed $\overrightarrow{v}_0$, axis orts $\overrightarrow{e}_i$, $i = \overline{1,3}$, and rotating at speed $\overrightarrow{\omega}_0$.

For relative velocity $\overrightarrow{u}$ of fluid particles, dynamic pressure p and dynamic density ρ we have initial boundary value problem

$$\frac{\partial\vec{u}}{\partial t} - 2\omega_0(\vec{u}\times\vec{e}_3) + \frac{1}{\rho_0}\nabla p = \vec{0}, \quad \frac{\partial p}{\partial t} + \beta^2 \operatorname{div}\vec{u} = 0 \quad (\text{в} \ \ \Omega), \tag{1}$$

$$u_n := \vec{u}\cdot\vec{n} = 0 \quad (\text{на} \ \ S), \quad 0 < \beta^2 := \rho_0 a^2, \quad p = a^2\rho, \tag{2}$$

$$\vec{u}(0,x) = \vec{u}^0(x), \quad p(0,x) = p^0(x) \quad (\text{в} \ \ \Omega). \tag{3}$$

Here Ω and S are the volume and the surface of the pipe. The considerations are conducted at the site $\widetilde{\Omega} := \Gamma\times[-h,h]$ that responses to the step $l = 2h = 2\pi v_0/\omega_0$ of the fluid motion.

Under the conditions of sufficient smoothness of S, existence of classical solution of (1)-(3) and periodic extendability of the functions $\overrightarrow{u}$, p along $O'x'_3$ with period l, we get the law of the total energy conservation for the region $\widetilde{\Omega}$:

$$\frac{1}{2}\rho_0\int_{\widetilde{\Omega}}|\vec{u}|^2 d\widetilde{\Omega} + \frac{1}{2}\beta^{-2}\int_{\widetilde{\Omega}}|p|^2 d\widetilde{\Omega} = \text{const}. \tag{4}$$

From (1)-(3) we obtain the Cauchy problem in a Hilbert space H:

$$dz/dt - 2\omega_0\widetilde{\mathcal{B}}z + a\widetilde{\mathcal{A}}_0 z = 0, \quad z\in\mathcal{H}, \quad \forall t\in[0,T], \tag{5}$$

which is reduced to the abstract Cauchy problem of the first order. The statements on the properties of operator coefficients and strong solvability of abstract problems as well as the original problem at the site of periodicity are proved. The properties of associated abstract spectral problem are investigated. To solve the original problem the using of the function of the state is proposed [1].

[1] N. D. Kopachevsky, S. G. Krein, Ngo Zuy Kan, *Operator Methods in Linear Hydrodynamics: Evolution and Spectral Problems*, Nauka, Moscow, 1989 (in Russian).

Symmetric subdifferentials and their applications to Fourier series

Inna Baran, *Simferopol, Ukraine*

We consider a generalization to the symmetric case of the concept of compact subdifferential (or K-subdifferential), which was introduced recently by I. V. Orlov (see [1], [2]). Such a generalization is produced by replacing of the usual difference quotient by the symmetric one of the corresponding order in the basic construction.

The main tools of the theory of symmetric K- subdifferentials, including symmetric analogs of mean value theorem (for the first order subdifferentials) and Schwarz theorem (for the second order subdifferentials) are researched.

This enabled us to generalize the classical Riemann method for summation of Fourier series by means of transition from the usual second symmetric derivative to the corresponding subdifferential in the framework of the method. The application of K-Riemann method to the problem of the uniqueness of the Fourier series is considered. Some perspectives of applications of K-Riemann method to harmonic analysis are discussed.

[1] I. V. Orlov, F. S. Stonyakin, *Compact subdifferentials: the formula of finite increments and related topics.* J. Math. Sc., 170(2010), Issue 2, 251–269.

[2] I. V. Orlov, Z. I. Halilova, *Compact subdifferential in Banach spaces and their application to the variational functionals.* Contemporary Mathematics. Fundamental directions, to appear (In Russian).

[3] I. V. Baran, *Symmetric compact subdifferentials of the first order.* Scientific Notes of Taurida National V. I. Vernadsky University, to appear (In Russian).

Sufficient condition for compact extremum of variational functional in Sobolev spaces $W^{1,p}(D)$ in terms of Hessian of integrand

Ekaterina Bozhonok, *Simferopol, Ukraine*

In the work a sufficient condition in terms of the Hessian of integrand for compact extrema (K-extrema) [1] of variational functional

$$\Phi(y) = \int_D f(x, y(x), \nabla y(x))dx$$

in Sobolev spaces $W^{1,p}(D)$, $p \in \mathbb{N}$, $p \geq 2$, where D is a compact domain in $\mathbb{R}^n$ with a Lipschitz boundary ∂D, is obtained. Here the integrand f belongs to, so called Weierstrass class $W^2K_p(z)$ [2].

The example of variational functional having non–local K–extremum at zero is considered.

At last, a connection between the compact characteristics and the classical analytical characteristics of variational functional acting in Sobolev spaces, which are connected with compact embedding is considered. Namely, the sufficient condition of K–extremum for functional acting in $W^{1,p}$ is the sufficient condition of local extremum in all spaces $W^{k+1,q}$, which are compact embedded in $W^{1,p}$.

[1] I. V. Orlov, E. V. Bozhonok, *Additional chapters of modern natural science. Calculus of variations in Sobolev space H^1: tutorial*, DIAYaPI, Simferopol, 2010 (in Russian).

[2] E. V. Bozhonok, E. M. Kuzmenko, *Generalized Euler–Ostrogradsky equation for K–extremal in $W^{1,p}(D)$*, Uchenye Zapiski Tavricheskogo Natsyonal'nogo Universiteta, 25(64), 2012, no. 2, 15–27 (in Russian).

On eigen oscillations of a thin-layered system of an ideal rotating fluid

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Consider the movements of a hydrosystem that consists of m thin immiscible homogeneous layers of various density. This system fills out the cylindrical domain Ω and rotates uniformly around vertical axis in undisturbed state. Together with problem of hydrodinamics we investigate the model problem where the coefficients of operators are unitary. Model problem is reduced to the following spectral problem:

$$\begin{pmatrix} 0 & 0 & \dots & 0 & \nabla & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \nabla & \nabla & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \nabla & \nabla & \dots & \nabla \\ \mathrm{div} & \mathrm{div} & \dots & \mathrm{div} & 0 & 0 & \dots & 0 \\ 0 & \mathrm{div} & \dots & \mathrm{div} & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \mathrm{div} & 0 & 0 & \dots & 0 \end{pmatrix} \begin{pmatrix} \vec{u_1} \\ \vec{u_2} \\ \dots \\ \vec{u_m} \\ p_1 \\ p_2 \\ \dots \\ p_m \end{pmatrix} = \lambda \begin{pmatrix} \vec{u_1} \\ \vec{u_2} \\ \dots \\ \vec{u_m} \\ p_1 \\ p_2 \\ \dots \\ p_m \end{pmatrix}$$

After certain substitutions and expansion of operator, the following problem arises:

$$\mathcal{A}\varphi = \mu\mathcal{J}\varphi,$$

where operator matrix $\mathcal{A} >> 0 = \mathcal{A}^*$ is unbounded, $\mathcal{A}^{-1} \in \mathfrak{S}_\infty$; and $\mathcal{J} = \mathcal{J}^*$ is bounded positively defined matrix. This problem has discrete positive spectrum with a cluster point in $\pm\infty$, while zero is an infinite-fold eigenvalue and the corresponding system of eigenelements forms an orthonormal basis in Hilbert space.

Analogously we consider the hydrodinamical problem with corresponding physical coefficients of operators. The existence and uniqueness theorem, as well as a spectrum theorem, are stated for the original problem.

[1] Yu. B. Ivanov, N. D. Kopachevsky, *On solvability of the initial boundary value problem on small movements of the rotating layer of an ideal fluid,* Taurida Journal of Computer Science Theory and Mathematics, Simferopol, Taurida National V.I.Vernadsky University (2003), no. 1, 61-77 (in Russian).

[2] N. D. Kopachevsky, *Operator methods of mathematical physics: Special lection course*, Simferopol, 2008, 140 (in Russian).

On the modeling of static equilibrium of the system "ideal fluid–barotropic gas"

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The problem of a static equilibrium of a hydrosystem "homogeneous ideal incompressible fluid–barotropic compressible gas" in a closed container Ω with the solid walls S and a characteristic size l in a gravitational field of the intensity $\overrightarrow{g} = -g\overrightarrow{k}$ is considered. We assume that the gas is above the fluid of a density ρ_1, geometric parameters of the container, the gas density $\rho_{2,0}$ at the equilibrium surface Γ at the initial point $z = 0$ of the coordinate system $Oxyz$, the volume V of fluid, a speed a of a sound in the gas are given, $\varepsilon = -gl/a^2$.

Our purpose is to find the equilibrium surface Γ, a depence of an angle δ of a moistening on the given parameters and the critical values of fluid Bond number B, at which the static stability of the equilibrium surface Γ is disturbed.

After transition to dimensionless variables the finding of the equilibrium profile $L(s)$ of the surface Γ is reduced to the solving of the Cauchy problem [2]:

$$z'' = r'k,\ k := C + Bz + bf_\varepsilon(z) - z'/r,\ z(0) = z_0,\ z'(0) = 0, \qquad (1)$$

$$r'' = -z'k, \quad r(0) = 0,\ r'(0) = 1, \quad f_\varepsilon(z) := \{\exp(-\varepsilon z) - 1 + \varepsilon z\}/\varepsilon. \quad (2)$$

We note that the solution $\{z(s), r(s)\}$ of problem (1)-(2) depends on the given parameters B, b, function $f_\varepsilon(z)$, but the parameters $z_0 = L(0)$, C (initial value of the curvature $L(s)$) and the final value s_k of the integration interval are not specified. It is known that $L(s)$, $s \in [0, s_k]$, should fit in Ω, have no self-intersections and conserve the volume V of fluid [1].

We show one-to-one correspondence between C and z_0. Taking into account of this facts and the differential analog of the conserving condition of fluid volume V and the conditions that listed above, an explicit iterative scheme for finding of L, that based on Runge-Kutta method, was built.

Using the spectral stability criterion based on the variational principle of the potential energy, the region of static stability of the system "gas-fluid" is found.

[1] A. D. Myshkis, V. G. Babskii, M. U. Zhukov, N. D. Kopachevsky, L. A. Slobozhanin, A. D. Tyuptsov, *The methods of the solving of the problems on fluid mechanics under the conditions of weightlessness*, Naukova Dumka, Kiev, 1992 (in Russian).

[2] E. L. Gaziev, *The problem on the static state of hydrosystem "fluid - barotropic gas" under conditions closed to weightlessness*, Proceedings of Applied Mathematics and Mechanics of NASU, 20 (2010), 39-47 (in Russian).

General form of the Maxwellian distribution for the model of rough spheres

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We consider the Boltzmann equation for the model of rough spheres [1] (or Bryan-Pidduck equation):

$$D(f) = Q(f,f);$$
$$D(f) \equiv \frac{\partial f}{\partial t} + V \cdot \frac{\partial f}{\partial x};$$
$$Q(f,f) \equiv \frac{d^2}{2} \int_{R^3} dV_1 \int_{R^3} d\omega_1 \int_{\Sigma} d\alpha B(V - V_1, \alpha)$$
$$\times \left[f(t, V_1^*, x, \omega_1^*) f(t, V^*, x, \omega^*) \; - f(t, V, x, \omega) f(t, V_1, x, \omega_1) \right],$$

where $B(V - V_1, \alpha) = |(V - V_1, \alpha)| - (V - V_1, \alpha)$ is the kernel of the collision integral.

The only exact solution of this equation, which is known in explicit form to date, is an expression similar to those obtained by Maxwell in 1859, in the case of hard sphere model, i.e. satisfying the system of equations $D = Q = 0$.

In [1] claimed that its natural logarithm has an representation:

$$\ln f = \alpha^{(1)} + \alpha^{(2)} V - \alpha^{(3)} \left(\frac{1}{2} V^2 + \frac{1}{2} I \omega^2 \right) + \alpha^{(4)} (I\omega + [x \times V]).$$

However, with direct substitution of this expression to the noted system it was found that in general this expression does not satisfy it that was verified in [2]. Found an inaccuracy is easily eliminated by removing of the term $\alpha^{(4)} I\omega$ from $\ln f$.

Further, we note that all the coefficients $\alpha^{(i)}, i = 1..4$ may depend on the time t and spatial coordinates x. This relationship was firstly defined in [2] and has a specific form.

The physical meaning of the found solution and the classification of its special cases is also investigated.

[1] S. Chapman and T. G. Cowling, *The mathematical theory of non-uniform gases* (Cambridge Univ. Press, Cambridge 1952 / publishing house foreign literature, Moscow, 1960).

[2] V. D. Gordevskyy, A. A. Gukalov, *Maxwell distributions for the model of rough spheres*, Ukr. Mathem. Journ., 63 (2011), no. 5, 629-639 (in Russian).

Continual distributions with screw modes

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Olena Sazonova, *Kharkiv, Ukraine*

The evolution of the rarefied gas of hard spheres is described by the kinetic integro-differential Boltzmann equation [1–2]:

$$D(f) = Q(f, f). \tag{1}$$

We will consider the continual distribution [3]:

$$f = \int\limits_{\mathbb{R}^3} \varphi(t, x, u) M(v, u, x) du, \tag{2}$$

which contains the local Maxwellian of special form describing the screw-shaped stationary equilibrium states of a gas (in short-screws or spirals) [4]. They have the form:

$$M(v, u, x) = \rho_0 e^{\beta\omega^2 r^2} \left(\frac{\beta}{\pi}\right)^{\frac{3}{2}} e^{-\beta(v-u-[\omega\times x])^2}. \tag{3}$$

The purpose is to find such a form of the function $\varphi(t, x, u)$ and such a behavior of all hydrodynamical parameters so that the uniform-integral remainder [3–4]

$$\Delta = \sup_{(t,x)\in\mathbb{R}^4} \int\limits_{\mathbb{R}^3} |D(f) - Q(f, f)| dv, \tag{4}$$

tends to zero.

Also some sufficient conditions to minimization of remainder Δ are founded. The obtained results are new and may be used with the study of evolution of screw and whirlwind streams.

[1] C. Cercignani, *The Boltzman Equation and its Applications*, Springer, New York, 1988.

[2] M. N. Kogan, *The dinamics of a Rarefied Gas*, Nauka, Moscow, 1967.

[3] V. D. Gordevskyy, E. S. Sazonova, *Continuum analogue of bimodal distributions*, Theor. Mat. Fiz., 171 (2012), no. 3, 483-492. (in Russian); English transl.: Theor. Math. Phys., 171 (2012), no. 3, 839-847.

[4] V. D. Gordevskyy, *Biflow Distributions with Screw Modes*, Theor. Mat. Fiz., 126 (2001), no. 2, 283–300 (in Russian); English transl.: Theor. Math. Phys., 126 (2001), no. 2, 234–249.

The second order K-subdifferentials and their application to variational extreme problems

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It is well known that there are a lot of types of subdifferential in modern convex and non-smooth analysis. Some times ago the so called compact subdifferential (or K-subdifferential) was introduced and researched in detail by I. Orlov and F. Stonyakin for the case of scalar argument (see, e. g. [3]).

Recently, in our joint with I. Orlov works the concept of K-subdifferential was transferred to the case of vector argument (see, e. g. [1, 2]). In this connection, the closed theory of the first order K-subdifferentials leads to the more general classes of the main objects, namely, to the Banach cones and multivalued sublinear operators.

The construction under consideration enabled us to obtain some applications to the extreme variational problems with non-smooth, in general, integrand. In particular, so called Euler - Lagrange inclusion is obtained.

The constructed first order theory enabled us also to introduce by induction a concept of the second order K-subdifferential. Application of the second order K-theory to variational problems leads to the multivalued analogs of the Legendre condition and Jacobi condition.

[1] Z. I. Khalilova, *Application of compact subdifferential in Banach spaces to variational functionals*, Academic Records of Tavricheskiy National University. Mathematics and Informatics Series, 25(64) (2012), no. 2., 140–160.

[2] I. V. Orlov, Z. I. Khalilova, *Compact subdifferential in Banach spaces and their application to the variational functional*, Modern mathematics. Fundamental directions. Journal of Mathematical Sciences (2013), in print.

[3] I. V. Orlov, F. S. Stonyakin, *Compact variation, compact subdifferentiability and indefinite Bochner integral*, Methods of Functional Analysis and Topology, 15 (2009), no. 1, 74 – 90.

On an implicit linear non-homogeneous differential equation in a vector space

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Let F be an arbitrary field of characteristic zero and V be a vector space over F. By $V[[x,\frac{1}{x}]]$ we denote the vector space of all formal Laurent series with coefficients from V. For a linear operator $T: V \to V$ and $f(x) = \sum\limits_{n=-\infty}^{+\infty} v_n x^n \in V[[x,\frac{1}{x}]]$ we set $(Tf)(x) = \sum\limits_{n=-\infty}^{+\infty} (Tv_n)x^n$.

Let now E be a vector subspace of $V[[x,\frac{1}{x}]]$, which is invariant with respect to the operator $\frac{d}{dx}$, and $f \in E$. Consider the following differential equation in the space E:

$$Ty' + f(x) = y \tag{1}$$

Theorem 1 *Let E be the vector space of formal Laurent series of the form $\sum\limits_{n=-k}^{\infty} \frac{c_n}{x^n}, k = 0, 1, 2, \ldots$. Then the series $y(x) = \sum\limits_{m=0}^{\infty} T^m f^{(m)}(x)$ is well defined and is a unique solution of Equation (1) in the space E.*

Theorem 2 *Let $\dim V < \infty$ and E be the vector space of all formal power series with coefficients from V, i.e. $E = V[[x]]$. Equation (1) has a unique solution in the space $V[[x]]$ for every $f \in V[[x]]$ if and only if the operator T is nilpotent.*

Complete Volterra integro-differential second-order equations

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Ekaterina Syomkina, *Simferopol, Ukraine*

In Hilbert space $\mathcal{H}$, we consider a Cauchy problem for Volterra integro-differential second-order equation of the form

$$\begin{aligned} &A\frac{d^2u}{dt^2} + (F+iG)\frac{du}{dt} + Bu + \sum_{k=1}^{m} \int_0^t G_k(t,s)C_k u(s)ds = f(t), \\ &u(0) = u^0, \qquad u'(0) = u^1. \end{aligned} \tag{1}$$

Such equations describe an evolution of dynamical systems with infinite number of freedom degrees with taking into account of relaxation effects. An unknown function $u = u(t)$ with values in $\mathcal{H}$ describes a field of system displacements relative to the equilibrium state. Physical meaning of operator coefficients in (1) are the following. A is a kinetic energy operator and therefore $A = A^* > 0$. Next, B is a potential energy operator; if the equilibrium state of the system is statically stable, then $B = B^* \geqslant 0$. Operator $F = F^* \geqslant 0$ takes into account energy dissipation while operator $G = G^*$ describes Coriolis (gyroscopic) forces action. Finally, integral terms take into account relaxation effects.

In this report, A is supposed to be a bounded operator ($A \in \mathcal{L}(\mathcal{H})$) and coefficients F, G, B, C_k are assumed to be unbounded noncommuting operators with domains of definition that are dense in $\mathcal{H}$. It is considered that these operators are compared by their domains of definition. Namely we consider such classes of equations which have a unique so-called main operator; it has the most restricted domain of definition by comparison with another operators.

Investigation of problem (1) is based on methods stated in [1] for the case $A = I$, where I is an identity operator. We mention also the monograph [2] where Cauchy problems are investigated for integro-differential and functional equations as well as correspondent spectral problems for the case when one of coefficients is a main operator while another are powers of this one.

[1] N. D. Kopachevsky, *Volterra's integro-differential equations in the Hilbert space: The specific lecture courses*, Simferopol:FLP "Bondarenko O.A.", 2012.

[2] V. V. Vlasov, D. A. Medvedev, N. A. Rautian, *The functional differential equations in the Sobolev spaces and theirs spectral analysis*, Actual problems of mathematics and mechanics. Mathematics, vol. VIII., issue 1 , 2011.

Lie group analysis of Moffatt's problem

Sergii Kovalenko, *Kyiv, Ukraine*

In this abstract we consider from group-theoretic point of view a model describing the "skin effect" in a thin surface layer of liquid metals immersed in a high-frequency magnetic field [1]:

$$\psi_y \psi_{xy} - \psi_x \psi_{yy} = \nu \psi_{yyy}, \ (x, y) \in \Omega, \tag{1}$$

$$\psi = 0, \ \psi_y = Ax^m, \quad \text{on} \quad y = 0, \tag{2}$$

$$\psi_y \to 0 \quad \text{as} \quad y \to +\infty, \tag{3}$$

where ψ is a stream function to be determined; $A, \nu > 0, m \neq -1$ are arbitrary real constants obeying the condition: $A > 0$ when $m + 1 > 0$ and $A < 0$ when $m + 1 < 0$; $\Omega = \{(x, y) : 0 < x, y < +\infty\}$.

Recently, N. H. Ibragimov [2] showed that among Lie's symmetry operators of the governing equation (1) there is only one operator

$$X = 2x\partial_x + (1 - m)y\partial_y + (m + 1)\psi\partial_\psi$$

being simultaneously the invariance operator of the boundary conditions (2) and (3) (an additional condition $m \neq 0$ is assumed).

We performed the complete group-theoretic analysis of the boundary value problem (1)–(3), namely, we found all Lie's symmetry operators of equation (1), which are compatible with the boundary value problem under study. In particular, it was shown that the case $m = 1$ gives rise to the invariant solutions of (1)–(3).

In this case, the boundary value problem (1)–(3) is compatible with the operator

$$X = \left(\alpha x + (\nu A)^{-1/2}\right)\partial_x + A^{-1}x\partial_y + (\alpha\psi + 1)\partial_\psi,\ \alpha \in \mathbb{R},$$

which leads to the following exact explicit solution of problem (1)–(3):

$$\psi = (\nu A)^{1/2}x\left(1 - e^{-(\nu^{-1}A)^{1/2}y}\right).$$

[1] H. K. Moffatt, *High-frequency excitation of liquid metal systems*, in: H. K. Moffatt, M. R. E. Proctor (Eds.), IUTAM Symposium: Metallurgical Application of Magnetohydrodynamics, Cambridge Univ. Press, Cambridge, 1984.

[2] N. H. Ibragimov, *Lie group analysis of Moffatt's model in metallurgical industry*, J. Nonlinear Math. Phys., 18 (2011), Suppl. 1, 143–162.

The classes of examples non–local compact extrema of variational functional in Sobolev spaces $W^{1,p}(D)$, $p \in \mathbb{N}$ on multi-dimensional domain

Ekaterina Kuzmenko, *Simferopol, Ukraine*

In the work a brief review of the theory of compact extrema of variational functional

$$\Phi(y) = \int_D f(x, y, \nabla y)dx$$

in Sobolev spaces $W^{1,p}(D)$, $p \in \mathbb{N}$, where D — compact domain in $\mathbb{R}^n$ with a Lipschitz boundary is represented.

The main results from the general theory of compact extremum and from the theory compact-analytical characteristics of variational functional are given. Both necessary conditions and sufficient of compact extremum of variational functional in Sobolev spaces $W^{1,p}(D)$, $p \in \mathbb{N}$ are considered.

By developed scheme, the extensive classes of variational functionals, having non–local K–minimum at zero are obtained.

[1] I. V. Orlov, E. V. Bozhonok, *Additional chapters of modern natural science. Calculus of variations in Sobolev space H^1: tutorial*. Simferopol: DIAYaPI, 2010 (in Russian).

[2] E. M. Kuzmenko, *Compactly - analytic properties of variational functionals in Sobolev spaces $W^{1,p}$ of functions of several variables*, Simferopol: Dynamic sistem, 2(30)(2012), no. 1-2, 89–120 (in Russian).

[3] E. V. Bozhonok, E. M. Kuzmenko, *Generalized Euler–Ostrogradsky equation for K–extremal in $W^{1,p}(D)$*, (in Russian) Uchenye Zapiski Tavricheskogo Natsyonal'nogo Universiteta, 25(64) (2012), no. 2, 15–27.

Bimodal approximate solutions of the Boltzmann equation in the weighted spaces

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Non-homogeneous, non-stationary linear combination of two Maxwellians with different hydrodynamic parameters is considered, i.e., it's the distribution $f = \varphi_1 M_1 + \varphi_2 M_2$, where the coefficient functions $\varphi_i = \varphi_i(t, x)$, $i = 1, 2$ are nonnegative and smooth, and M_i have the form: $M_i = \overline{\rho}_i \cdot \left(\frac{\beta_i}{\pi}\right)^{3/2} \cdot e^{\beta_i(2\overline{u}_i x - v^2 + 2v(\overline{v}_i - \overline{u}_i t))}$, where $\overline{\rho}_i = const$, $\beta_i = \frac{1}{2T_i}$ are inverse temperatures (T is the absolute temperature).

The norm of the difference between the parts of the Boltzmann equation $D(f) = Q(f, f)$ in the weighted spaces is considered for construction such a bimodal distribution, describing the interaction between two Maxwell flows of hard spheres of "accelerating - packing" type.

We have found such functions φ_i, $i = 1, 2$ and such the behavior of all available parameters for which "mixed weighted error"

$$\widetilde{\Delta} = \sup_{(t,x)\in R^4} \frac{1}{1+|t|} \int_{R^3} |D(f) - Q(f, f)|\, dv$$

approaches to zero.

Theorem 1 *Let*

$$\varphi_i(t, x) = \psi_i(t, x) \exp\{-\beta_i(\overline{v}_i - \overline{u}_i t)^2\}, \quad i = 1, 2, \tag{1}$$

and $t\psi_1\psi_2 \exp\{2\beta_1\overline{u}_1 x + 2\beta_2\overline{u}_2 x\}$; $\frac{\partial\psi_i}{\partial t} \exp\{2\beta_i\overline{u}_i x\}$; $t\psi_i \exp\{2\beta_i\overline{u}_i x\}$; $\left|\frac{\partial\psi_i}{\partial x}\right| \exp\{2\beta_i\overline{u}_i x\}$; $t\left(\overline{u}_i, \frac{\partial\psi_i}{\partial x}\right) \exp\{2\beta_i\overline{u}_i x\}$, $i = 1, 2$, *are bounded with the weight* $\frac{1}{1+|t|}$. *Let besides prediction*

$$\overline{u}_i = \overline{u}_{oi}\beta_i^{-n_i}, \quad i = 1, 2, \tag{2}$$

where $\overline{u}_{oi} \in R^3$ *are arbitrary fixed vectors, takes place for*

$$n_i \geq 1. \tag{3}$$

Then holds true the estimation $\widetilde{\Delta} \leq \widetilde{\Delta}'$, *and the finite limit of the value* $\widetilde{\Delta}'$

exists. Besides for $n_i > 1$

$$\lim_{\beta_1,\beta_2\to+\infty} \widetilde{\Delta}' = \sum_{i,j=1,i\neq j}^{2} \overline{\rho}_i \sup_{(t,x)\in R^4} \frac{1}{1+|t|} \left| \frac{\partial\psi_i}{\partial t} + \overline{v}_i \frac{\partial\psi_i}{\partial x} + \psi_1\psi_2\pi d^2\overline{\rho}_j|\overline{v}_1-\overline{v}_2| \right| +$$
$$+2\pi d^2\overline{\rho}_1\overline{\rho}_2|\overline{v}_1-\overline{v}_2| \sup_{(t,x)\in R^4} \frac{1}{1+|t|}(\psi_1\psi_2),$$

and for $n_i = 1$

$$\lim_{\beta_1,\beta_2\to+\infty} \widetilde{\Delta}' = \sum_{i,j=1,i\neq j}^{2} \overline{\rho}_i \sup_{(t,x)\in R^4} \frac{1}{1+|t|} \left| \mu_i(x)\left(\frac{\partial\psi_i}{\partial t} + \overline{v}_i \frac{\partial\psi_i}{\partial x}\right) + \right.$$
$$\left. +\psi_1\psi_2\mu_1(x)\mu_2(x)\pi d^2\overline{\rho}_j|\overline{v}_1-\overline{v}_2| \right| + 2\pi d^2\overline{\rho}_1\overline{\rho}_2|\overline{v}_1-\overline{v}_2|\times$$
$$\times \sup_{(t,x)\in R^4} \frac{1}{1+|t|}[\mu_1(x)\mu_2(x)\psi_1(t,x)\psi_2(t,x)]+$$
$$+2\sum_{i=1}^{2} \overline{\rho}_i|(\overline{u}_{oi},\overline{v}_i)| \sup_{(t,x)\in R^4} \frac{1}{1+|t|}\{\mu_i(x)\psi_i(t,x)\},$$

where $\mu_i(x) = \exp\{2\overline{u}_{oi}x\}$, $i = 1,2$.

Corollary 1 *Let the parameters* n_i, $i = 1,2$ *from the formula (2) are satisfied the inequality (3). Let besides the additional condition:* $\overline{u}_i \perp \overline{v}_i$, $i = 1,2$, *is valid and the prediction (1) remained. Let the functions* ψ_i *have the form:* $\psi_i = C_i(x - \overline{v}_i t)$ *or* $\psi_i = C_i([x \times \overline{v}_i])$, $i = 1,2$, *where* $C_i \geq 0$ *are smooth functions of compact support. Then:*

1. *if* C_1, C_2, $\overline{v}_1$, $\overline{v}_2$ *satisfy the following conditions:* $suppC_1 \cap suppC_2 = \oslash$ *or* $\overline{v}_1 = \overline{v}_2$, *then* $\lim_{\beta_1,\beta_2\to+\infty} \widetilde{\Delta} = 0$;

2. *for arbitrary* $C_1, C_2, \overline{v}_1, \overline{v}_2 :\quad \lim_{d\to 0}\lim_{\beta_i\to+\infty,i=1,2} \widetilde{\Delta} = 0.$

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Characterization of the Haar distribution on a compact Abelian group

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Let X be a compact Abelian group, ξ_i, $i = 1, 2, \ldots, n$, $n \geq 2$, be independent random variables taking values in X, α_{ij}, $i, j = 1, 2, \ldots, n$, be continuous endomorphisms of the group X. Consider the linear forms $L_j = \sum_{i=1}^{n} \alpha_{ij}\xi_i$, $j = 1, 2, \ldots, n$.

In the paper [1] it was shown that if α_{ij} are integers, X is a compact connected Abelian group and there are some restrictions on ξ_i, then the independence of the linear forms L_j implies that all ξ_i have the Haar distribution on X.

In the paper [2], in particular, it was shown that in the case when $n = 2$, X is an arbitrary compact Abelian group and there are some restrictions on the random variables ξ_i (different from used in [1]) the answer is the same: the Haar distribution on X is characterized by the independence of the linear forms L_j.

We continue the researches on the characterization of the Haar distribution in the case of an arbitrary compact Abelian group X and arbitrary n. We prove the result that generalizes the results above. Moreover we show that the conditions of this theorem are sharp in some sense.

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Riemann-Hilbert problem and mKdV equation with step like initial data: Parametrixes for elliptic wave region

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We consider the modified Korteweg – de Vries equation on the line. The initial function is step-like, that is it tends to some constants when $x \to \pm\infty$. Our goal is to study the asymptotic behavior of the solution of the initial value problem as $t \to \infty$. For elliptic wave region the parametrixes in Airy function and its derivative are constructed. Main methods are method of Riemann – Hilbert problem, the g-function method and the nonlinear steepest – descent method.

Independent linear statistics on the cylinders

Margaryta Myronyuk, *Kharkiv, Ukraine*

V.P. Skitovich and G. Darmois proved the following characterization theorem for a Gaussian distribution on the real line $\mathbb{R}$ ([1, Ch. 3]): Let ξ_j, where $j = 1, 2, \dots, n$, and $n \geq 2$, be independent random variables. Let α_j, β_j be nonzero constants. If the linear forms $L_1 = \alpha_1\xi_1 + \dots + \alpha_n\xi_n$ and $L_2 = \beta_1\xi_1 + \dots + \beta_n\xi_n$ are independent, then all random variables ξ_j are Gaussian.

A number of works has been devoted to generalizing of the Skitovich–Darmois theorem for different classes of groups (see e.g. [2], where one can find references). As have been noted in [3], an analogue of the Skitovich–Darmois theorem fails for the circle group $\mathbb{T}$ even for two linear forms. Nevertheless, as we prove, an analogue of the Skitovich–Darmois theorem holds for the cylinder $X = \mathbb{R} \times \mathbb{T}$ in the case of three linear forms of three independent random variables (coefficients of the forms are topological automorphisms X) under assumption that the characteristic functions of random variables do not vanish.

Theorem 1 *Let $X = \mathbb{R} \times \mathbb{T}$, and α_{ij} be topological automorphisms of X, where $i, j = 1, 2, 3$. Let ξ_j, where $j = 1, 2, 3$, be independent random variables with values in X and distributions μ_j such that their characteristic functions do not vanish. Assume that the linear forms $L_1 = \alpha_{11}\xi_1 + \alpha_{12}\xi_2 + \alpha_{13}\xi_3$, $L_2 = \alpha_{21}\xi_1 + \alpha_{22}\xi_2 + \alpha_{23}\xi_3$, and $L_3 = \alpha_{31}\xi_1 + \alpha_{32}\xi_2 + \alpha_{33}\xi_3$ are independent. Then all μ_j are either degenerate distributions or Gaussian distributions such that their supports are cosets of subgroups topologically isomorphic to $\mathbb{R}$.*

This work was done in conjunction with G.M. Feldman [4].

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Conditions on a submanifold $F^n \subset E^{n+p}$ to lie in E^{2n-1}

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Questions accessories surface F^2 with degenerate ellipse of normal curvature are discussed in [1], [2]. In the work [3] we obtain conditions on $F^2 \subset E^n$ with a non-degenerate ellipse of normal curvature to lie in E^4.

We consider a submanifolds F^n with the n principal directions in space E^{n+p} where $p \geq n-1$.

Lemma 1 *If $F^n \subset E^{n+p}$ is a submanifold with n principal directions, then the indicatrix of the normal curvature is the $(n-1)$-dimensional simplex Δ or degenerates into subsimplex.*

Theorem 1 *Suppose that at each point P of a submanifold $F^n \subset E^{n+p}$ with the n principal directions and with non-degenerate Δ. Let $(n-1)$-dimensional subspace N^{n-1} containing the Δ passes through the point $P \in E^n$. Let every $n-1$ principal vectors of the normal curvature, passes through point P, are not in the $(n-2)$-dimensional subspace N^{n-2}. Then the submanifold F^n lies in the subspace E^{2n-1}.*

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Scale of overcompact sets and its application to theorems about convexity for range of vector measure

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There is a well-known Lyapunov Theorem about convexity for range of non-atomic vector measure $\overrightarrow{\mu} : \Sigma \longrightarrow R^n$, where Σ is a subsets σ-algebra of some space Ω. But this result, in general, is not true for vector measures with values in infinite dimensional Banach spaces [1, 2, 3]. The most known approach to this problem is to select the special class of Banach spaces with the so-called Lyapunov property. In spaces E with this property for each σ-additive non-atomic measure $\overrightarrow{\mu} : \Sigma \longrightarrow E$ the set $\overline{\overrightarrow{\mu}(\Sigma)}$ is convex [2, 3]. For example, the spaces c_0, ℓ_p $(p \in [1;2) \bigcup (2;+\infty))$ possess this property. However, the space ℓ_2 doesn't possess the Lyapunov property. We propose a new approach to Lyapunov problem and introduce the concept of overcompact set in Banach space.

Definition 1 *Let's call absolutely convex set $\overline{C} \subset E$ overcompact in Banach space E, if $p_{\overline{C}}(a) = 0 \iff a = 0$ in E and each arbitrary bounded set in E is precompact in some space $E_{\overline{C}} = (\operatorname{span} \overline{C}, p_{\overline{C}}(\cdot))$. Here $p_{\overline{C}}(\cdot)$ is Minkovskiy* *functional of absolutely convex set* $\overline{C} \subset E$. *Also we suppose that* $E_{\overline{C}}$ *is complemented by norm* $\|\cdot\|_{\overline{C}} = p_{\overline{C}}(\cdot)$. *Let's denote* $\overline{\mathcal{C}}(E)$ *a collection of all overcompact subsets of Banach space E.*

The properties of scale $E_{\overline{C}}$, $\overline{C} \in \overline{\mathcal{C}}(E)$ are investigated in detail. On the base of this theory the following analogue of Lyapunov Convexity Theorem for vector measures with values in separable Hilbert spaces is obtained.

Theorem 1 *Let H be a separable Hilbert space. For each non-atomic vector measure* $\overrightarrow{\mu} : \Sigma \longrightarrow H$ *with weak bounded variation there exists overcompact set* $\overline{C} \in \overline{\mathcal{C}}(H)$ *such that the closure of the set* $\overrightarrow{\mu}(\Sigma)$ *in* $H_{\overline{C}}$ *is convex.*

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The sufficiency of the strengthened Legendre condition for variational problems under a possible restriction on an area measure

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The classical approach to the the solution of extreme variational problems, as is known, requires as sufficient condition of an extremum, to check the strengthened Legendre condition and Jacobi condition for Jacobi equation. The second step in the non-quadratic situation is complicated enough and therefore the possibility to "get around" checking the Jacobi condition has long attracted the attention of mathematicians.

Recently, the works by I. V. Orlov [3], [4] have developed a new method of elimination of Jacobi equation and Jacobi condition in one-dimensional variational problems.

In this report, this method is generalized to the case of a multi-dimensional area [1], [2]. It is shown that the minima problem for the classical Euler-Lagrange variational functional $\Phi(y) = \int\limits_D f(x, y, \nabla y)dx$, $D \subset \mathbb{R}^n$ can be solved under the fulfilment of Euler-Ostrogradski equation, strengthened Legendre condition, the positivity of f_{y^2} (in the non-smooth case), and (in one of two possible cases) additional restrictions on an area measure which weakens with growth of dimension of space. There are considered both a case of a local minimum in $C^1(D)$, and a case of a compact minimum in $W^{1,p}(D)$.

As an application, some square estimates from below for at the speed of tending of variation functional to its minimal value are received.

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On the spectral problem arising in superlens theory

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We consider the following spectral problem

$$-\beta\Delta u_1(x) = \lambda u_1(x), \qquad x \in \Omega_1, \tag{1}$$

$$-\alpha\Delta u_2(x) = \lambda u_2(x), \qquad x \in \Omega_2, \tag{2}$$

$$-\beta\Delta u_3(x) = \lambda u_3(x), \qquad x \in \Omega_3, \tag{3}$$

$$u_1(x) = u_2(x), \quad \beta\frac{\partial u_1}{\partial n}(x) = \alpha\frac{\partial u_2}{\partial n}(x), \quad x \in \Gamma_1, \tag{4}$$

$$u_2(x) = u_3(x), \quad \alpha\frac{\partial u_2}{\partial n}(x) = \beta\frac{\partial u_3}{\partial n}(x), \quad x \in \Gamma_2, \tag{5}$$

$$u_3(x) = 0, \qquad x \in \Gamma_3. \tag{6}$$

Here $u_k(x)$ $(k = 1, 2, 3)$ are unknown functions given in the corresponding domains Ω_k, where Ω_1 is the circle of radius $r_1 > 0$, Ω_2 is the middle ring of radius $r_2 > 0$ and Ω_3 is the outward ring of radius $r_3 > 0$. We suppose that circle Γ_1 is common boundary of Ω_1 and Ω_2, circle Γ_2 is common boundary of Ω_2 and Ω_3, circle Γ_3 tends to infinity. We set outward normal derivatives $\partial/\partial n$ on Γ_k. The number $\lambda \in \mathbb{C}$ is the spectral parameter, the numbers $\alpha, \beta \in \mathbb{C}$ $(0 \le \arg\beta < \arg\alpha \le \pi)$ are given.

Corresponding evolution problem describes the solutions for the electromagnetic fields in a coated cylinder where the core radius is bigger than the shell radius (see [1] and [2]). This physical model has interesting properties connected with anomalous resonance and cloaking effect observed in composite materials with appropriate dielectric constants α and β.

We prove that problem (1)–(6) has the discrete spectrum that consists of eigenvalues $\lambda_n \to \infty$ $(n \to \infty)$ situated in the sector: $\arg\beta \le \arg\lambda \le \arg\alpha$.

For any $\varepsilon > 0$ there exists a number $R > 0$ such that for $|\lambda| > R$ there are no eigenvalues in the sector: $\arg\beta + \varepsilon \le \arg\lambda \le \arg\alpha - \varepsilon$. We find asymptotic characteristic equations and asymptotic formulas for λ_n. As a consequence, eigenvalues λ_n situate along two parabolas with axes of symmetry $\lambda = c\beta$ and $\lambda = d\alpha$ $(c, d \in \mathbb{R}_+)$ when $n \to \infty$.

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